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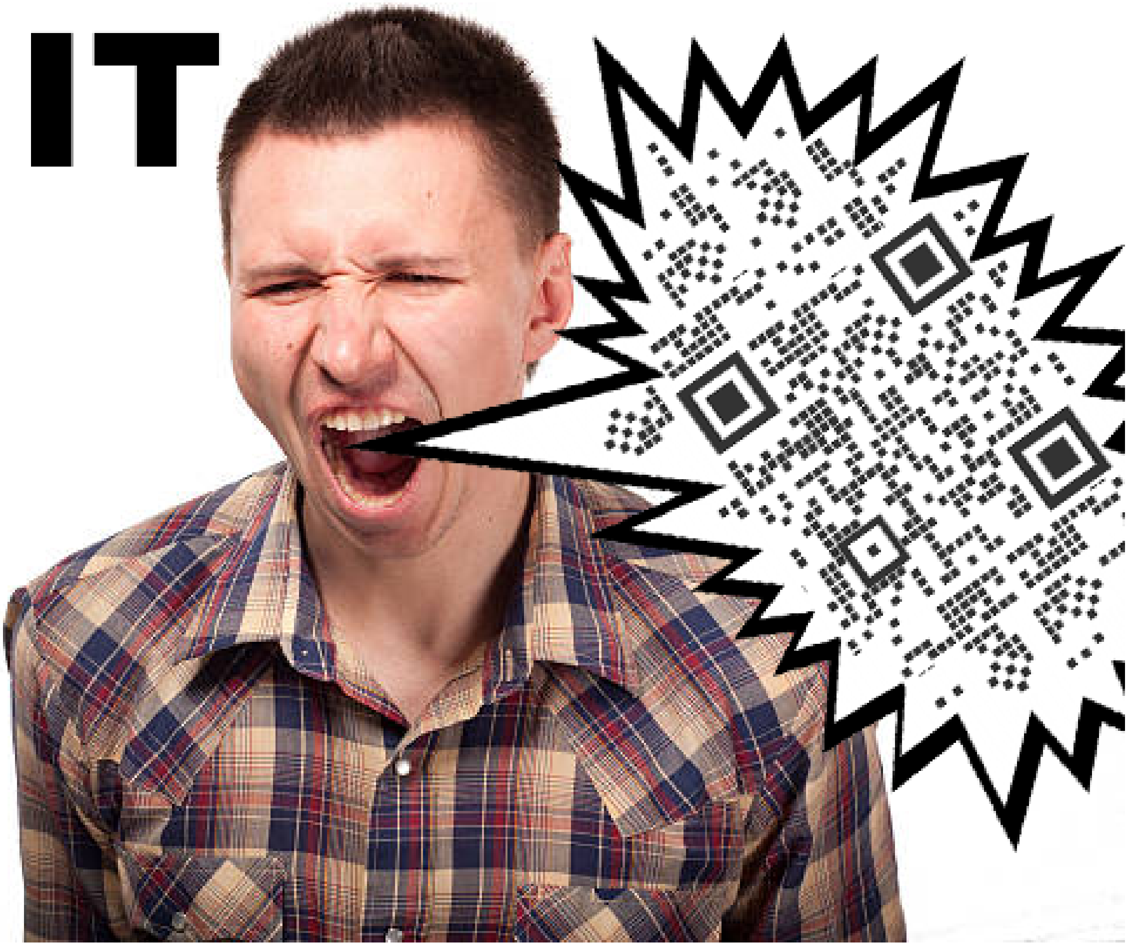

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PROCEEDINGS
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RECENT PROGRESS IN THE THEORIES OF MODULAR AND FORMAL INVARIANTS AND IN MODULAR GEOMETRY

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Presented to the Academy, November 7, 1914

1. *Contrast between algebraic and modular invariants.* By way of introduction we recall the argument, made in certain texts on invariants, to prove that a linear form $l = ax + by$ has no invariant. For, the vanishing of an invariant $I\ (a, b)$ of l would imply a property of those forms l for which $I = 0$, not possessed by the forms l for which $I \neq 0$. But all forms l are equivalent since each can be transformed into x. This argument is erroneous since the identically vanishing form l (with $a = b = 0$) cannot be transformed into x. Nor is the conclusion correct. The function $I\ (a, b)$, defined in the sense of Dirichlet to be unity if $l \equiv 0$ and zero if l is not identically zero, is evidently an invariant of l, since it has the same value for all equivalent forms l.

In the number-theoretic case in which the coefficients of l and of the linear transformation are integers taken modulo p (a prime), the bizarre Dirichlet function $I\ (a, b)$ employed in the algebraic case is no longer necessary, since it may now be replaced by the polynomial invariant

$$I = (a^{p-1} - 1)(b^{p-1} - 1),$$

with the value unity if $a \equiv b \equiv 0 \pmod{p}$ and the value zero if a and b are not both congruent to zero. Hence I is an invariant of l modulo p. It is called a *modular* invariant of l.

2. *Formal invariants and their construction.* Let the coefficients a and b of l be independent variables as in the theory of algebraic invariants. But let the coefficients of the linear transformations be integers taken modulo p as in the theory of modular invariants. Invariants arising in this composite case are called formal invariants and were first intro-

duced by Hurwitz.[1] Although a remarkably simple theory of modular invariants has been given,[2] no headway was made towards a theory of formal invariants before the very recent discovery[3] of a simple effective method for their construction. This method will be illustrated for the linear form l and the modulus 2. The real points (i.e., those with integral coördinates) modulo 2 are (1, 0), (0, 1) and (1, 1). The values of l at these points are a, b, $a + b$. Any real linear transformation induces a permutation of these three values since it merely permutes the three real points. Hence any symmetric function of these three values is a formal invariant of l. The elementary symmetric functions reduce modulo 2 to zero, $i = a^2 + ab + b^2$ and $j = ab\,(a + b)$. We pass to modular invariants by taking a and b to be integers modulo 2. Then $j \equiv 0, i \equiv I + 1$, where I is the invariant in § 1.

In treating similarly the formal invariants of l modulo 5, we would employ the symmetric functions of the fourth powers of our values a, b, $a + b$, and not the values themselves. For, (1, 0), (2, 0), (3, 0), (4, 0) give the same point and yet lead to the values a, $2a$, $3a$, $4a$ of l; we take the fourth power to secure a value uniquely defined by the point. In the case of a quadratic form modulo 5, we need only take the squares of the values.

The method is applicable to invariants of several forms in any number of variables and to semi-invariants, as shown in the paper cited, which gives also a novel method of deriving modular invariants from semi-invariants.

3. *Modular plane curves for modulus* 2. Let $f\,(x, y, z)$ be a homogeneous form of degree n with integral coefficients. A point for which the three first partial derivatives of f are zero modulo 2 shall be called a *derived point*. If n is even, it need not be a singular point of the curve, since it need not lie on the curve; the argument in the algebraic case, based on Euler's theorem

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + z\frac{\partial f}{\partial z} = nf,$$

does not apply modulo 2 when n is even, since the vanishing of the left member does not require that of f. A non-singular derived point shall be called an *apex* of the curve; its linear polar is indeterminate.

For example, any non-degenerate conic (i.e., having no linear factor modulo 2) can be transformed linearly into $x^2 + yz = 0$. The only derived point is $p = (1, 0, 0)$ and is an apex. Any line through p is tangent to the conic; this is evident for $z = 0$ and follows for $y = kz$ since the elimination of y leads to $x^2 + kz^2 = 0$, with a double root modulo 2. For the theory[4] of quadratic loci modulo 2 in space of any number of dimensions, see the *Madison Colloquium Lectures*, p. 65.

The next case, $n = 4$, for which apices occur presents other remarkable peculiarities.[5] Whereas in the algebraic case a quartic curve has 28 bitangents in general, one in the case of modulus 2 has at most 7 bitangents and usually exactly 7. The bitangents intersect at derived points and usually all of the derived points are intersections of bitangents. An interesting example is given by

$$K = x^4 + y^4 + z^4 + x^2y^2 + x^2z^2 + y^2z^2 + xyz\,(x + y + z),$$

an invariant under all real linear transformations. Here real is used in the sense of integral; likewise for the real points $(1, 0, 0), \ldots, (1, 1, 1)$ modulo 2. The bitangents to $K = 0$ are the 7 real lines in the plane and their interesections are the 7 real points, the latter being apices and not singular points.

A quartic curve containing all seven real points and having no linear factor modulo 2 can be transformed into

$$x^3y + x^2y^2 + xz^3 + x^2z^2 + y^3z + yz^3 = 0.$$

It has no singular point and has the 7 apices $(1, z^3, z)$, where $z^7 + z^3 + 1 = 0$. Its 7 bitangents are $x = (b^3 + 1)\,y + bz$, where $b^7 + b + 1 = 0$; they intersect at apices. Each apex is on three bitangents, while three apices are on each bitangent. The configuration of the apices and bitangents, here all imaginary, is entirely similar to that for K, composed of real elements.

The classification of quartic curves is similar to that next illustrated for the simpler case of cubic[6] curves modulo 2.

A cubic curve containing all seven real points is of the form

$$a\,(x^2y + xy^2) + b\,(x^2z + xz^2) + c\,(y^2z + yz^2) = 0.$$

If not zero identically, it can be transformed into $x^2y + xy^2 = 0$. A cubic curve containing just two real points can be transformed into one containing $(1, 0, 0)$ and $(0, 1, 0)$; the transformations leaving the latter fixed or permuting them are available for the specialization of the parameters in the coefficients of the cubic. In this way we find the 21 types of non-equivalent cubics, including degenerate curves, and see that they are completely characterized by the number of real points, real inflexion points, real and imaginary singular points,—geometrical invariants easily expressed by modular invariants.

For the determination of the inflexion points of the cubic $n\,(x_1, x_2, x_3) = 0$ modulo 2, the Hessian of n is not available, being identically zero modulo 2. In its place we may employ the function

$$C = \xi_1\xi_2\xi_3 + \Sigma\, \xi_i\eta_i^2 + k\Sigma x_1\xi_1\eta_1 + k\Sigma x_1\eta_2\eta_3 + k^2\Sigma x_1x_2\eta_3 + k^3x_1x_2x_3,$$

in which $n = v + kx_1x_2x_3$, while the quotients

$$\xi_i = \tfrac{1}{2}\frac{\partial^2 v}{\partial x_i^2},\ \eta_1 = \tfrac{1}{2}\frac{\partial^2 v}{\partial x_2\,\partial x_3},\ \cdots,\ \eta_3 = \tfrac{1}{2}\frac{\partial^2 v}{\partial x_1\,\partial x_2}$$

have integral coefficients. The points of inflexion of $n = 0$ are its intersections with $C = 0$. Although C is not a covariant, it forms with n a covariant pencil, since C is transformed into a linear function of n and C.

[1] Hurwitz, *Arch. Math., Leipzig*, ser. 3, 5, 25, (1903).

[2] Dickson, *Madison Colloquium Lectures*, American Mathematical Society, (1914).

[3] Dickson, *Trans. Amer. Math. Soc.*, 15, 497, (1914).

[4] An advance in the theory of seminvariant leaders of covariants of quadratic forms has been made recently by the writer, *Bull. Amer. Math. Soc.*, January, 1915.

[5] Dickson, *Trans. Amer. Math. Soc.*, April, 1915.

[6] MS. offered Aug. 4, 1914 to *Amer. J. Math.* To appear April, 1915.

THE SYNTHESIS OF TRIAD SYSTEMS Δ_t in t ELEMENTS, IN PARTICULAR FOR t = 31

By Henry S. White

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Presented to the Academy, December 3, 1914

Purely theoretical interest first led to the study of triad systems Δ_t in t elements; systems of threes or triads, that is, in which every possible pair of elements is found in some one triad, but only one. Their relation to other objects of research in algebra and geometry began to appear when Noether (1879) pointed out the peculiar nature of a resolvent equation of the seventh degree which had been found by Betti, Hermite, and Kronecker in discussing the transformation of the seventh order of elliptic functions. This modular equation has roots related in triads like the Δ_t. Hesse had shown earlier, in plane curves of the third order, that the nine inflexion points lie by threes on twelve lines, thus exemplifying a Δ_9. Noether succeeded also in connecting the Δ_7 with the important sets of double tangents to the plane quartic curve which Aronhold had introduced under the name of *Siebenersysteme*. With those important applications in hand, and ten or twelve known Δ_{15}'s to serve as further data, mathematicians took up with renewed interest the question whether there are actual triad systems for every suitable member t of elements, i.e., for t = 13, 15, 19, 21, 25, 27, etc.; or precisely, t = 6k + 1 or 6k + 3.

Although Reiss in 1859 had answered this question in the affirmative, constructing one system for every value of t, his work was overlooked until the same question was settled independently in 1893 by E. H. Moore. Moore's methods, based on more penetrating analysis than Reiss's, established at least two distinct sorts of systems for every t above 13, and led him to forecast definitely that the number of such systems would be found a rapidly increasing function of the number of elements, t. As to the sole doubtful number, $t = 13$, Zulauf, a pupil of Netto, found that there are two different systems, and others soon proved that there are no more than two.

After the lapse of ten years or more, Miss L. D. Cummings has now shown conclusively (1914) the existence of at least 24 distinct triad systems in 15 elements. These 24 include all that had been found before and as many more new systems, with their differences now for the first time rigorously demonstrated. All but one (viz., Heffter's) of these 24 exhibit what I call odd-and-even structure. The odd part are the elements appearing in seven triads that constitute an included triad system Δ_7, which may be termed the head in its Δ_{15}. Heffter's Δ_{15} is at present the only headless system in 15 elements whose description has been published.

Since the appearance of Miss Cummings' dissertation, I have applied a new method for constructing all possible Δ_{15}'s which can be transformed into themselves by any substitution among their elements,—all whose group is above the identity. By this means I find a considerable additional number of systems, all headless. These new Δ_{15}'s I now employ in attacking the question, how many distinct systems Δ_{31} are there in 31 elements? If there were but few, then it would be desirable to compile a complete census of them as further substratum for a general theory. But what I find is that even the restricted class selected for this study are far too numerous for detailed exhibition, their number being greater than 10^{13}. This result is attained through a new theorem, whose generality is significant of further possibilities.

The theorem, specialized for application, is this. *If among the triads of a system* Δ_{31} *there occur two complete systems* Δ_{15} *and* Δ'_{15}, *then there is a* Δ_7 *whose seven triads, and no other triads or elements, are common to* Δ_{15} *and* Δ'_{15}. Conversely, *if a* Δ_{31} *contains a headless* Δ_{15}, *it can contain no other triad system* Δ'_{15}; nor indeed any other larger than a Δ_7, and even such a Δ_7 will have one triad from the Δ_{15}.

Odd-and-even structure in a Δ_{31} consists in this: its 155 triads include 35 that form a Δ_{15} in 15 elements, and of the remaining 16 elements two are found in each of the other 120 triads, along with one element from the 15. Thus every triad has an odd number, 3 or 1, of the *odd set* of 15, and an even number, 0 or 2, of the *even set* of 16. Any one of the odd set is found therefore in triads with 8 pairs from the even set, and these pairs can be arranged in 15 columns, an 8 by 15 array. Every odd element is found also with 7 pairs from the odd set. This leads to the tabulation of 15 columns of 7 pairs each, ranged above the columns of the 8 by 15 array. Every column is marked then by one odd element above it; the upper partial columns exhibit the head, or Δ_{15}.

Head and array form a convenient mode for constructing Δ_{31}'s that are to have odd-and-even structure. If the head, the Δ_{15}, is itself headless, this tabulation is unique for that Δ_{31}. *I study here exclusively these odd-and-even* Δ_{31}*'s whose head is a headless* Δ_{15}. Given any one such Δ_{31}, tabulated, many others can be obtained by shifting the columns of its 8 x 15 array while the head is kept stationary. To apply this method and to count the distinct Δ_{31}'s that will be produced, one must know the groups $G_{d'}$ and $G_{d'}$, belonging to the head and to array respectively.

The number of resulting Δ_{31}*'s is certainly not less than 15! divided by the product, d d', of the orders of the groups belonging to the head and to the array respectively.* These orders are small, whence the resulting Δ_{31}'s are very many.

Incidentally, if d and d' are relative primes, the resulting Δ_{31}'s must be of the peculiar kind having no automorphic substitutions; i.e., their group is reduced to the identity. Such cases occur, e.g., with $d = 2$ and $d' = 3$. Full details are to appear in the *Transactions of the American Mathematical Society* for January, 1915.

THE Φ-SUBGROUP OF A GROUP OF FINITE ORDER

By G. A. Miller

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Presented to the Academy, November 19, 1914

A set of λ operators $s_1, s_2, .., s_\lambda$ of a finite group G is called a set of generators of G provided there is no subgroup in G which includes each of these operators. When these operators satisfy the additional condition that G can be generated by no $\lambda - 1$ of them the set is said to be a *set of independent generators of G.* Those operators of G which can appear in none of its possible sets of independent generators constitute a characteristic subgroup, which was called by G. Frattini *the* ϕ*-subgroup of G.* See *Rend. Acc. Lincei*, ser. 4, 1, 281 (1885).

Among the results of the present paper which are supposed to be new are the following: The number of the different operators in each of the possible sets of independent generators of a group whose order is a power of a prime number is the same,—that is, if the order of a group is a power of a prime number, the number of its independent generators is an invariant of the group. The ϕ-subgroup of every direct product is the direct product of the ϕ-subgroups of the factors of this direct product. In every group whose order is a power of a prime number the ϕ-subgroup includes the commutator subgroup of the group. By constructing the ϕ-subgroup of a group and of its successive ϕ-subgroups we can always arrive at the identity.

The ϕ-subgroup of a Sylow subgroup of the symmetric group of degree n is the commutator subgroup of this Sylow subgroup. In particular, the number of the different operators in a set of independent generators of the Sylow subgroup of order p^m of the symmetric group of degree p^n, p being any prime number, is n. In particular, the subgroup of order p^{p+1} which is contained in the symmetric group of degree p^2 has exactly 2 independent generators irrespective of the prime number represented by p. No ϕ-subgroup can contain a complete set of conjugate subgroups, or a complete set of conjugate operators, involving more than one subgroup or more than one operator, when this complete set of conjugates is also a complete set of conjugates under the entire group. An important special case of this theorem is that every ϕ-subgroup whose order is not a power of a prime number is the direct product of its Sylow subgroups. This special case was noted by G. Frattini in the article to which reference has been given.

With respect to abelian groups a set of λ independent generators is commonly defined so that the group generated by every $\lambda - 1$ of these generators has only the identity in common with the group generated by the remaining operator. For an abelian group whose order is a power of a prime number the number of the different operators in a possible set of independent generators is the same under both of the given definitions of a set of independent generators. The fact that the number of independent generators of a group is not always an invariant of the group becomes evident if we observe that when we generate the symmetric group of degree n by transpositions there will always be $n - 1$ independent generators. On the other hand, this symmetric group can also be generated by a cyclic substitution of degree $n - 1$ and a transposition involving the remaining letter. Complete proofs of these results are contained in a paper having the same heading, which has been offered for publication in the *Transactions of the American Mathematical Society.*

ON THE RADIAL VELOCITIES OF NEBULAE

By W. W. Campbell

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Presented to the Academy, October 31, 1914

The discovery, early in 1910, that the velocities of the stars are functions of their spectral types—the stellar velocities increase rapidly, on the average, as we pass from the blue stars through the yellow stars and on to the red stars—and the prevailing opinion that the stars have been evolved from nebulae, gave special interest to the search for possible relationships existing between nebular and stellar velocities. The nebular velocities then available were the velocities of approach and recession (radial velocities) of 13 planetary nebulae and of the Orion nebula, as observed by Keeler in 1890–1891 with a visual spectroscope attached to the 36-inch refracting telescope. The average radial velocity of the 14 nebulae, as observed by Keeler, was 25 km. per second; or omitting the Orion nebula, whose velocity is close to zero, $26\frac{1}{2}$ km. per second for the 13 planetaries on his list. The average radial velocity of the helium stars, very generally supposed to be the stars most recently formed from nebulae, was found to be only 6 km. per second. The average nebular velocity was four times as great. Were Keeler's results, only 14 in number, too few to furnish a safe statistical basis of comparison? The basis was at least too small to support a superstructure of any pretensions to reliability. A knowledge of a greatly increased number of nebular velocities was strongly demanded.

During the past three years, at the Lick Observatory and at the associated D. O. Mills Observatory, Santiago, Chile, more than 200 measures of nebular radial velocities have been secured by spectrographic methods. These relate to 61 nebulae, but as 7 of these objects have been observed only once each, this note is limited to the velocities of 54 nebulae.

The distribution of the velocities, as to magnitude, tabulated below is remarkable. No distinction between velocities of approach and velocities of recession is made in this table.

Velocities, km./sec.	0–10	10–20	20–40	40–60	60–80	80–up
Number of nebulae	11	6	17	11	2	7
Average velocity	5	16	28	50	77	132

Total number of nebulae 54. Grand average velocity 42 km./sec.

It is now well known that the radial velocities of four-fifths of the so-called helium stars are under 10 km. per second, and that the average for all helium stars is about 6 km. per second. Of the observed nebular velocities only one-fifth are under 10 km. per second, and the average for the 54 observed objects is 42 km. per second, or 7 times that of the helium stars. The velocities of the helium stars follow pretty closely the so-called 'probability curve'—a great number of small velocities and very few large velocities. The nebular velocities do not follow the probability curve at all.

If we omit velocities greater than 60 km. per second, as abnormal, we find the *average radial velocity of 45 nebulae to be 26.4 km. per second,* which is still 4 times the average for all the helium stars.

Omitting 12 extended and ring nebulae we find that *the average radial velocity of 42 planetary nebulae is 46.1 km. per second.*

The velocities of the extended and ring nebulae seem to be habitually low, but the number of these included in the program thus far is too small to serve as a safe basis for drawing conclusions. The nebulae N. G. C. 6644 ($\alpha =$ 18h 26m, $\delta = -25.2°$) and N. G. C.$_2$ 4732 ($\alpha =$ 18h 28m, $\delta = -22.7°$), which are less than 3° apart, have velocities 202 km. per second recession, and 141 km. per second approach, respectively—a relative radial motion of 343 km. per second.

In general, assuming that the motions of celestial bodies are substantially at random in direction and speed, the average radial velocity of a considerable number of objects is exactly one-half of the average space velocity of the objects. We shall probably not be far wrong in assuming that the average radial velocity of 42 km. per second for the 54 nebulae means that the average space velocity of the same objects is approximately 84 km. per second.

All of the above results have been freed from the effects of the sun's motion as determined from spectrographic observations of the stars.

The algebraic mean of the 54 observed velocities, after correcting for the effects of the sun's motion, is -2 km. per second. There is thus no indication of a systematic tendency of the planetary nebulae for motions of approach or of recession.

These observations refer exclusively to nebulae whose spectra are composed chiefly of bright lines, and there are no spiral nebulae on this list.

With reference to stellar motions, the planetary nebulae are rapid travellers. Does this fact throw any light upon their origin or their possible relation to stars? A few of the stars are known to be travelling rapidly. These rapid travellers are found mostly amongst the yellow and red stars, which are usually held to be furthest removed from the nebular condition. Perhaps the best explanation of the so-called novae, or new stars—stars which suddenly flash up where previously no stars, or at the best, very faint stars, were known to exist—is that they are due to the rushing of dark or relatively faint stars through invisible nebulae or resisting media. The resulting collisions, in effect a bombardment of the stellar surfaces, seem to generate sufficient heat to convert the surface strata of the stars into incandescent gases and vapors. Spectrographic observations have shown for all recent novae that in the course of a few weeks or months the spectra of the novae are converted into the nebular type. Later the nebular spectra disappear and a certain type of stellar spectrum takes their place. These changes occur rapidly, perhaps because the bombardment effects have been but skin deep.

That planetary nebulae may have resulted in some such way from the collisions of stars and the resisting media seems quite possible. Exactly those stars which are travelling with very high speeds would have the greatest chance to encounter resisting media: and further, the disturbances would be deeper and more permanent the higher the speed of collision. Conversely, if this speculation has a basis in fact, that is, if the planetary nebulae have been formed in this manner, the prevailing high velocities would find a natural explanation.

In securing the difficult long-exposure observations upon which this note is based, I have been greatly assisted by several members of our staff, especially by J. H. Moore, E. S. Haynes, and P. W. Merrill.

PRELIMINARY NOTE ON NEBULAR PROPER MOTIONS

By H. D. Curtis

LICK OBSERVATORY, UNIVERSITY OF CALIFORNIA

Presented to the Academy, October 31, 1914

The great number of nebulae in the sky, their well-known grouping in regions distant from the Milky Way, the spiral structure so frequently occurring, and the question as to the place of these bodies in the evolution of the sidereal universe, all present a series of problems as yet only partially solved.

A knowledge of the proper motions or of any rotational movements which these bodies may have would be of very great value in investigations as to the size and distance of the nebulae, and therefore as to their place in the structure of the visible universe. Owing to the hazy and diffuse character of the great majority of nebulae when observed visually, the visual observations of the past, just as they have proved nearly valueless in giving an adequate conception of the form and structure of these remarkable objects, so they are also inadequate as records of position for the determination of proper motions and rotations. Trustworthy results can be secured only by the aid of photography.

Sixteen years have now elapsed since Keeler inaugurated his program of nebular photography with the Crossley Reflector of the Lick Observatory; and Keeler's plates, in connection with recent photographs secured with the same instrument, afford a large mass of material for the investigation of nebular motions. Some of the diffuse objects of the Keeler program do not admit of precise measurement; some also of the old plates are rather poor; but when the program of repetition, commenced early in 1914, shall be completed, in 1915, sets of photographs, early and recent, with an average interval of time of about thirteen years, will be available for the determination of the proper motions or internal movements and changes of between 80 and 100 nebulae.

This program is now about one-third complete, as regards the taking of the modern plates, and about one-fourth of the measurements have been made. The work to date comprises most of the larger spirals, such diffuse nebulosities as the Orion and Network nebulae, several planetaries, and several very faint small nebulae

As the early and late plates of this investigation have been secured with the same instrument, the methods of measurement have been made entirely differential. Five or six symmetrically-placed small stars are selected as a common reference system for all the plates of a given nebula, and the nucleus and numerous knots or condensations are measured in each nebula as available.

Detailed results will be published upon the completion of the program, but from the observations already made we may draw the following general conclusions, which will probably not be seriously modified by the inclusion of more objects:

1. In this average interval of thirteen years it has not been possible to detect any evidence of internal movement, rotatory or otherwise, in the nebulae measured.

2. The derived proper motions are so small that they may well be ascribed very largely to the difficulty and uncertainty of measuring these difficult objects. The largest proper motions found are for the planetary nebulae N. G. C. 6905 and 7009: 0.056″ and 0.054″ per year, respectively. Those for the other objects measured will average two or three hundredths of a second of arc per year. As the plates are exposed from two to four hours, and as the nebular condensations are much less sharp than star images, the errors of measurement are manifestly much larger than they would be in the case of short-exposure stellar images.

So far as the program has gone, therefore, all the evidence points to a great distance for these objects. As the spirals are undoubtedly in revolution—any other explanation of the spiral form seems impossible—the failure to find any evidences of rotation would indicate that they must be of enormous actual size, and at enormous distances from us. At a distance of one-thousand light-years, supposing mass conditions to be the same as in our solar system, a nebular structure one minute of arc in apparent distance from its nucleus, or 60 times the distance of Neptune from the Sun, would have a yearly angular movement of only $53''$, or a maximum yearly displacement in a given component with reference to the nucleus of only $0.015''$. A much greater time interval will probably be necessary before nebular rotations can be definitely established by measures of position, unless a nebula abnormally close to us be found.

Wolf and Slipher have reported the observation of rotation in the spiral nebulae Messier 81 and N. G. C. 4594, respectively, by means of the velocity-displacements of lines in the spectra of these bodies. The spectrographic method, here as elsewhere, is independent of the distance or absolute size of the object.

DISCOVERY OF THE NINTH SATELLITE OF JUPITER

By S. B. Nicholson

LICK OBSERVATORY, UNIVERSITY OF CALIFORNIA

Presented to the Academy, October 31, 1914

The Ninth Satellite of Jupiter was discovered on photographs taken with the Crossley Reflector of the Lick Observatory, on July 21 and 22, 1914. The photographs were of two and one-half hours exposure, and were taken to secure observations of the Eighth Satellite. The new satellite is fainter than the Eighth, and is estimated as nineteenth magnitude. The identity of the object as a satellite was established from the observations of July 22, 27, and 31 by Leuschner's *Method of Direct Solution of Orbits of Disturbed Bodies*. The motion is retrograde, and the first estimate of the period places it at about three years. The remaining elements of the orbit resemble those of the Eighth Satellite. Additional observations of the satellite have been secured in August and September. A detailed account of the discovery and observations, and of the investigation of the orbit, will be published in a *Lick Observatory Bulletin*.

SPHERICAL ABERRATION IN ASTRONOMICAL OBJECTIVES DUE TO CHANGES OF TEMPERATURE

By Frank Schlesinger

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In September of this year the 30-inch objective for the Thaw Photographic Refractor was installed at the Allegheny Observatory. The objective was made by the John A. Brashear Company of this city. Before it was removed from the optical shop it was tested in a number of ways by the makers and by me. These tests implied an almost perfect figure, and also indicated that the material in the objective is of unusual excellence.

When the objective was first tested at the telescope on actual images of stars, a considerable amount of positive spherical aberration was found to be present; that is, the focal lengths of the outer zones were found to be shorter than for the inner and intermediate zones. The tests were repeated on as many different occasions as possible, with the result that while positive aberration was found to be always present, it varied in amount from night to night and even in the course of a single night. A study of the data thus obtained indicates that the aberration arises from effects of temperature and that it is more pronounced when the temperature is falling rapidly.

To remove this aberration the following expedients were employed. During the day-time, when the telescope is not in use, the objective is enclosed by a metallic cover. With the tube of the telescope this cover acts as a kind of temperature-case, throughout which the temperature is roughly uniform. The observer removes this cover about one hour before beginning to observe in the evening, and at the same time he starts a small electric fan mounted near the upper end of the tube. This fan blows a continuous current of air into the tube through a large vent just below the rear surface of the objective; two similarly placed vents permit the air to escape from the tube. In this way the warmer air that has accumulated during the day is expelled and the rear surface of the objective cools as rapidly as the front. The upper (flint) lens of the objective is separated from the lower (crown) lens by a little less than half an inch. The iron cell in which the two are mounted has been pierced by six holes half an inch in diameter; these permit the air to gain access to the space between the two components.

At the close of the night the observer covers the objective, closes the three vents and inserts cork stoppers in the six lateral holes.

The fan is on the same circuit with the electric lights that illuminate the setting circles of the telescope. As a consequence the fan is in operation at frequent intervals throughout the night. As the lights must be turned off whenever an exposure is being made, the fan is out of operation and cannot cause the tube to vibrate at any time that this could be harmful.

These devices have proved successful in removing the aberrations due to temperature. The Hartmann test has since been applied on a number of occasions and has invariably indicated an excellent figure for the objective.

It is not to be supposed that the Thaw objective is peculiar in its behavior with respect to changes in temperature. An examination of the literature on this subject with regard to other objectives reveals the fact that some of them, at least, are similarly affected and exhibit aberrations that change from time to time, precisely as the Thaw objective did before the application of the ventilating devices. It therefore appears to me that the installation of similar arrangements in the case of other telescopes might bring about a considerable improvement in their performance.

The details of this investigation are to appear in Volume 4 of the Publications of this Observatory.

THE RELATIONS BETWEEN THE PROPER MOTIONS AND THE RADIAL VELOCITIES OF THE STARS OF THE SPECTRAL TYPES F, G, K, AND M

By J. C. Kapteyn and W. S. Adams

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Presented to the Academy, November 16, 1914

Among the stars observed for radial velocity with the 60-inch reflector of the Mount Wilson Observatory, there are many of the spectral types *F*, *G*, *K*, and *M* which have either very large or very small astronomical proper motions. The two classes of stars show some very marked differences which indicate a close relationship between proper motion and radial velocity. It is the aim of the present note to bring out this relationship as clearly as possible.

For this purpose it is necessary to supplement the Mount Wilson data for the stars of extreme proper motion with similar data for the rest of the stars. There is, of course, no better source for this purpose than the catalogue of radial velocities published recently by the Lick Observatory.[1] The Bonn observations by Küstner came to hand too late for any extended use. The classification used is that of the Harvard College Observatory, and the spectra given in Campbell's catalogue are, with a very few exceptions, those of the Harvard Revised Photometry.[2] The Mount Wilson stars have been classified by the method described in the paper by Adams and Kohlschütter.[3] Occasionally large differences have been found from the estimates of the Harvard observers for the same stars.

The observed radial motions were first reduced to velocities (ρ) relative to the center of gravity of the stellar system by applying the component of the sun's motion through space with reversed sign; that is, the correction $V \cos \theta$, where V is the sun's velocity, and θ the angular distance between star and apex. For the apex we adopted the position α (1900) = 18h, 0m; $\delta = +31°$, and for V the value 20.0 km. per second. The values of ρ thus obtained were grouped: (1) according to amount of proper motion; (2) according to distance λ from the nearest true vertex of the two star-streams. The positions of these vertices, according to the determinations of Kapteyn, Eddington, and others are:

Stream I, 18h 12m $-12°$; Stream II, 6h 12m $+12°$

The results are summarized in Table I. The letter μ denotes the total proper motion, and W and C refer to the radial velocity determinations by the Mount Wilson observers and by Campbell, respectively. Average values are indicated by dashes over the letters, and figures in parentheses denote the number of stars on which each average is based. The two columns under λ are, as a rule, limited by values of λ 60° to 90° ($\bar{\rho}_1$), and 0° to 49° ($\bar{\rho}_2$). A third group between the narrow limits 50° to 59° was formed, but is omitted from the table. In a few cases where the number of stars was small this middle group was used to increase the weight of the extreme groups. Six stars of abnormal velocity have been excluded out of a total of 1106 stars. These are:

Lal. 15290	−249.4	A.Oe 20452	−163.0	Boss 130	− 80.7
Boss 315	+ 62.8	Boss 1511	+163.0	A.Oe 14320	+299.4

An inspection of Table I shows clearly: first, that the average velocity of the stars near the vertices is considerably greater than that of the stars far away from these points; second, that the average value of ρ increases rapidly with the proper motion.

The first of these conclusions is indicated most concisely by the values of the ratio $\bar{\rho}_2/\bar{\rho}_1$ in the last column. It is a necessary consequence of the existence of two star streams with opposite motions. Thus, it is evident that if all of the stars moved accurately in the direction of these streams the radial velocity would be zero at those points where the line of sight is at right angles to the stream motion; that is, where λ = 90°. On the other hand the motion would be altogether radial at the vertices themselves; that is, ρ should be large for λ = 0°. Since the motions of the stars do not in general coincide with the stream motion, but only show a preference for directions differing but little from it, the contrast in the amount of the radial motion at λ = 90° and λ = 0° will be less marked. This amount can, of course, be calculated as soon as we have derived complete elements for the two streams from a consideration of the proper motions. As it is, different theories give slightly different values,[4] and eventually these differences may serve to determine the theory to be

TABLE I

Average Values of the Radial Velocity ρ in km.

Type	Proper Motion	Authority	λ 60° to 90°		λ 0° to 49°		Total		$\bar{\rho}_2/\bar{\rho}_1$	
	μ		$\bar{\lambda}$	$\bar{\rho}_1$	$\bar{\lambda}$	$\bar{\rho}_2$	$\bar{\lambda}$	$\bar{\rho}$	Obs.	Theory
F	0.000″ to 0.029″	W+C	79°	8.9 (25)	32°	13.6 (22)	56°	10.6 (54)	1.53	1.63
	0.030 0.069	C	71	8.5 (9)	29	12.5 (11)	48	10.15 (22)	1.47	1.56
	0.070 0.149	C	78	12.3 (17)	34	20.5 (17)	53	14.65 (38)	1.67	1.58
	0.150 0.249	C	72	11.5 (22)	32	18.9 (12)	58	14.3 (40)	1 64	1.55
	0.250 0.499	C	74	16.7 (24)	33	21.1 (8)	62	16.2 (38)	1.26	1.54
	≧0.500	W+C	73	18.05 (30)	43	34.9 (18)	62	24.4 (48)	1.93	1.39
G	0.000 to 0.026	W+C	73	6.9 (37)	32	12.8 (32)	54	9.6 (80)	1.86	1.55
	0.027 0.049	C	76	9.6 (10)	37	12.8 (13)	54	11.8 (28)	1.33	1.53
	0.050 0.099	C	74	9.5 (18)	27	12.3 (7)	61	10.3 (25)	1.30	1.63
	0.100 0.499	C	76	14.95 (13)	44	23.6 (11)	60	19.0 (28)	1.58	1.41
	≧0.500	W+C	76	22.9 (51)	37	45.6 (31)	60	32.1 (91)	1.99	1.51
K	0.000 to 0.025	W+C	76	10.9 (47)	36	12.3 (39)	57	11.7 (91)	1.13	1.52
	0.026 0.039	C	73	12.15 (37)	37	11.9 (24)	58	11.7 (68)	0.98	1.47
	0.040 0.059	C	75	14.9 (21)	38	21.3 (21)	56	17.9 (50)	1.43	1.49
	0.060 0.079	C	75	10.7 (24)	36	15.1 (16)	58	12.3 (41)	1.41	1.50
	0.080 0.099	C	74	15.7 (26)	34	14.0 (10)	62	15.2 (36)	0.89	1.52
	0.100 0.119	C	76	20.7 (12)	32	22.6 (15)	52	23.4 (31)	1.09	1.58
	0.120 0.149	C	73	16.5 (24)	39	23.8 (10)	62	19.0 (36)	1.44	1.43
	0.150 0.199	C	74	15.6 (24)	39	24.5 (10)	62	18.2 (34)	1.57	1.45
	0.200 0.299	C	75	16.5 (18)	32	24.3 (13)	56	20.4 (37)	1.47	1.56
	0.300 0.599	C	73	33.3 (12)	33	47.05 (7)	58	38.35 (19)	1.41	1.52
	≧0.600	W+C	74	23.5 (25)	34	23.0 (28)	53	22.5 (58)	0.98	1.52
M	0.000 to 0.029	W+C	76	12.9 (18)	36	18.1 (15)	58	14.4 (40)	1.40	1.52
	0.030 0.089	C	79	15.0 (19)	36	21.0 (11)	62	17.0 (34)	1.40	1.55
	0.090 0.499	C	77	17.9 (14)	35	28.9 (7)	62	20.7 (24)	1.61	1.56
	≧0.500	W	71	53.2 (4)			71	53.2 (4)		

preferred. If we use that developed by Kapteyn[5] we obtain the comparison shown in Table I. Except for some of the K-type stars the agreement of the values is very satisfactory. The theory, of course, requires that ρ be proportional to $1 + \cos^2 \lambda$.

The values of $\bar{\rho}_2/\bar{\rho}_1$ are no smaller for the stars having the very smallest proper motions than for the other stars in the list. This proves that the two star streams extend to the greatest distances for which we have means of judging. This fact has been doubted by Eddington[6] who proposed an attractive explanation of the behavior of the helium stars, which show hardly a trace of the second stream, on the supposition that the greater part of them are beyond the region where we must admit the existence of two streams. Several objections have already been raised against this hypothesis,[7] and we believe that Eddington has abandoned it. It is interesting, however, to find in the radial motions of these stars so strong a proof of the great extent of the star streams.

The low value of $\bar{\rho}_2/\bar{\rho}_1$ for some of the K stars, particularly for some of those of very small proper motion, seems difficult of explanation. A preliminary investigation of the proper motions for the stars having values between 0.026″ and 0.039″ indicates that members of the second stream are not wanting. Evidently the anomally will require much further study.

The second conclusion indicated by Table I, that ρ increases with increase in proper motion, may be explained in any one of at least three ways. We may assume that:

a. The real velocity of the stars decreases with the distance.

b. The more luminous stars move more slowly than the fainter ones.

c. The distribution of the velocities of the stars is not in accordance with Maxwell's law, the large velocities being in excess.

The application of the first two of these explanations is evident. In the consecutive groups for each spectral type the average magnitude is roughly the same. Hence, with the average proper motions increasing regularly from one group to another, the average distances must decrease, and the luminosities become less.

The possibility of the third explanation, *c*, may seem somewhat less evident. Its consideration, however, is essential since it has already been shown by Schwarzschild[8] that the distribution of velocities, using values relative to the sun, cannot agree with Maxwell's law, and that the larger values must be in excess. The radial velocities used here afford the data for the derivation of the velocity-law relative to the center of the stellar system. Such a derivation we have actually carried out in the case of the K stars, and we hope in the course of a more extended discussion to describe it in detail, as well as to obtain similar expressions for the stars of other types. A very brief summary of the method used is as follows:

1. All of the stars between the limits $\lambda = 60°$ and $\lambda = 90°$ were selected from Campbell's catalogue and arranged according to amount of radial velocity. This selection was made in order to eliminate the effect of stream motion so far as possible. The application of Maxwell's law to this material at once showed a large preponderance of high velocities. Thus there are 17 stars with velocities above 40 km. per second where Maxwell's law requires less than one-third of this number.

2. A satisfactory expression for representing the observed distribution of velocities was found in the sum of two Maxwell equations with different moduli. A peculiar feature of this result is that if all of the stars are used, and not alone those on which the stream motion has little influence, the exponential constants remain essentially unchanged.

3. With the aid of this expression the relationship was determined between the average radial velocity and the component of the linear velocity at right angles to the line of sight.

4. The stars were then divided into groups, each within a narrow range of proper motion, so that they may all be regarded as at the same distance. The components of the linear velocities at right angles to the line of sight were then computed by the aid of the parallaxes given in *Groningen Publications* No. 8. A factor was applied to the parallaxes such as to make the total average linear velocity for all of the stars equal to the total average radial velocity ρ. Table II shows the final results for ρ.

It appears from these results, therefore, that our assumption c is quite adequate to explain the variation of radial velocity with proper motion. There remains, however, the possible effect of a or b, which we may designate briefly as the effect of distance or of absolute magnitude. With more extensive data it might perhaps be possible to separate these two effects. As it is we shall suppose the distance effect to be negligible and try to determine the absolute magnitude effect.

For stars of approximately the same distance we shall assume that $\rho = a + b\,M$, where M is the absolute magnitude derived from the formula,[9] $M = m + 5 + 5 \log \pi$. In general we have kept the same groups used in Table I, the variation in distance not being excessive, but in a few cases consecutive groups have been combined. These groups were then further divided according to apparent magnitudes, an attempt being made to keep the numbers of the stars in the extreme subdivisions about equal. The values of the parallax π were then taken from *Gronin-*

gen Publications No. 8, and the absolute magnitudes were computed. From the resulting equations the values of b were determined by least squares.

TABLE II

COMPUTATION OF THE RADIAL VELOCITY AS A FUNCTION OF THE PROPER MOTION

PROPER MOTION μ		NO. STARS CAMPBELL	PARALLAX π GRON. NO. 8	$\bar{\rho}$ OBS.	$\bar{\rho}$ COMP.	DIFFERENCE
seconds (″)			*seconds* (″)	*km.*	*km.*	*km.*
0.000 to	0.025	27	0.0064	10.9[1]	12.1	−1.2
0.026	0.039	37	0.0112	12.15	12.5	−0.35
0.040	0.059	21	0.0148	14.9	12.9	+2.0
0.060	0.079	24	0.0185	10.7	13.3	−2.6
0.080	0.099	26	0.0218	15.7	13.7	+2.0
0.100	0.119	12	0.0248	20.7	14.3	+6.4
0.120	0.149	24	0.0283	16.5	14.8	+1.7
0.150	0.199	24	0.0332	15.6	15.9	−0.3
0.200	0.299	18	0.0412	16.5	17.7	−1.2
≦0.300		19	0.108	26.7[1]	24.5	+2.2
		232				

[1] These numbers have the Mount Wilson results for the corresponding proper motions included in them.

This solution requires two corrections. The first is due to the difference in the average value of λ for the several groups. This correction has been applied on the very probable assumption that the change of ρ with λ is proportional to $1 + \cos^2 \lambda$. The second correction is due to the fact that while the values of μ are equal, or practically so, the values of π are somewhat unequal owing to the differences in magnitude. As a result the average linear motions and hence the radial velocities will be affected. This effect is systematic in character, though small, and can be applied only in the case of the K stars.

If we combine the results for the separate groups into means for each spectral type we obtain Table III. The corrected value for the K stars is denoted by $b2$. On the assumption, which seems probable, that the correction in the case of the other spectra is the same as for the K stars we find, therefore, *a change of radial velocity per unit of absolute magnitude of* 1.1 km. ± 0.4.

TABLE III, VALUES OF THE CONSTANT b IN KM.

TYPE	$b1$	$b2$	PROB. ERROR
F	+2.1	.....	±0.73
G	+1.0	.....	0.82
K	+1.6	+1.1	0.62
M	+2.6	.....	2.5
Mean	+1.6	.....	±0.40

The reality of this conclusion that the more luminous stars move more slowly than the fainter stars must be accepted with considerable reserve. Certain features of this investigation are necessarily somewhat crude, and the data upon which it rests were not collected with a view to such a question. Moreover, of two direct observational points of evidence one appears to be opposed to this conclusion.

The stars denoted as *c* stars by Miss Maury were shown by Hertzsprung to be exceedingly luminous and very distant. According to an unpublished investigation by Kapteyn and Hertzsprung these stars, of average magnitude 4.5, must be 300 times more luminous and 4.5 times as far away as the average *A* stars of the fifth magnitude. We should, therefore, expect a low velocity in their case. In reality the average radial velocity of 28 of these stars is 12.8 km. For the average of all the *A* stars Campbell finds 11.1 km. In considering this result it should be borne in mind that the effect found here is for the second type stars and may not apply to the *A* stars; also that the *c* stars may actually constitute a separate spectral class with which the *A* stars are not directly comparable.

It is known from the researches of Hertzsprung that the variable stars of the *δ Cephei* type are stars of very high luminosity. He finds for their average absolute magnitude the value -2.2, using for this purpose their parallactic motions and apparent magnitudes. The average absolute magnitude of the 198 *F* stars given by Campbell is $+1.8$. The average radial velocity, freed from the sun's motion, of 11 of the *δ Cephei* variables, of average type *F*8, is 9.0 ± 1.2 km. The average radial velocity of Campbell's *F* stars is 14.75 km. This would give 1.5 km. as the value for our coefficient *b*.

SUMMARY OF RESULTS

1. The radial velocities furnish a very thorough test of the theory of the star streams. The results found for the *F*, *G*, *K*, and *M* stars are in close agreement with those we should expect from the theory as derived from proper motions.

2. The radial velocities of the stars of the smallest proper motions show the effects of the two star-streams with the same certainty as those of the other stars. The existence of the two star-streams is, therefore, proved at the greatest distances for which we have adequate data.

3. The *K* stars behave in general like the other stars, but there are a few exceptional cases. These do not appear to be due to the absence of the second stream.

4. For all of the spectral classes the average radial velocities show a regular increase with the proper motion.

5. Such a change of radial velocity is a necessary consequence of a velocity distribution (for the peculiar motions) different from that given by Maxwell's law.

6. A first approximation to the velocity distribution has been derived for the K stars. It explains the change of velocity with proper motion in a satisfactory manner.

7. Some positive indications have been found of a change of radial velocity with absolute magnitude, the brighter stars moving more slowly than the fainter stars.

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[1] *Lick Obs. Bull.*, **7**, 113 (1913).

[2] See *Annals Harvard Coll. Obs.*, vol. **50**.

[3] Adams and Kohlschütter, "Some special criteria for the determination of absolute stellar magnitudes," *Contrib. Mt. Wilson Solar Obs.*, No. 89; *Astrophys. J.* **40** (1914).

[4] For example, *Astrophysical Journal*, **31**, 266 (1910).

[5] Kapteyn, *Monthly Notices*, 72, 743 (1912).

[6] Eddington, *Observatory*, 34, 355 (1911).

[7] *Proc. Amsterdam Acad. Sciences*, 14, 524 (1911).

[8] Schwarzschild, *Astron. Nachr.*, No. 4557 (1912).

[9] See *Publ. Astron. Lab. Groningen*, No. 11, p. 12.

CRITIQUE OF THE HYPOTHESIS OF ANOMALOUS DISPERSION IN CERTAIN SOLAR PHENOMENA

By Charles E. St. John

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According to the theory of anomalous dispersion, the interpretations of solar observations in terms of absorption, motion, pressure, and level are generally misleading since the observed phenomena, such as prominences, flash spectra, flocculi, and displacements of the Fraunhofer lines are from this point of view mainly the effects of anomalous refraction in the solar atmosphere, so that in their study we are facing optical illusions. The establishment of such a point of view would revolutionize or render futile many of the present lines of solar and stellar observation, and would make practically impossible the solution of many problems which confront the investigator. It is therefore of the highest importance to determine the form and extent of the influence of anomalous refraction in the solar atmosphere, if it obtains to a detectible amount.

There is a degree of vagueness in the deductions from the theory, due to its extreme flexibility, that makes a quantitative examination of its claims difficult. During the present year, however, W. H. Julius, its originator and chief supporter, announced a deduction that offers an opportunity of making such a quantitative test, namely: the mutual influence of Fraunhofer lines upon each other.[1] The observational data available for a crucial test are available,[2] and consist of the displacements of 506 Fraunhofer lines at the edges of the penumbra of eccentrically located sun-spots observed by me. In such observations the slit of the spectrograph is parallel to the radius of the solar image passing through a spot whose position for best observation is between the limb and the midpoint of the radius. The displacements are a maximum at the peripheral edges of the penumbra. At the edge directed toward the limb they are in general in the direction of longer wave-length, while at the edge directed toward the center of the disk they are in the direction of shorter wave-length. These displacements were interpreted by Evershed and myself as effects of motion in the line-of-sight, but are considered by Julius to be due to anomalous refraction.

In this phenomenon the particular form in which mutual influence would manifest itself is that a weak line on the violet side of and near to a stronger line is displaced less, but, if on the red side, more than the average amount. Julius, in making the announcement of the deduction to which reference is made, gave a discussion based upon a portion of the available data and obtained a result apparently confirming the deduction and considered by him a veritable proof of the theory. In fact, the deduction follows so directly from the theory that its confirmation would greatly strengthen the position of the theory as a working hypothesis, while a failure of confirmation would discredit it. The questions raised by Julius are elsewhere considered by me in detail.[3]

These displacements are well suited for a definitive test of the theory, as they are purely differential and free from possible observational bias, since they were made without any knowledge of what they are now used to prove. Moreover, a large number of other lines present upon the same plates are available for standards of reference. It is evident that much depends upon the derivation of these standards.

The 506 lines in my original paper do not form a homogeneous series of observations. From this it follows that standard displacements for different spectral regions cannot be determined with high precision by deducing them for each region by any smoothing-out process involving the results for all the segregated regions. There are six series:

(1) λ 3624 – λ 3724; (2) λ 3879 – λ 4410; (3) λ 4634 – λ 4829;
(4) λ 5123 – λ 5349; (5) λ 5598 – λ 6065; (6) λ 6393 – λ 6643.

The measurements represent with high precision the relative displacements in any single region; but the data are not sufficient to obtain the relation between the absolute values for the separate regions to the same degree of precision, depending as they do upon varying observational conditions.

To obtain residuals capable of representing any systematic behavior of lines within the refractive influence of stronger lines, the standard displacements should be derived from the limited homogeneous series of observations in which they are to be used and, as far as the data permit, they should refer to the particular element under consideration. Such standard displacements have been obtained by determining the mean displacements for the lines of each element of a given intensity for each of the homogeneous series.

With these standards the displacements of 131 Fraunhofer lines within 0.5 A of stronger ones are compared, 67 on the violet side of stronger lines and 64 on the red side. The hypothesis of mutual influence requires that for the lines to the violet the mean residual be negative and for the lines to the red, positive. The result of the comparison applied to this large number of lines is that 67 lines to the violet give a mean residual − 0.0003 A, and 64 lines to the red give a mean residual − 0.0004 A.

Stated in another form, the sum of the favorable residuals is 0.212 A; of the unfavorable, 0.218 A. Tested, then, by standards deduced from the homogeneous series of observations to which they are to be applied, the lines under the influence of stronger ones show no systematic difference within the limits of measurement from lines not so situated.

Another criterion is whether the effects produced by the influencing lines depend upon their power to produce anomalous dispersion in the laboratory. A comparison of the residuals given by lines under the influence of those having strong anomalous dispersion with the residuals given by others under the influence of lines having very weak anomalous dispersion shows no greater effect in one case than the other.

It would seem that the effect of mutual influence must increase with nearness of the influenced line to the controlling line. The change should then be especially manifest for those in the near neighborhood of such powerful lines as H, K, and 4226.9 of calcium and the strong aluminum lines, all of which show large anomalous dispersion in the laboratory and are bordered by broad shadings or wings in the solar spectrum. From the point of view of the anomalous dispersion theory, these wings are considered dispersion bands. In making the original series of measures, the lines near these very strong lines were in general omitted from the observing list, as they present special difficulties. A few of the best plates have now been remeasured with the view of obtaining data relative to such lines, and also a few plates of a new series of observations on spots belonging to the approaching spot-maximum. The displacements of 144 lines have been measured; of these 16 are within one angstrom of very strong lines showing great anomalous dispersion. For standards of reference the mean displacements for all lines of a given intensity are used. Within one angstrom of these strong lines there are eight lines to the violet which give a mean residual of + 0.0006 A based upon 51 measures, and eight lines to the red which show a mean residual of + 0.0003 A based upon 62 measures. The result is practically zero. There is, moreover, no systematic variation, as the influencing lines are approached from a distance of five angstroms to their near neighborhood.

The displacements so far considered are of lines in the reversing layer and indicate an outflow from the spot-vortex tangential to the solar surface. The important chromospheric lines, H and K of calcium, the hydrogen lines, the D lines of sodium and the b group of magnesium, give displacements of the opposite sign, interpreted as inflow of the high level vapors. Up to the present the theory of anomalous dispersion has not been able to suggest an explanation of these negative shifts.

All these displacements are obtained with the slit of the spectrograph parallel to the radius of the solar image passing through the spot. Displacements of the high level lines are also found when the slit is normal to the radius, which on the line-of-sight interpretation indicate vortex motion. These displacements have always been troublesome to the dispersion theory and are disposed of as occasional phenomena due to unequal refraction at the opposite edges of spots under exceptional conditions. The displacements are, however, a practically constant feature of all spots, persist for many weeks, and characterize regular spots during at least the greater part of their existence.

It is probably possible to imagine a density distribution capable of explaining these displacements, "Since almost any peculiarity in the appearance of spectral lines may be explained by anomalous dispersion, if only we are at liberty to assume the required distributions," as Julius remarks; but it seems questionable that such an artificial condition characterizes a regular spot and the mind accepts more readily the obvious explanation based upon motion in the line of sight.

There is one more consideration bearing upon an explanation of these displacements based upon the anomalous-dispersion theory, namely, the differences characteristic of the elements. In examining the standing of a theory, facts that cannot be explained by it are of particular interest. The existence, then, of such differences becomes of great importance, since as yet it has not been possible to suggest an explanation of them in harmony with the dispersion theory. As standards of reference the displacements of the iron lines of like solar intensity measured upon the same plates have been used. The residuals (that is, displacement for the element minus the mean displacement of iron lines of the same intensity) are given in thousandths of angstroms in the accompanying table:

Element	Ba	Ce	Cd	La	Nb	Nd?	Pb	Yt	T
Number of lines	2	1	1	9	1	3	1	1	47
Atomic weight	137	142	112	138	94	144	207	173	48
Mean residual in 0.001A.	+5	+7	+5	+2	+10	0	+6	+6	−3

The residuals for the group of heavy elements are systematically positive and are undoubtedly real, while the residuals for titanium are systematically negative and likewise exceed the errors of measurement.

The displacements considered in this paper may all be interpreted in terms of motion and level as shown in contributions already published.[4] The purpose of the present discussion has been to show their bearing upon the theory of anomalous dispersion in its application to certain solar phenomena of which the theory has suggested partial explanations and to which it has appealed for evidence of its influence in the solar atmosphere. The only quantitative criterion, mutual influence, fails to confirm the theory, and in my opinion there are well established results of these observations for which the theory has no explanation.

[1] W. H. Julius: Radial motion in sun-spots. *Astrophys. J.*, **40**, 1 (1914).

[2] *Contrib. Mt. Wilson Solar Obs.*, No. 69.

[3] Ibid, No. 93.

[4] Ibid, Nos. 69 and 74; *Astrophys. J.*, **37**, 322 (1913); **38**, 341 (1913).

A REVISION OF THE ATOMIC WEIGHT OF CADMIUM

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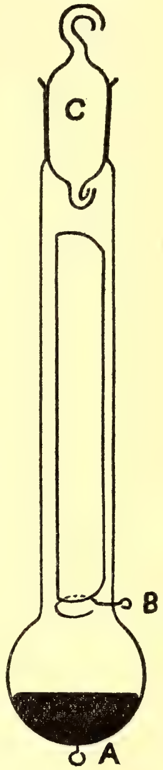
In several recent investigations by Hulett and his collaborators[1] concordant values have been obtained for the atomic weight of cadmium by the electrolytic deposition, in a mercury cathode, of the metal in hydrated and anhydrous cadmium sulphate and in anhydrous cadmium chloride and bromide, and by the comparison of simultaneous electrolytic deposits of cadmium and silver. The average value so obtained for the constant in question is 112.3. On the other hand, Baxter, Hines, and Frevert[2] had previously found a considerably higher value, 112.42, by the estimation of the halogen-content of cadmium chloride and bromide. So wide a discrepancy is disturbing and obviously needs attention. We therefore decided to repeat a portion of the work of Quinn and Hulett by analyzing anhydrous cadmium chloride through the electrolytic deposition of the cadmium in a mercury cathode.

Cadmium material was purified (1) by electrolytic deposition with a dissolving anode (yielding samples *A* and *B*), (2) by fractional precipitation with hydrogen sulphide (yielding samples *C* and *D*), and (3) by crystallization of cadmium bromide (yielding samples *E* and *F*); these operations being followed by crystallization of cadmium chloride (except in the case of sample *E* where the bromide was converted into chloride by fusion in a current of dry chlorine). Another sample (*G*) was obtained by combining the mother-liquors of the other specimens of cadmium chloride and, after crystallizing the salt, fusing the mixture in dry chlorine. In order to follow the purity of the different samples, the metal was deposited electrolytically from the mother-liquor of the last chloride crystallization in each case; and, after conversion into electrodes by fusion, it was used for the production of spark-spectra which were photographed in a Féry quartz spectograph. Careful examination of the spectrograms in the visible and ultraviolet regions showed that some of the samples were wholly free from impurities, and that, although some of them contained traces of copper and lead, the proportions of these impurities were too small to have any perceptible effect upon the outcome. Furthermore, the analyses of the chloride showed no apparent differences between the different samples. On the whole, crystallization of cadmium bromide seemed to be the most rapid and efficient method of purifying cadmium material.

For the electrolytic deposition of the metal the special form of cell shown in the figure was designed. In this cell both the cathode *A* and the anode *B* were fused into the glass and weighed with the cell, in order that any metal dissolved from the anode during electrolysis might immediately be deposited upon the cathode. This made it possible to electrolyze the chloride solution directly without previous removal of the chlorine. Another advantage of this form of cell lies in the possibility of introducing bodily the boat in which the salt was fused preparatory to weighing, so that no quantitative transference of solution was necessary. The mercury and the amalgams were prepared for weighing by washing with water and alcohol and drying in a high vacuum. Spurting of the amalgams out of the cell during the drying was prevented by the specially constructed stopper *C*, which was always weighed with the cell.



Blank determinations in which hydrochloric acid solutions were electrolyzed showed that the cell with a charge of mercury could be depended on to remain constant in weight. Furthermore, no evidence could be found of appreciable oxidation of the amalgams.

The experimental procedure of an analysis was as follows: The cadmium chloride, in a quartz boat contained in a quartz tube, was heated in a current of pure dry hydrochloric acid gas, gently at first so as to expel as much as possible of the water at a low temperature, but finally for some minutes at the fusing-point. After the tube had been allowed to cool, the hydrochloric acid was displaced by pure dry nitrogen, which was in turn displaced by dry air. Then the boat was transferred to the weighing bottle, in which it was originally weighed, by means of a 'bottling apparatus,'[3] without exposure to moisture, and weighed. Next the boat was inserted in the cell, in which it was supported upon the anode. The weighing bottle was rinsed, and the rinsings were used to dissolve the salt in the cell. For the purpose of retaining spray formed during the electrolysis, a column of bulbs, which had been ground into the mouth of the cell, was inserted. Electrolysis was then commenced, at first with a current of about 0.5 ampere. Later, when the greater part of the cadmium had been deposited, the current-strength was increased until finally the eight storage cells used as the source of electricity were short-circuited through the cell. The whole operation extended over about eight hours. At the end chlorine had nearly ceased to be evolved at the anode, although some residual conductivity remained, owing to the formation of a small amount of perchloric acid. Toward the end of the electrolysis the column of bulbs and the walls of the cell were several times rinsed into the cell. At the completion of the electrolysis the electrolyte was displaced with water which had been freshly saturated with hydrogen; and the water in turn was displaced with pure alcohol, the maximum voltage being maintained between the electrodes during the operations, and care being taken not to break the electrolytic circuit. The cell was then placed in a desiccator containing fused potassium hydroxide, the pressure in the desiccator was reduced to a few millimeters of mercury with a mechanical pump, and the cell was allowed to stand until the alcohol had evaporated. The cell was next transferred to a second desiccator which contained the counterpoise of the cell, and the two were allowed to stand in a low vacuum for some hours before being weighed. Further standing in an exhausted desiccator produced no change in weight, showing both that the cell and amalgam were dry and that the amalgam had little tendency to oxidize. The rinsings, both aqueous and alcoholic, were evaporated in a quartz dish with a very small amount of redistilled sulphuric acid; and, after transference to a weighed platinum crucible, the residue was ignited at dull redness. In this way a residue, in most cases weighing less than one milligram, was obtained which was shown to be cadmium sulphate. A correction to the weight of cadmium was applied upon this basis. Complete precipitation of the cadmium was never secured, although, according to the experience of Quinn and Hulett, no cadmium is left undeposited, even when a relatively large amount of sulphuric acid is present. The small amount of free chlorine contained in the residual electrolyte in our experiments may have been the cause of the incomplete deposition of the metal.

The weighing-bottle containing the boat and the electrolytic cell were both weighed by substitution for a counterpoise very similar in construction and of the same total volume.

The experimental data and the results calculated from them (assuming the atomic weight of chlorine to be 35.457) are shown in the accompanying table. The weights here given have been reduced to the vacuum standard by applying the following corrections for each gram of substance: cadmium chloride, + 0.000152 g, cadmium dissolved in mercury, − 0.000016 g.

It will be seen that this research has yielded a value (112.417) for the atomic weight of cadmium in very close agreement with the earlier one

(112.42) obtained by Baxter, Hines, and Frevert. In other words the determinations of the cadmium and chlorine in cadmium chloride sum up to almost exactly one hundred per cent. The disagreement of the outcome of this research with that of Quinn and Hulett's work also is marked.

ANALYSIS NUMBER	SAMPLE	GRAMS OF $CdCl_2$	GRAMS OF Cd	RATIO Cd:Cl_2	ATOMIC WEIGHT
1	A	6.08570	3.73181	1.58538	112.426
2	A	4.20489	2.57863	1.58562	112.443
3	A	5.36203	3.28817	1.58553	112.436
4	B	7.50512	4.60221	1.58538	112.426
5	B	6.71591	4.11839	1.58551	112.435
6	B	5.91556	3.62763	1.58555	112.438
7	C	5.49323	3.36805	1.58483	112.387
8	B	3.14416	1.92791	1.58513	112.408
9	C	7.58705	4.65173	1.58474	112.381
10	B	2.26738	1.39006	1.58444	112.359
11	B	5.93501	3.63962	1.58562	112.443
12	F	6.04122	3.70490	1.58578	112.454
13	D	4.07400	2.49821	1.58537	112.425
14	E	9.00004	5.51879	1.58529	112.419
15	G	6.56891	4.02808	1.58534	112.423
16	G	7.12956	4.37174	1.58522	112.414
17	E	8.57291	5.25679	1.58522	112.415
18	G	7.76294	4.76011	1.58521	112.413
Average				1.58529	112.419
Average of analyses 14 to 18				1.58526	112.417

It is a great pleasure to express our gratitude to the Carnegie Institution of Washington and to the Elizabeth Thompson Science Fund for very generous assistance in providing indispensable apparatus.

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[1] *J. Phys. Chem.*, **15**, 1579 (1911); *Trans. Amer. Electrochem. Soc.*, **22**, 385 (1913); *J. Phys. Chem.*, **17**, 780 (1913).

[2] *J. Amer. Chem. Soc.*, **27**, 222 (1905); **28**, 770 (1906).

[3] Richards and Parker, *Proc. Amer. Acad.*, **32**, 59 (1896).

PHORADENDRON

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Several years ago my attention was seriously called to the need of a revision of our leafy mistletoes through inability to understand the basis of characterization that could admit to one species such different appearing plants as those from the southeastern, southwestern, and arid United States—not to mention California and Yucatan—to which the name *Phoradendron flavescens* is currently applied Among their manifold differences, a diligent search was made for characters; the types of related species and varieties that have been held to be differentiable from *flavescens* were examined with care, and every form occurring in the United States was traced to the known limits of its range, sometimes south of our national border. As political boundaries do not often form satisfactory limits to such a study as I had begun, I was quickly lured into an examination of the Mexican species which approach our border, and of others which reach into the field of these, so that no arbitrary geographical limit, even, could be fixed short of the Isthmus.

In the course of this study it became apparent that the great conservatism of Engelmann, who seems never to have given this genus the care that marked his study of the related genus *Arceuthobium* or *Razoumofskya*, had not only caused him to withdraw segregates of *P. flavescens* that he admitted at one time, but had reacted on his early colleague in the study of our southwestern plants, Torrey, to the extent of causing a number of forms which had been designated in the Torrey herbarium as new species to lie there, as they still do, without publication.

At the New York meeting of the National Academy of Sciences, in November, 1911, and at a meeting of the Academy of Science of St. Louis on December 18, 1911, the preliminary results of this study of the northern species were outlined, and this was followed at the Washington meeting of 1912 by presentation to the National Academy of the manuscript of a revision of all of the forms of *Phoradendron* recognized as occurring in continental North America. As I was then on the eve of departing for a year in the great herbaria of Europe, this revision was withheld from immediate publication in the hope that several obscure Mexican species could be cleared up certainly, through authentic specimens, and in the hope that they might be illustrated from the types. Though the admission of Torrey's long neglected manuscript names had quite prepared me for an apparently inordinate increase in the number of differentiable species in the genus, I was not a little surprised to find, when casting my results then into classified form, that on an average nearly two new named forms appeared for each one already admitted to our flora.

Notwithstanding an intention to limit my investigation to the species of continental North America, the temptation to learn the characters of the South American species proved irresistible when, at Brussels, I examined the specimens in the personal herbarium of von Martius, whose collections have done so much to make known the flora of Brazil; and it was not long before the genus as a whole engaged my attention, though West Indian material was given less care than the other until at Dahlem I reached the collection of Professor Urban, who in 1897 had published a revision of all of the West Indian Loranthaceae. To my keen satisfaction, I then found that for the Antillean region very few forms were to be differentiated from those admitted by Urban, confirming my judgment that the large increase in our own flora rested upon previous neglect of application to them of characters which appear to be really differential, rather than to excessive optimism concerning their separability. The thorough study of tropical forms by Eichler in his revision of Loranthaceae for the Flora Brasiliensis, in 1868, supplemented by a reëlaboration of available material when Urban monographed the West Indian forms, has also prevented an increase in the number of South American species at all comparable with that within our own flora, though the number of names added is relatively greater than for the Antilles.

The essential characters of *Phoradendon* in its group of Loranthaceous genera are chiefly its axillary spikes of sessile unisexual and monochlamydeous small flowers often sunken in hollows of the frequently swollen internodes of the rachis, normally trimerous, with 2-celled longitudinally dehiscent anthers. A very few species, like *P. cymosum*, present the phenomenon of a terminal spike corresponding to the 1- or few-flowered cyme of the old world *Viscum*, but in addition to axillary spikes. Except in the species taken by Hooker for *falcatum*, the receptacular cups, which range from so shallow as hardly to surround the base of the flower to a depth covering a noticeable part of the mature fruit, are essentially even on their margin; but in this species the cup is parted so as to present sometimes the appearance of a deeply divided calyx. The flowers, with a small vestigial nectar-gland and apparently adapted to pollination by such short tongued insects as flies and small bees, are usually yellowish green when expanded, but in *P. Brittonianum* and some of its relatives the sepals are blood-red even before anthesis. Some species are known to be apogamous and apogamy is to be expected in many others, a circumstance very probably connected, as in *Taraxacum* and *Hieracium*, with polymorphism or close affinity in species as now understood.

In all of our own species the plants are strictly dioecious and, as a rule, staminate spikes are longer than pistillate and bear more flowers. This is known to be true also of a number of tropical species, such as *P. Wattii*: in others, prevailingly if not exclusively staminate and pistillate spikes showing something of the same dimorphism occur monoeciously on the same plant. Though usually not too closely applicable as between related species, the number and arrangement of the flowers on a given spike present equally characteristic differences, but with the qualification that flowers of the uppermost joints may be fewer in number and simpler in grouping than below, while one or two of the lower joints may be partly or entirely without flowers, the lowermost almost universally being reduced to a sterile peduncle. The greater number of tropical species differ from those of the north in being androgynous through the occurrence of a number of staminate flowers on spike-joints that are otherwise pistillate, or, less commonly and sometimes differentiated by the term 'gynandrous,' through the occurrence of a few pistillate flowers on otherwise staminate joints,—as many of Eichler's accurately drawn plates show very beautifully. Except in a broad way, these differences do not appear to be practically applicable in contrasting species though representing in part morphological differences of fundamental taxonomic value. The prevailing grouping of the flowers is in 2, 4, or 6 series on each joint of the spike; i.e., in 1, 2 or 3 ranks over each of the two scales by which it is subtended. Examples of the first and last are given by *P. laxiflorum* (2), and *P. flavescens* (6), and where the joints are unisexual these numbers commonly prevail, though four series may be found by reduction and as many as ten by increase when the number is typically six. When the joints are androgynous, the staminate flowers often occur at top between the normal two ranks over each scale, and this condition is usually accentuated on luxuriant spikes and sometimes on all by the downward intrusion of a partial or complete third series over each scale. For the separation of the groups into which tropical species fall, I have found it most convenient to use the prevalence of 2 or 6 series of flowers on the joint as a differential, providing as an intermediate the prevalence of the interjected two series under the designation 4 + 2. A glance at *P. domingense* (2), *P. trinervium* (4 or 4 + 2), *P. hexastichum* (6) and *P. Lindavianum* (6–10) will make these distinctions evident,—

more than 6 ranks being very exceptional except in some tropical species with leaves venulose above and dull beneath, and in some of our northern forms.

One of the characters most available and significant in the classification of the species of *Phoradendron* is a fundamental difference in their leaves. By far the larger number of species have unmistakable leaves, but our western group to which *P. californicum* and *P. juniperinum* belong have their foliage reduced to short thin scales which resemble the leaves of the related genus *Arceuthobium* or *Razoumofskya* so closely that species of either genus are commonly to be found in herbaria as representative of the other. Unlike typical foliage leaves, these scales do not disarticulate, though a constriction at the base of the scales in two forms affords partial ground for their specific recognition: one species of the Mexican mountains, *P. minutifolium,* has almost equally small though fleshy disarticulating leaves, and two of the South American species, *P. tunaeforme* and *P. fragile,* are characterized by bearing small scale-like leaves only. Such species are very likely to be mistaken for some representatives of the related genus *Dendrophthora,* which differs technically in its 1-celled anthers.

If any species of the United States, for example *P. Eatoni* of the everglades of Florida, is compared with any West Indian or South American species, for example *P. rubrum* of the Bahamas, the latter will be found to possess constantly in addition to its foliage one or more pairs of scale-leaves at least on the lowermost foliage internode of every branch. Comparable with the scales of the flowering spikes and with the stem-scales of *P. juniperinum,* etc., these cataphyls afford by their presence or absence what proves to be one of the most important of characters for the primary division of the genus *Phoradendron.* Usually cataphyls do not subtend flowers or spikes, apparently serving no function further than the protection they may afford the shoot in its earliest development; but in *P. crassifolium* and *P. craspedophyllum* spikes are regularly and characteristically found in the axils of some of the cataphyls, and less characteristically in a few other cases.

Never found on any species of the United States, absent from three-fourths of those of Mexico and Central America, but invariably present on all of the South American and West Indian species, these scales are usually confined in the latter to the basal joint of each branch, though in cases of true or cymose forking they are found on all joints—since only basal joints are then present. In a very small percentage, only, of the tropical species with percurrent or monopodial branching, e.g., *P. flavens* and *P. crassifolium* and their allies, cataphyls are found on all foliage internodes; and in a single known species, *P. paradoxum*, the stem is made up of rather terete joints with cataphyls and ancipital joints without them, in regular alternating succession.

In the geographic distribution of its species, *Phoradendron* is rather unusually instructive. The genus is strictly American and extends from Washington, Colorado, the mouth of the Ohio River, and southern New Jersey to the Southern Argentine region on the continent, and through the entire West Indian chain; one species occurs in the Pacific island Guadalupe, and two are found in the Galapagos group of Pacific islands,—both oceanic but with American floras. None of its many species of fairly homogenous character possesses a very wide geographic range. Marked examples of widespread occurrence are afforded only by a few such polymorphous species as what is usually called *P. latifolium*, or an assemblage of intricately related if differentiable species like that usually known as *P. rubrum* or *P. quadrangulare*, both of which range from Brazil to Central Mexico and well through the West Indies. Few species, indeed, equal in range our native *P. flavescens*, which occurs from southern New Jersey to the lower Wabash, Oklahoma and eastern Texas, reaching southeast to the Gulf and ocean.

Admirably endowed with means of free dissemination through its edible berries with extremely viscid pulp, which leads to their dispersal by birds, these mistletoes seem limited nevertheless to a surprising extent by ordinary barriers to plant migration. Like the similar European *Viscum album*, with its scarce-definable races capable of effective germination only on the host-species from which the seed came, our eastern *P. flavescens* though attacking a large variety of host plants is usually found confined to a single host in a given region, and such experiments as have been made on it show that it can be transferred from one host to another with difficulty if at all. How far this may be concerned in the polymorphism of this species and how far its like may serve to limit the dispersal of most species is at present a matter of conjecture only.

Viewed on broad geographic lines, the species of *Phoradendron* usually occupy areas that present severally an assemblage of fairly uniform meteorologic features with limiting environment,—in this respect agreeing with most other plants and with animals. In the main, the regions affected by individual species of *Phoradendron* are the following:

North America: (1) Atlantic, (2) Rocky Mountain, (3) Pacific, (4) Great Basin or Sonoran,—in the United States; (5) Table-land, (6) Eastern or Western Sierra Madre, (7) Western riparian, (8) Yucatecan, and (9) Cordilleran,—in Mexico and Central America.

South America: (10) Andean, (11) Peruvian and Bolivian, (12) Argentine, (13) Brazilian upland, apparently with less differentiation than would be thought between the arid and humid portions, (14) Amazonian, (15) Cayenne, and (16) Venezuelan.

West Indies: (17) Caribbean, (18) Antillean, to the north of the Anegada passage, and (19) Bahamian.

Few species range throughout any one of these regions, and it is very rare for a species to reach from one into the other.

Briefly summarized, the purely taxonomic part of my study of the genus leads to the conclusion that *Phoradendron* may be best divided into two primary groups, respectively constantly without and constantly with cataphyls on their foliage shoots: for the first I am using the name BOREALES, since its species alone are represented in the north; and for the other, AEQUATORIALES, since its species only are found in the equatorial region. Species destitute of expanded foliage are found in each group in small numbers. Those of the first group are pubescent for the most part, while only two of the second group are more than papillately roughened. The Boreales appear to be strictly dioecious; the Aequatoriales for the most part, though not exclusively, are monoecious, usually with some or all of their spikes androgynous.

So far as shown by the material now contained in the great herbaria at Washington, New York, St. Louis, Chicago, Brussels, Copenhagen, Kew, Munich (where von Martius' official collection is), Geneva, Buda-Pest, Prag, and Dahlem, and in many smaller collections, I find a total of 262 differentiable forms, most of which I regard as species, and of which 60, or 23 percent, are of the Boreales and 202, or 77 percent, Aequatoriales. Of the Boreales, 41, or two-thirds, and of the Aequatoriales, 85, or two-fifths, are characterized as new.

The distribution of the main groups (species which occur in more than one region being included in each) is:

	U. S.	*Mexico*	*Centr. Amer.*	*W. Indies*	*S. Amer.*
BOREALES	26	45	1	0	0
AEQUATORIALES	0	25	23	34	124
	26	70	24	34	124

THE MORPHOLOGY AND RELATIONSHIPS OF PODOMITRIUM MALACCENSE. (STEPH.)

By Douglas H. Campbell

DEPARTMENT OF BOTANY, LELAND STANFORD UNIVERSITY

Presented to the Academy, October 30, 1914

There are but two species of the liverwort, Podomitrium, *P. phyllanthus*, a common species of the Australian region, and *P. malaccense*, which according to Stephani is known only from Singapore and New Caledonia. The writer collected the latter species in Sarawak, Borneo, in February, 1913, and also at one station in Luzon, in May of the same year. The plant is common in Sarawak, and sufficient material was secured to make a pretty complete study of its structure, which has not been done before.

Podomitrium is often included in the genus Hymenophytum, but it is certainly generically distinct.

The sterile plants of *P. malaccense* resemble exactly certain species of Blyttia, and the plant can only certainly be recognized when it bears the reproductive organs. Fertile plants, however, are immediately distinguishable by the position of the reproductive organs, which instead of being borne on the dorsal surface of the ordinary shoots, are formed upon the special branches which grow from the side of the midrib upon the ventral side of the main shoot, very much as in Metzgeria.

As in Blyttia, the thallus has a very conspicuous, and sharply delimited midrib, and wings which consist of a single layer of cells. In *P. malaccense*, the thallus is usually unsymmetrical, one of the wings being almost obliterated. Rather short rhizoids occur upon the ventral side of the midrib, but these are not uniformly distributed. They occur in dense patches separated by considerable intervals which are quite free from rhizoids.

In the form of the apical cell, and in the character of the thick-walled cells which occupy the axis of the midrib, Podomitrium closely resembles Blyttia.

The fully developed plant may reach a length of about 4 cm. with a maximum width of about 4.5 mm. No cases of dichotomous branching were seen, all of the branches appearing to be of adventitious origin, springing like the sexual shoots from the sides of the midrib.

The antheridia arise in somewhat irregular rows, one on each side of the midrib of the special shoot upon which they are borne. They arise in acropetal succession, and each is covered by a scale with laciniate margins.

Both in structure and position the antheridia are much like those of some species of Blyttia, and perhaps still more like those of *Calycularia radiculosa.*

The archegonial branches are very short, and bear but one receptacle which is relatively very large. The receptical is very much like that of Blyttia, having a similar large involucre, within which is formed a second envelope, the perianth. The latter can be seen in the young receptacle as a shallow ring enclosing the group of archegonia. If one of these is fertilized, the perianth begins to grow very rapidly, and soon becomes a conspicuous tubular sheath enclosing the developing sporophyte.

The young embryo shows a conspicuous suspensor, and very closely resembles corresponding stages of the embryo of *Blyttia radiculosa.* As it develops, however, it is more like the embryo of Mörkia or Calycularia. This is shown in the development of a distinct foot, which is not so evident in Blyttia, and the capsule is less elongated than in Blyttia, resembling more nearly that of *Calycularia radiculosa,* with which it also agrees in the dehiscence of the capsule. In *P. malaccense,* the capsule opens by two completely separate valves, while in Blyttia (and it is said also in *P. phyllanthus*), there are four valves which remain united at the apex.

In the structure of the wall, and the markings of the spore membrane, *P. malaccense* much resembles a typical Blyttia.

It is evident that Podomitrium is much more like Blyttia, than like Metzgeria, and should be placed in the Blyttiaceæ, rather than Aneuraecæ.

THE FEEBLY INHIBITED. I. VIOLENT TEMPER AND ITS INHERITANCE

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Presented to the Academy, November 12, 1914

Recent studies have shown that the impulse to general mental development and the nervous 'strength' which successfully resists the stress of untoward conditions and emotional shocks have a clear hereditary basis. The hereditary factor behaves, indeed, in typical fashion. But it has often been pointed out by sociologists, who for the most part hold differences in 'conditions of life' responsible for the differences between men, that there is no evidence that the elements of moral or social behavior have a hereditary basis, and while there have not been wanting those who have insisted that 'criminality is hereditary' yet no one has successfully determined the method of such inheritance. It appears, indeed, improbable that so complex a thing as criminality should prove to have a single hereditary factor. An attempt has been made to study the hereditary behavior of some of the elements of moral action—to analyze the family history of persons who have marked emotional traits.

Opportunities for such study have been afforded by 165 family histories of wayward girls in state institutions gained by trained field workers who visited the homes of the patients and got as full an account as possible of the behavior of all of the close relatives. The data were gathered without prejudice; indeed, it was impossible for the 'field worker' to know what laws of inheritance the histories might yield. Also, many other sources of information at the Eugenics Record Office have been drawn upon for additional data as to the inheritance of special traits.

Violent and more or less periodic outbursts of temper occur in families which are characterized by prevalence of epileptic attacks, also in those exhibiting cases of mania, also in others in which 'hysterical' attacks are common. The special form of the attacks differs somewhat in these classes of families, but the method of inheritance of the tendency is the same in all, and it seems probable that in each class the simple symptoms of the emotional outburst are modified by the differences in these three classes of the nervous condition.

The method of inheritance is indicated at once by the fact that, in the 66 family histories studied, the tendency to outbursts does not, typically, skip a generation. In one history it is traced through 5 generations; in a large proportion of the histories it is traced through 3 consecutive generations. The few cases in which neither parent of an affected individual is reported to have the tendency to outbursts are explained by obvious insufficiency in the record.

The fact that the tendency to outbursts of temper does not skip a generation indicates that it is a positive or dominant trait. That segregation of this tendency occurs is shown by the ratio of affected offspring in any fraternity to the total number of offspring whose emotional history is fully described. From the mating of an uncontrolled and a normal person expectation is that 50 percent of the children will be uncontrolled. A summation of all such children gives a total of 106 affected among 219 sufficiently described, or close to the 50 percent expected on the hypothesis that the tendency to outbursts of temper is a simple, positive trait.

The detailed investigation will appear in the *Journal of Nervous and Mental Diseases* and in the *Bulletin of the Eugenics Record Office.*

THE ENGLISH RABBIT AND THE QUESTION OF MENDELIAN UNIT-CHARACTER CONSTANCY

By W. E. Castle and Philip B. Hadley

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Whatever the theoretical importance of Mendel's law, its practical utility depends largely upon the purity of the gametes. If Mendelian unit-characters can through hybridization be recombined in desirable ways *without essential modification* during the process, Mendel's law is evidently a distinct acquisition to the practical breeder. Nevertheless if crossing is likely to produce considerable changes in the characters which it is desired to combine in a new race, it is evident that Mendelian crosses must be used judiciously and with caution by the practical breeder.

Considerations such as these have led W. E. Castle for several years to concentrate his studies of genetic problems upon the question of gametic purity. As a crucial experiment he conceived the plan of deriving an entire race of animals, not from a single *pair* of ancestors, but from a single gamete, so far as concerns a particular unit-character. It was thought that in a race so derived, if the principle of gametic purity holds, there should be no variation whatever in the particular unit-character concerned.

Color patterns of mammals seemed especially well adapted for such studies, since they are early differentiated and clearly Mendelize in crosses. The so-called 'English' piebald rabbit presents an especially fine example of such a color pattern. The figures give a good idea of this striking pattern in which white and colored areas are interspersed much as in the 'coach-dog.' It would be a distinct gain to breeders if they could reduce the variation in details of the English pattern so that 'prizewinners' could be bred without the production of so many 'wasters,' which depart in essential points from the standard pattern adopted for the breed. This was an additional reason for undertaking work with the English rabbit.

The first standard-bred English rabbits which Castle had under observation, when mated *inter se*, produced young of three sorts. About half the young were fairly good 'standard' English extensively marked with colored spots (see fig. 3). About one-fourth were much whiter than the standard demands, their spots being fewer and smaller (see fig. 1). And the remaining fourth were without spots, that is were self colored. This last class was found to be recessive and not to produce English offspring, if mated *inter se*.

The *whiter-than-standard* English proved to be homozygous for the pattern, the 'standard' English being heterozygous and breeding like their parents.

From these observations it was clear (1) that the English pattern is a Mendelian dominant and (2) that the breeding of English rabbits resembles that of blue Andalusian fowls. For the standard bred animal is a heterozygote in the production of which there is bound to be a constant production of 'wasters' unless either the standard is changed or the homozygote can be changed to conform with the standard, producing an animal with more color. In the latter case homozygotes could be bred with each other and wasters eliminated. The question whether the pattern can be changed becomes therefore one of practical as well as theoretical interest.

In making crosses of English with other breeds of rabbits, there was found to be considerable variation among the heterozygous English produced, some being much whiter than others; i.e., having less extensive colored spots. Plus (dark) and minus (light) selections were made to see to what extent the pattern was capable of modification. These selection experiments are still in progress but will be reported upon at another time.

The single-gamete experiment, with which this report will deal, was placed in the hands of P. B. Hadley, who has carried it out at the Rhode Island Agricultural Experiment Station.

As foundation stock for the experiment a single *heterozygous* English rabbit of standard character (grade 2, fig. 5) was selected. To mate with him, it was desired to obtain a distinct breed of rabbits, free from the English pattern, and as pure (uniform) in all respects as possible. For this purpose the 'Belgian hare' was chosen. A buck and two does obtained from G. W. Felton, Cliftondale, Mass., were found to breed very true. From them was bred a stock of does very uniform in character, twelve of which, together with one of the parents (2A) were mated with the selected English buck which we may henceforth call by his record number ♂21A. The young thus produced will be called 'Series I' offspring. About half of them were self (non-English), the remainder (187 in number) were English.* The latter, although all undoubtedly heterozygous, varied in whiteness from grade 1 to grade 4, figures 1 to 4, the *modal* or commonest condition being about the same as that of the father (grade 2). The distribution of the young in relation to our grades is shown in the Table. Statistical treatment of the table gives the *average grade* of the young as 2.43, that is somewhat *darker* than the father. Inspection of the table shows that more than half of the young are darker than the father, which supports in a general way the statistical average grade. If we consider separately the average grade of the young produced by each mother, we find that it ranges from 2.15 in the case of ♀18F, which had 5 English young, to 2.79 in the case of ♀16D which had 14 English young. The average number of young to a mother is 14.4.

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* The total number of young obtained from ♂ 21A, when mated with Belgian hare does has been to the time of writing 436. The English young now number 210, the non-English (self) number 226. For Series II matings presently to be described the corresponding numbers of young are: English 219, non-English 196, total 415. For Series I and II combined the numbers are : English 429, non-English 422, total 851. This is unmistakably a 1 : 1 Mendelian ratio.

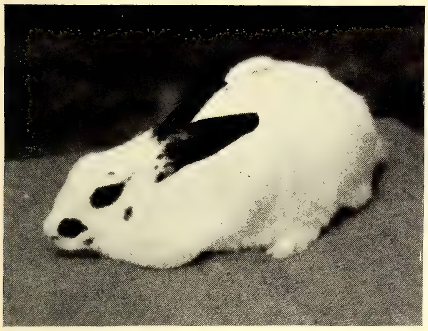
TABLE SHOWING THE DISTRIBUTION OF GRADES OF OFFSPRING IN THE FIRST AND SECOND SERIES OF MATINGS FOR EACH INDIVIDUAL MOTHER

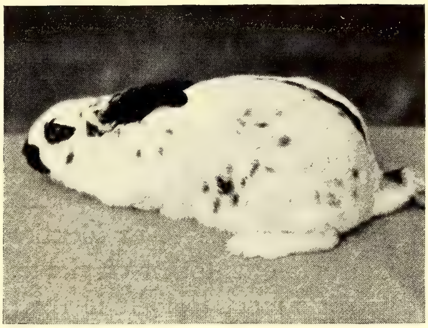
MOTHER	SERIES	GRADES OF YOUNG													TOTALS		AVERAGE
		1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	Ser. I	Ser. II	
16A.........	I					1	2	2							5		2.30
	II					**3**	**1**	**1**	**2**	**1**	**7**	**3**	**3**			**21**	**3.03**
16B.........	I		1		1	3	1	1	1	2			1		11		2.39
	II			**1**	**1**	**2**	**1**		**1**	**2**	**1**	**3**				**12**	**2.67**
16D.........	I					2	5			1	2	3	1		14		2.79
	II								**3**	**1**	**6**		**2**			**12**	**3.19**
16E.........	I			2		5		2	1	1			1		12		2.29
	II										**1**	**3**	**1**			**5**	**3.50**
16F.........	I	1				2	1	5	3	2		3			17		2.62
	II				**1**	**2**	**1**	**3**	**3**			**1**				**11**	**2.48**
16G.........	I		1	2		2	3	1	2	3	1				15		2.35
	II						**1**	**2**	**2**	**3**	**1**	**2**	**1**	**1**		**13**	**3.06**
16H.........	I			1		3	3	1	2	2	2	1	2	1	18		2.76
	II					**3**		**2**	**1**	**1**	**8**	**2**	**2**	**1**		**20**	**3.06**
17E.........	I		1	1	3	4	3	3	1	1	2	2			21		2.36
	II					**1**	**2**	**5**	**5**	**3**	**4**	**4**	**2**	**1**		**27**	**2.97**
17G.........	I			1	9	6	3	3	2	1	2	1	1		29		2.27
	II		**1**	**1**		**1**	**1**	**1**	**1**	**1**	**1**	**1**				**9**	**2.44**
18D.........	I		1			3	3	3		1	2	2			15		2.53
	II					**2**	**1**	**2**	**3**	**1**	**2**	**5**				**16**	**2.91**
18F.........	I				3	1						1			5		2.15
	II				**1**	**1**	**2**		**4**	**2**	**1**	**1**	**2**	**2**		**16**	**2.97**
18H.........	I			1	1		3	1	1	3					10		2.43
	II				**1**		**3**	**3**	**4**	**2**	**3**		**1**	**2**		**19**	**2.87**
2A.........	I		1	2	1	1	4	2	3	1					15		2.22
	II				**1**	**2**				**1**	**2**	**2**				**8**	**2.78**
Totals......	I	1	5	10	18	33	31	24	16	18	11	13	6	1	187		2.43
	II		**1**	**2**	**5**	**17**	**13**	**19**	**29**	**18**	**37**	**27**	**14**	**7**		**189**	**2.92**

After this series of matings had been completed, a second series was begun in which the same 13 females were mated with one of the darkest bucks produced in the Series I matings (a son of ♀16 E). The selected buck was ♂40A (fig. 6) grade 3.75, considerably darker than his father (fig. 5). This series of matings produced 189 English young, together with a like number of self (non-English) young. The grade distribution of the English young is shown in Table 1, Series II. All of the 13, mothers except one (♀16F) produced darker offspring in the Series II than in the Series I matings. The lowest average grade was shown by the young of ♀17G, viz. 2.44. For Series I matings, the lowest average was 2.15. The highest average grade in the Series II matings was given by the young of ♀16E, viz. 3.50. For Series I matings, the highest average was 2.78. Consequently both maximum and minimum averages were higher in the Series II than in the Series I matings. The grand average of all the 189 Series II offspring was 2.92, as compared with 2.43, the average grade of the Series I young. The modal grade of the Series II young is 3.25. The modal grade for Series I was 2.00. Since the mothers were identical in both series, the difference in the young can be attributed only to the difference in the fathers. The male used in the Series II matings differed genetically as well as somatically from his father who sired the Series I young. Not only was he darker, but he also produced darker English young. Yet the father contained only a single dose (one gamete) of English pattern and the son derived his English pattern exclusively from this same source. Hence the English unit-character had changed quantitatively in transmission from father to son. This seems to us conclusive evidence against the idea of unit-character constancy, or "gametic purity." If unit-characters are not constant, selection reacquires much of the importance which it was regarded as possessing in Darwin's scheme of evolution, an importance which many have recently denied to it.

The question whether an imaginary 'unit-factor' for English pattern has or has not changed in correlation with the visibly changed English unit-character is not here discussed. We recognize that it has an academic interest, which however scarcely affects the practical question whether the visible Mendelizing characters of animals are subject to change through crossing or through selection or both.

1





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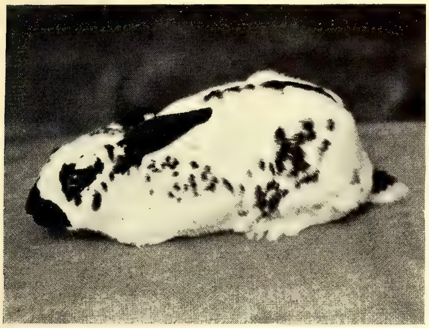

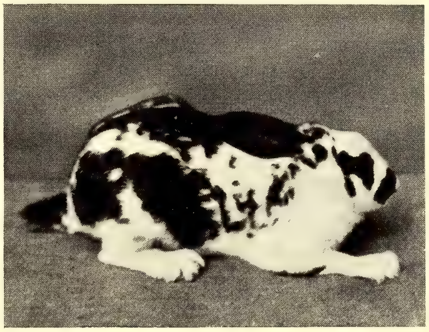

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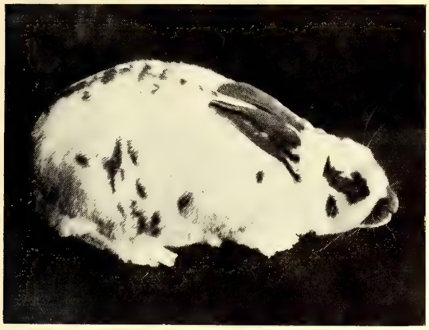

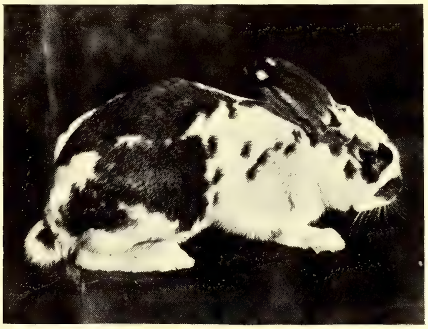

6

FIGS. 1-4, PHOTOGRAPHS OF FOUR ENGLISH RABBITS FORMING A GRADED SERIES IN AMOUNT OF COLORED FUR.

These were adopted as our standard Grades 1-4, in terms of which all the rabbits described in these experiments have been classified.

FIG. 5, RABBIT ♂21A, FATHER OF THE SERIES I YOUNG.

FIG. 6, RABBIT ♂40A, FATHER OF THE SERIES II YOUNG.

ON THE IDENTITY OF HELIOTROPISM IN ANIMALS AND PLANTS

By Jacques Loeb and Hardolph Wasteneys

ROCKEFELLER INSTITUTE FOR MEDICAL RESEARCH, NEW YORK

Presented to the Academy, November 12, 1914

1. Since 1888 Loeb has in a number of publications expressed the idea that the apparent attraction of animals by light is in reality a phenomenon of automatic orientation of the animals by a source of light, comparable to or identical with the well-known phenomena of heliotropic orientation of plants. Thus he proved that sessile animal organisms react to light in the same way as sessile plants, namely, by bending or growing towards (or away from) a source of light; while motile animals move towards (or away from) a source of light as do the motile swarm spores of certain algae.

He based his theory of these reactions in animals on the following three assumptions. First, the light acts chemically upon the photosensitive elements of the surface of the body (eyes or skin). Second, symmetrical elements of the surface are identical chemically, so that if one source of light is given and symmetrical elements are struck at the same angle by the rays emanating from one source of light, the velocity of the photochemical reactions in the symmetrical elements is identical; if, however, they are struck at different angles, the velocity of chemical reactions is no longer the same in symmetrical elements.

Third, the velocity of photochemical reactions in the eyes or the skin influences (through the nerves or, as the case may be, other protoplasmic conductors) the tension of the muscles (or other contractile elements) connected with the photosensitive elements at the surface of the body. If symmetrical photosensitive elements are struck by the light at the same angle (and only one source of light exists), the symmetrical muscles of the body are influenced by the light in the same way and no change in the direction of the motion of the animal will occur. If, however, symmetrical photosensitive elements are struck at different angles the velocity of chemical reactions will not remain the same in the symmetrical elements and hence the tension of symmetrical muscles connected with these elements will not be the same; as a consequence, when the animal moves it must show a tendency to deviate from the straight line until finally its axis or plane of symmetry goes through the source of light again. When this happens the symmetrical elements of the photosensitive surface are again struck at the same angle by the light and there is now no more reason for the animal to deviate from this direction.

If this theory of heliotropism were correct, it was necessary to show that the law of photochemical action should hold for the heliotropic reactions of plants as well as of animals. This law (which was first established by Hankel for a narrow range of light intensity and extended by Bunsen and Roscoe over a wider range) says, that within certain limits the photochemical effect of light is equal to the product of intensity into the duration of illumination. Five years ago it was shown independently by Blaauw[1] and by Fröschl that the heliotropic reactions of plants obey the law of Bunsen and Roscoe, i.e., that the time required to bring about the heliotropic curvature of plants changes inversely with the intensity of illumination. A year ago Ewald and Loeb[2] showed that the same law holds also for the heliotropic curvature of an animal, namely, that of the polyps of *Eudendrium.*

2. Although the question of wave length does not enter into Loeb's theory of heliotropic orientation, it seemed of interest to compare the relative efficiency of the various parts of the spectrum upon the production of heliotropic curvatures in *Eudendrium* with that found for the heliotropic curvatures in plants. The most exact measurements on plants are those by Blaauw on the seedlings of *Avena.* Blaauw used a carbon arc spectrum, and found that the most efficient part of the spectrum was a region in the blue between 466 and 478 $\mu\mu$. In this region an exposure of only 4 seconds sufficed to call forth heliotropic curvatures in 50 percent of the stems of the seedlings. For longer waves a longer exposure was required, thus for a wave length of 499 $\mu\mu$ an exposure of 120 seconds, and for a wave length of 534 $\mu\mu$ an exposure of 6300 seconds. The yellow and red parts of the spectrum seemed absolutely ineffective.

We undertook last summer a series of experiments to ascertain the relative efficiency of the various parts of a carbon arc spectrum on the newly regenerated polyps of *Eudendrium.* The spectrum was thrown on a glass trough with parallel walls which contained a row of *Eudendrium* stems with a number of newly regenerated polyps. The position of each polyp was noted at the beginning of the experiment. Loeb and Ewald had found that the minimal time of exposure to produce heliotropic curvatures in more than 50 percent of the polyps is, for the same intensity of light, considerably greater than in the case of the seedlings of *Avena.* It required an exposure of at least 5 minutes in order to call forth heliotropic curvatures of the polyps of *Eudendrium* in the spectrum. The experiments were rendered difficult by the fact that the young polyps are very delicate and suffer easily. If a stem with sickly polyps is included in an experiment it is liable to disturb the result. It was found that for the polyps of *Eudendrium* the most efficient region of the spectrum was in the blue in the neighborhood of 4735 angstrom units, which coincides with the region of maximal efficiency found for the seedlings of oats by Blaauw; while the region from 4900 to 5300 and from 4690 to 4400 or below was still effective, but less than the region in the blue near 4735. The red and yellow rays were practically ineffective, at least an exposure of over 5 hours to the red and yellow rays induced no curvature. The following table may serve to illustrate this statement.

The fact that in the first series (5-minute exposure) one polyp bent to the light in the red was, as the other experiments showed, merely an accident. The region of maximal efficiency was in the blue of about 4735 A.

TIME	WAVE-LENGTH	COLOR	POLYPS BENT TO THE LIGHT		TIME	WAVE-LENGTH	COLOR	POLYPS BENT TO THE LIGHT	
	Angstroms		Fraction	Per cent		Angstroms		Fraction	Per cent
5 min.	6500 A	red	1/29	4	15 min.	6700–5400	red to yellowish green	0	0
	6000	yellow	0/4	0		5400–4900	yellowish-green and green	14/37	38
	5700	yellow	0/13	0		4900–4100	blue and violet	72/95	76
	5300–5345	yellowish-green	5/15	33		4100–3700	violet	14/30	46
	5100	green	3/12	25	$5\frac{1}{2}$ hrs.	6560–6000	red and orange	0/32	0
	4900	blue	11/32	35		5720	yellow	0/21	0
	4735	blue	30/49	62		5500–5300	green	3/13	24
	4690	blue	4/21	19		4950	blue	1/6	16
	4600	blue	5/22	23		4800	blue	7/16	44
	4400	blue	5/52	10		4700	blue	24/24	100

In the second series (15-minute exposure) no bending occurred in the red and yellow, in the green and bluish-green less than 50 percent bent to the light, in the blue 76 percent, and in the violet less than 50 percent.

The final series ($5\frac{1}{2}$-hour exposure) is given to illustrate the fact that even an exposure of $5\frac{1}{2}$ hours does not induce any bending in the red and yellow, while in the blue in the region of 4700 all the polyps were bent to the light. It should be mentioned that this effect was already reached in the region of 4700 A. in 160 minutes; after an exposure of 10 minutes already 73 percent were bent to the light.

These experiments show that the relative efficiency of the different parts of the spectrum of a carbon arc light for the production of heliotropic curvatures in the animal *Eudendrium* and in the seedlings of the plant *Avena* is practically identical.

[1] Blaauw, *Rec. des Trav. botaniques Néerlandais*, 5, 209 (1909).

[2] Loeb and Ewald, *Zentralbl. f. Physiol.*, 27, 1165 (1914).

ARCHAEOLOGY OF BARBADOS

By J. Walter Fewkes

BUREAU OF AMERICAN ETHNOLOGY, SMITHSONIAN INSTITUTION

Presented to the Academy, October 30, 1914

Our knowledge of the extent, character, and relationship of the prehistoric population of Barbados is very indefinite. It is not known whether the island was inhabited when discovered by the Portuguese in 1505, but there is every reason to believe it was, for a few aborigines still remained when it was colonized by the English over a century later. The existence of a native population is shown by Lignon's map published in 1657, thirty-one years after the English landed at Holetown, on which we find legends referring to Indian settlements. Several writers assert that while a limited number of Caribs were found by the first English colonists, these should be regarded as transient visitors from neighboring islands, rather than permanent occupants.

Archaeological evidences of a considerable prehistoric population in Barbados before the advent of Europeans are somewhat more definite than historical. These have already been presented by Greville T. Chester, and other archaeologists who have described many shell celts collected on Barbados. They have also brought to the attention of students numerous village sites that show evidences of a long continued occupation.

In a brief reconnaissance* made on Barbados by me in 1902, an examination was made of the archaeological evidence and data gathered bearing on the age and nature of the culture it revealed. An attempt is here made to determine the relation of this material to that found on the other West Indies. I am convinced from this examination that Barbados had a large local population in prehistoric times, and that its culture, with certain significant differences, resembled in general that on the other West Indies.

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*These studies were made under the auspices of the Heye Museum of New York and the Bureau of American Ethnology of the Smithsonian Institution; a more complete account, amply illustrated, will be published later by the Bureau.

The presence of an aboriginal population in Barbados is indicated by the numerous prehistoric objects collected in caves, natural and artificial, mounds, middens, and other village sites. The number of prehistoric implements found in these localities on Barbados, is so large that there can be no doubt that the island once had a great aboriginal population. These objects were made of shell, and were formerly so abundant that, according to several informants, they were ground up and used for road beds. Even now a considerable number can be secured from the natives at a few pennies apiece, for the asking.

There are several well-marked types of shell implements, so characteristic that there is no doubt they are of local manufacture. Stone celts are very rare for the reason that suitable material for their manufacture does not exist in any place on the island. The few that occur were evidently brought to Barbados from over the sea. This abundance of shell implements and relative paucity of those made of stone has an important bearing on whether the aborigines were transient visitors or permanent inhabitants. They must have been locally made by the latter, for shell implements are rare on the nearest islands where stone implements are very abundant.

Evidences that the aborigines of Barbados formerly inhabited natural caves are many. On the roofs of these caves there still remain marks of smoke indicating former fires, while in their floors, especially near the entrance, numerous artifacts have been excavated. Joseph Forte found over a hundred implements in one cavern, 350 feet above the level of the sea. Although the material out of which they were made is different, the forms of these objects show that the culture of the cave dwellers resembled that of cave people found elsewhere in the West Indies, although much more varied.

While there is nothing in the natural caves and their artificial contents which shows that the aboriginal culture of Barbados was exceptional, the claim that artificial caves exist is important; for if it can be successfully proved that the aborigines made these caves, Barbados is the only West Indian island where artificial cave dwellings were made. The existence of this kind of cave from its unique character is naturally doubted by many, and while some writers deny their artificial origin, or ascribe them to Europeans, others take the opposite view. The examinations made by me lead me with some hesitation to adopt the latter conclusion.

Three groups of artificial caves, supposed to have been the work of historic aborigines, were seen by me on my visit to Barbados. One of these, and the most problematical, has been known for several centuries from its site as Indian Castle; the others are pit-like excavations near Bridgetown on the right bank of Indian River, and at Freshwater Bay.

The so-called Indian Castle is situated northeast of Speightown, on the Pleasant Hill Estate, about three miles due east of Six Mens Bay. As one leaves the town above mentioned the road rises gradually to a hill and passes the 'Castle,' revealing the entrance to the cave on the right-hand side in the cliff above the road. The cave opens through the *archway with a key-stone* on which is cut a rude figure in relief. Both entrances and arch have smooth perpendicular walls with well made angles in each corner, the whole chamber cut out of the solid rock. The subterranean room has lateral recesses in its walls and smaller mural niches. An opening in the right-hand wall communicates with a well, also excavated in rock, but open above to the sky.

The whole appearance of this excavation, especially the conical apex of the second chamber, led me at my first visit to regard it a lime-kiln made by Europeans, but it greatly differs from one of these pits in several particulars. This cave has been known for many years. The Reverend Griffith Hughes speaks of this place in 1750, or about 125 years after the island was colonized, as follows: "A very commodious one [cave] in the side of a neighboring hill called to this day Indian Castle and almost in a direct line from Six Mens Bay, ..." "Among several broken fragments of idols said to be dug up in this place," continues the Reverend Hughes, "I saw the head of one which alone weighed 60 pds. weight." An adjacent pond, he writes, "since the memory of oldest inhabitants has been called Indian Pond." We have in fact circumstantial evidence that the cave is aboriginal, and yet not ample data to prove that the cave was excavated by Indians in prehistoric times.

There is, however, more trustworthy evidence that the other excavated chambers in the Barbados ascribed to prehistoric man were made by the aborigines. Unlike the Indian Castle cave and natural caves above mentioned, the 'Indian excavations' are dug into the surface rock like pit dwellings. They lie near undoubted Indian middens or sites of aboriginal settlements which tells in favor of their aboriginal character. Like the Castle cave they also are ascribed by some authors to early colonists, but the majority call them Indian excavations and Carib graves. It is instructive to note that abundant evidences of aboriginal occupation occur in their immediate vicinity, and there appears no good reason to question the theory that their local name, Indian excavations, correctly ascribes them to Indians.

Mounds or middens of considerable size, indicating aboriginal settlement, occur at many points on Barbados, especially the lee shore. The majority are situated on the plains skirting the coast, but many are found inland, especially where the soil invites the agriculturalist. Among these may be mentioned: (1) Near St. Luke's Chapel; (2) Indian River; (3) Freshwater Bay; (4) Codeington Estate Springs; (5) Maxwells; (6) New South Point Lighthouse; (7) Three Houses; (8) Marl Hill; (9) Speightown; (10) Holetown, and many other places. On account of long cultivation of the fields the size and appearance of many aboriginal sites have been more or less obscured, although their former existence can still be discovered.

A few undescribed specimens of aboriginal pottery found in these places are valuable in determining the culture of those who inhabited these localities. One of the most perfect of these having a graceful form like the well known Arawak "monkey vase," was found by Mr. Taylor at Indiantown near Bridgetown. This object can hardly be distinguished from Arawak pottery found in Grenada and St. Kitts. Another large flat fragment of pottery, from the same locality has the form of a griddle, resembling those on which the Arawaks bake their cassava cakes. A clay idol resembling one found in Porto Rico is suggestive of similar forms of idolatry in the two islands.

The general characters of aboriginal life in Barbados as revealed by these and many other archaeological remains, show that the culture was somewhat different from that of Porto Rico, the neighboring Lesser Antilles, or the coast of South America, but that the prehistoric people belonged to the same Carib-Arawak stock that inhabited these localities in prehistoric times. The number and variety of the artifacts show that the prehistoric population of Barbados was large and implies that it had diminished to such an extent that when England began to colonize the island, only a few survivors remained. These prehistoric inhabitants of the Barbados were practically cave dwellers and although they had not reached the stage of culture characteristic of Porto Rico and Hayti, they were probably the only Antilleans who made artificial caverns for habitations or other purposes, and carried the manufacture of shell implements to the highest degree of perfection. Their culture was not the most advanced among the Antilleans, but it was characteristic, showing in a marked manner the influence of their geographical environment. Although there are minor differences in artifacts, indicating well marked sub-areas—of which Barbados is one—in the different West Indies there is a certain unity which stamps them as belonging to one great culture area—the Carib-Arawak—extending through the islands from the heart of South America to Florida. This resemblance extending along the east coast of Central America to the Maya area, with which it has close affinities, and along the north coast of South America to the mouth of the Amazon, is practically identical with the Arawak.

CULTURE OF THE NORTH AMERICAN INDIANS OCCUPYING THE CARIBOU AREA AND ITS RELATION TO OTHER TYPES OF CULTURE

By Clark Wissler

AMERICAN MUSEUM OF NATURAL HISTORY, NEW YORK

Presented to the Academy, November 2, 1914

The anthropology of North America has now reached a stage in its development where larger and broader problems can be successfully pursued. In the past, for want of data and tried methods, investigations were of necessity confined to tribal units and it was not until a considerable number of these units had been studied that any positive conclusions could be formed as to the continent as a whole. The following brief statement is a mere summary of investigations bearing upon the origin and significance of the observed distribution of certain culture traits in the northern half of the continent. The method is to study in detail the collections in museums and to correlate the results with the field-data of anthropologists and the observations of travelers.

A brief general discussion of the problem may be found in the review of the 'Material Culture of the North American Indians' in the *American Anthropologist*, vol. 16, no. 2, and in special studies, as 'The Influence of the Horse in the Development of Plains Culture,' *American Anthropologist*, vol. 16, no. 1, and the 'Material Culture of the Blackfoot,' *Anthropological Papers*, American Museum of Natural History, vol. 5, part 1. These, however, are but preliminary to the more exhaustive treatment of the problems now under investigation.

1. *Caribou Culture Defined.* The material cultures of the Indian tribes occupying the interior of Canada present striking similarities. Their chief food was animal, the flesh of the caribou. Excepting the Pacific drainage and the prairie section, the entire area of Canada, including the interior of Alaska and the Arctic islands of the north, constituted the range of the caribou. Their range and that of the

bison overlapped in the Saskatchewan and Mackenzie Basins. Throughout this extensive territory the Indians were more or less completely dependent upon the caribou for food and clothing. Even the Eskimo hunted them in summer and made their winter clothing of their skins. Naturally, the material life of the Indians, and to a large extent that of the Eskimo, was adjusted to the necessities of a caribou hunter's career. For this reason, we speak of the culture of these Indians as the caribou culture. Some of the traits of this culture are: the use of birch and other bark for canoes, vessels, etc., the toboggan and the snowshoes, dog traction, the general use of snares for taking game both large and small, the high development of the net, the soft moccasin and the hooded coat, special methods of dressing and smoking skins, and the more or less exclusive use of a tipi-like shelter.

2. *The Relation of Caribou Culture to Bison Culture.* The Indian tribes formerly occupying the plains and prairies in the interior of North America constitute a highly individualized cultural group. They were almost entirely dependent upon the bison. As we have just noted, the bison and caribou areas overlapped, the ranging habits of the two animals have certain similarities; thus they move in great herds, and in the open country, and may be captured en masse by being driven into pounds or other enclosures. Our main problem is the analytic comparison of these two cultures with a view to determining their possible historical connection.

(*a*) *Bison and Caribou Drives.* From Alaska to Greenland the methods of driving herds of caribou in pens, snares, or between the lines of hunters are in general the same. The last method is rarely used in winter, while the former is almost exclusively a winter method. Long lines of poles are set up in the snow to drive the running caribou, which show a tendency to run along such lines, but men and women may be placed behind screens to turn the leaders in case of need. By long converging lines a herd may be run over a bank into an enclosure, into snares, or on to thin ice. In a precisely similar manner the Indians of the plains impounded buffalo in winter and occasionally in summer. From the detailed accounts, it is clear that even many minutiae of manipulation are the same for the bison and caribou. Another point is that the farther north one goes among the plains Indians the more highly developed was the method of impounding, and according to early literature the Cree were the most skillful. Now it so happens that the Cree living in the forests and tundras are also skillful in driving the caribou, giving us the probable connecting link.

(*b*) *Dog Traction.* Though the dog was quite generally distributed in America it was chiefly in the bison and caribou areas that he was used for transport. The use of sledges was practically confined to the caribou area but in summer the loads were made up as packs. In the bison area and the Barren Grounds tent poles were dragged by dogs, and in parts of the former area developed into a packing drag known as a travois. In this connection we have investigated the relation of the horse to bison culture making it clear that one of the reasons why the use of the horse spread so rapidly in this area was that he could be quickly substituted for the dog without developing new culture traits. Our data show that the horse quickly reached the tribes of Canada, via the Spanish settlements of Mexico, and was carried as far north as the climate permitted.

(*c*) *The Tipi-like Shelter.* The development of this problem has been the detailed study of the structure of tipis and tents of the two areas. The most intense development of the true tipi is found in the bison area, the Mackenzie Valley, and among the Algonkin speaking tribe of Canada. The definitive points of comparison are the arrangements of the poles forming the conical foundation and the detailed cut of the skin or bark covers. An analytic comparison of structure along these lines suggests that the tipi was introduced into the bison area from the caribou area to the north.

(*d*) *Costume.* Another important part of the investigation is the study of museum collections of costume, the related techniques of sewing and skin dressing. In this case we have a contemporary, Gudmund Hatt, a young Danish anthropologist who has made an intensive study of Lapp and Siberian costumes and pointed out some correspondences in America. Our own data for the bison area in contrast to that for the eastern and western parts of the United States make it clear that we have in this area alone a development of skin clothing comparable to that of the caribou area. Further, the detailed study of forms in both men's and women's clothing, shows that in general, as we proceed northward in the bison area, the resemblances to the culture of the caribou area become more numerous. In brief, the conclusion is that in America, the use of skin clothing is practically confined to the combined bison and caribou areas but is a more intense and universal trait in the latter. We find a similar distribution for the true moccasin and certain methods of dressing skins.

(*e*) *Harpoons and Hunting Points.* The investigation of these traits of culture has not gone far enough to make very definite statements. One characteristic of the caribou area is the large use of bone points, many of which are barbed. While barbed points are rarely used in the bison area, there was a strong tendency to use bone points, especially in the north.

In addition to the preceding, the investigation is concerned with such traits as textile arts, birchbark technique, weapons, and special manufacturing processes. The distribution of the traits enumerated above indicates a fundamental similarity between the material cultures of the caribou and bison areas. The interpretation of this observation is an important theoretical problem. The experience of anthropologists to date is that in all such cases we have two major alternatives, diffusion from a single center or independent development in two or more localities. It remains to be seen which of these will be the more satisfactory interpretation for the above noted similarities in culture.

Finally, this study has developed the problem of caribou, or New World reindeer, culture in its relation to the reindeer culture of the Old World in both modern and paleolithic times. It now appears probable that in the great area of the reindeer and caribou (for they are geographically continuous) we have a concomitant human culture which may be as old as paleolithic man in Europe. This will be more fully discussed when our detailed studies of caribou culture are published.

REPORT ON THE AUTUMN MEETING

Prepared by the Home Secretary

The Autumn Meeting of the National Academy of Sciences was held in the Botanical Laboratory of the University of Chicago on December 7, 8, and 9, 1914, twenty-one members of the Academy being present.

BUSINESS SESSIONS

Business sessions were held on December 8 and 9, at which the following business was transacted:

The President announced that since its Annual Meeting in April the Academy had lost by death two members, Theodore Nicholas Gill, elected in 1873, who died on September 25, 1914, and Charles Sedgwick Minot, elected in 1897, who died on November 19, 1914; also two foreign associates, Edouard Suess, elected in 1898, who died on April 26, 1914, and August Weismann, elected in 1913, who died on November 5, 1914.

The President made also the following announcements:

That Mr. Ira Remsen was acting as Chairman of the Board of Directors of the Bache Fund for the time being in the place of Charles S. Minot, deceased.

That by the death of Theodore N. Gill a vacancy was created in the Finance

Committee, and that this had been filled by the appointment of Mr. Arnold Hague.

That the following standing committees of the Academy had been chosen by the Council in accordance with article 4, section 4, of the Constitution:

COMMITTEE ON NOMINATIONS

Noyes, A. A. (Chairman)
Chittenden, R. H.
Comstock, G. C.
Conklin, E. G.
Hague, Arnold
Holmes, W. H.
Moore, E. H.
Trelease, William
Woodward, R. S.

COMMITTEE ON MATHEMATICS

Moore, E. H. (Chairman)
Bocher, Maxime
Bolza, Oskar
Dickson, L. E.
Moulton, F. R.
Osgood, W. F.
Story, W. E.
Van Vleck, E. B.
Webster, A. G.
Woodward, R. S.

COMMITTEE ON ASTRONOMY

Comstock, G. C. (Chairman)
Barnard, E. E.
Campbell, W. W.
Elkin, W. L.
Frost, E. B.
Hale, G. E.
Leuschner, A. O.
Moulton, F. R.
Pickering, E. C.
Woodward, R. S.

The President also announced that he had appointed the following committees:

COMMITTEE ON THE PROGRAM FOR THE MEETINGS

Boltwood, B. B. (Chairman)
Cattell, J. McK.
Mall, F. P.
Frost, E. B.

AUDITING COMMITTEE

Clarke, F. W.
Dall, W. H. (Chairman)
Day, A. L.

COMMITTEE ON THE REVISION OF THE CONSTITUTIONAL RULES

Day, A. L.
Cross, Whitman (Chairman)
Walcott, C. D.

The President further announced that committees on trust funds had been reappointed in accordance with Rule 27 as follows, the numbers following the names showing the date at which the term of service expires.

THE HENRY DRAPER FUND

Hale, G. E. (Chairman), 1918
Wright, A. W., 1915
Trowbridge, John, 1916
Michelson, A. A., 1917
Campbell, W. W., 1919

THE J. LAWRENCE SMITH FUND

Morley, E. W. (Chairman), 1919
Pumpelly, R., 1915
Remsen, Ira, 1916
Hague, Arnold, 1917
Dana, E. S., 1918

THE COMSTOCK FUND

Nichols, E. L. (Chairman), 1918
Thomson, Elihu, 1915
Crew, Henry, 1916
Noyes, A. A., 1919
Webster, A. G., 1917

THE MURRAY FUND

Dall, W. H., 1915
Hague, Arnold, 1916
Parker, G. H., 191

The Committee on the Amendment to the Act of Incorporation of the Academy reported through Mr. Charles D. Walcott, its Chairman, that the following amendment had been passed by both houses of Congress during the last session and approved by the President on May 27, 1914:

> (Public—No. 109—63d Congress. S. 4096)
>
> An Act to amend the Act authorizing the National Academy of Sciences to receive and hold trust funds for the promotion of science, and for other purposes.
>
> Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That the Act to authorize the National Academy of Sciences to receive and hold trust funds for the promotion of science, and for other purposes, approved June twentieth, eighteen hundred and eighty-four, be, and the same is hereby, amended to read as follows:
>
> "That the National Academy of Sciences, incorporated by the Act of Congress approved March third, eighteen hundred and sixty-three, be, and the same is hereby, authorized and empowered to receive, by devise, bequest, donation or otherwise, either real or personal property, and to hold the same absolutely or in trust, and to invest, reinvest, and manage the same in accordance with the provisions of its constitution, and to apply said property and the income arising therefrom to the objects of its creation and according to the instructions of the donors: Provided, however, That the Congress may at any time limit the amount of real estate which may be acquired and the length of time the same may be held by said National Academy of Sciences."
>
> Sec. 2. That the right to alter, amend, or repeal this Act is hereby expressly reserved.
>
> Approved, May 27, 1914.

At the business session on December 8, 1914 the Committee on the Revision of the Constitution and Rules reported a series of amendments to the constitution of the Academy, which, after discussion, were referred to the Council for consideration and report. The Council presented its report, with further amendments, to the Academy at the stated session on December 9, 1914; and the amended report was adopted by the Academy, sitting as a committee of the whole. This report will come up for final consideration by the Academy at the stated meeting in April 1915.

A new Rule was adopted, reading as follows: The holders of the Medal for Eminence in the Application of Science to the Public Welfare shall be notified, like members, of the meetings of the Academy, and invited to participate in its scientific sessions.

Rule III was amended, by replacing the second and third sentences, so as to read as follows: That it shall be the duty of the Auditing Committee to verify the records, receipts, and disbursements maintained by the Treasurer and the agreement of book and bank balances; to examine all securities in the custody of the Treasurer, and to compare the stated income of such securities with the receipts of record; to examine all vouchers covering disbursements for the account of the Academy and the authority therefor, and to compare them with the Treasurer's record for expenditures; to examine and verify the account of the Academy. The auditing committee may employ the services of an expert accountant to assist in the examination of the books of the Treasurer.

The recommendation of the Committee on the Henry Draper Fund to award the Draper medal in 1915 to Prof. Joel Stebbins, of the University of Illinois, in recognition of his work on Application of the Selenium Cell to Stellar Photometry, was approved.

A resolution of the Council providing for a special business meeting and a special scientific meeting of the Academy on April 19, 1915, to be followed by the stated meeting on Tuesday and Wednesday, April 20 and 21, 1914, as required by the constitution, was unanimously adopted.

It was voted that the thanks of the National Academy of Sciences be given to the Chairman, Mr. E. H. Moore, and to the members of the local committee for their most successful arrangements for the Autumn Meeting held in Chicago, December 7, 8, 9, 1914.

It was further voted that the thanks of the Academy be extended to President Judson, to the University of Chicago, to the Quadrangle Club and to the Chaos Club for their aid in arranging the social features which contributed so much to the enjoyment of the Autumn Meeting.

SCIENTIFIC SESSIONS

Two public lectures on the William Ellery Hale Foundation were given on December 7 and 8 by William Wallace Campbell, Director of the Lick Observatory, on Stellar Evolution and the Formation of the Earth.

Four public scientific sessions were held on December 7, 8, and 9, at which the following papers were presented:

1. W. W. Campbell: On the radial velocities of nebulae.
2. Heber D. Curtis (introduced by W. W. Campbell): Preliminary note on nebular proper motions.
3. R. A. Millikan (introduced by A. A. Michelson): The coefficient of slip in gases and its relation to the nature of the impact between a molecule of a gas and the surface of a solid or liquid.
4. W. D. Harkins and E. C. Humphrey (introduced by Julius Stieglitz): The capillary and electrical forces at the interface between two liquids.
5. Herbert N. McCoy (introduced by Julius Stieglitz): The solubilities of radium compounds as indicated by the solubilities of analogous compounds of calcium, strontium, and barium.
6. L. A. Bauer (introduced by George E. Hale): Present status of the general magnetic survey of the globe.
7. Charles J. Chamberlain (introduced by J. M. Coulter): A phylogenetic study of cycads.
8. C. M. Child (introduced by J. M. Coulter): A dynamic conception of the organic individual.

9. S. W. Williston (introduced by T. C. Chamberlin): The American land vertebrate fauna and its relations.

10. A. J. Carlson (introduced by J. M. Coulter): Some new observations on the physiology of the stomach in man.

11. Leonard E. Dickson: Recent progress in the theories of modular and formal invariants.

12. Henry S. White (introduced by L. E. Dickson): The synthesis of triad systems Δ_t in t elements, in particular for $t = 31$.

13. E. J. Wilczynski (introduced by E. H. Moore): Conjugate systems of space curves with equal Laplace-Darboux invariants.

14. Edwin B. Frost: An interesting stellar system.

15. George E. Hale: The direction of rotation of solar storms.

16. A. A. Michelson: Behavior of metals and other substances near the rupture point.

17. C. W. Balke and George W. Sears (introduced by W. A. Noyes): The atomic weight of tantalum.

18. E. W. Washburn (introduced by W. A. Noyes): Our systematic knowledge of the properties and behavior of solutions of non-electrolytes.

19. T. C. Chamberlin: The fundamental segmentation of the earth.

20. William Trelease: Phoradendron.

21. Charles E. Allen (introduced by E. B. Van Vleck): Development of the male germ cells of polytrichum.

22. C. T. Knipp (introduced by W. A. Noyes): Experimental data on the stability of positive and negative ions.

23. S. W. Parr (introduced by W. A. Noyes): The development of an acid-resisting alloy for a bomb calorimeter.

24. W. L. Tower (introduced by J. M. Coulter): Experimental production of a new ordinal character.

25. G. A. Miller (introduced by L. E. Dickson): The ϕ-subgroup of a group of finite order.

26. E. E. Barnard: Explanation of certain phenomena of the tail of comet Morehouse (III, 1908).

27. Philip Fox (introduced by E. B. Frost): The rotation-period of the sun.

28. J. C. Kapteyn and W. S. Adams: On the relations between the proper motions and the radial velocities of the stars of the spectral types F, G, K, and M.

29. S. B. Nicholson (introduced by W. W. Campbell and A. O. Leuschner): Discovery of a ninth satellite of Jupiter.

30. Frank R. Lillie (introduced by J. M. Coulter): The fertilizing power of sperm dilutions.

31. Gilbert Ames Bliss (introduced by E. H. Moore): A generalization of a theorem of Gauss concerning geodesic triangles.

32. F. R. Moulton: An extension of the process of successive approximations for the solution of differential equations.

33. Julius Stieglitz: Molecular rearrangements of triphenylmethyl derivatives.

34. William Crocker and J. F. Groves (introduced by J. M. Coulter): Methods of determining the life-duration of seeds.

35. Edwin O. Jordan (introduced by J. M. Coulter): Variation in bacteria.

36. Shiro Tashiro (introduced by J. M. Coulter): On the nature of nerve impulse.

37. Eliakim H. Moore: On the integration by successive approximations of the ordinary differential equation of the first order in general analysis.

38. W. H. Wright (introduced by W. W. Campbell): On the occurrence of the line 4886A and the related series of lines in the spectra of the planetary nebulae.

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CONJUGATE SYSTEMS OF SPACE CURVES WITH EQUAL LAPLACE-DARBOUX INVARIANTS

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It is the object of this note to provide a new geometrical interpretation for the condition that the Laplace-Darboux invariants of an equation of the form

$$\frac{\partial^2 z}{\partial x \partial y} + a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} + cz = 0 \tag{1}$$

be equal. These invariants are

$$h = a_x + ab - c, \quad k = b_y + ab - c, \tag{2}$$

where we have put

$$a_x = \frac{\partial a}{\partial x}, \quad b_y = \frac{\partial b}{\partial y}, \tag{3}$$

a notation which we shall employ throughout this paper.

Let $z^{(1)}$, $z^{(2)}$, $z^{(3)}$, $z^{(4)}$ be any four linearly independent solutions of (1), functionally independent in the sense that no two of their three ratios can be expressed as a function of the third. If $z^{(1)}, \ldots, z^{(4)}$ be interpreted as the homogeneous coordinates of a point P of space, variation of the parameters x and y will cause P to generate a non-degenerate surface S which is not a plane. Let us assume further that S is not developable. Then $z^{(1)}, \ldots, z^{(4)}$ will satisfy, besides (1), just one other linear homogeneous differential equation of the second order, of the form

$$z_{yy} = mz_{xx} + nz_x + pz_y + qz, \tag{4}$$

where

$$m = \frac{D''}{D}, \quad D \neq 0, \quad D'' \neq 0, \tag{5}$$

D and D'' being two of the fundamental quantities of S of the second order. The third of these quantities, D', is equal to zero on account of the fact that the curves x = const. and y = const. form a conjugate system on S.

Let us now apply the Laplace transformations to the surface S, by putting

$$z_1 = z_y + az, \quad z_{-1} = z_x + bz. \tag{6}$$

These expressions determine two points P_1 and P_{-1} whose loci give rise to two further surfaces S_1 and S_{-1}. The surface S_1 is the second sheet of the focal surface of the congruence formed by the lines which are tangent to the curves x = const. on S; the surface S_{-1} is connected in similar fashion with the curves y = const. on S.

Consider now the line which joins the points P_1 and P_{-1} of the 1st and −1st Laplace transformed nets. There is one such line for every point P of the original surface S. Consequently, the totality of these lines forms a *congruence*, whose developables we proceed to determine. For this purpose, let us give increments, δx and δy, to x and y. The coordinates of those points of S_1 and S_{-1}, which correspond to the point $z + z_x\,\delta x + z_y\delta y$ of S, will be

$$Z_1 = z_1 + \frac{\partial z_1}{\partial x}\,\delta x + \frac{\partial z_1}{\partial y}\,\delta y, \qquad Z_{-1} = z_{-1} + \frac{\partial z_{-1}}{\partial x}\,\delta x + \frac{\partial z_{-1}}{\partial y}\,\delta y.$$

Now we find, making use of (1),

$$\begin{aligned} z_1 &= z_y + az, & z_{-1} &= z_x + bz, \\ \frac{\partial z_1}{\partial x} &= -bz_y + (a_x - c)\,z, & \frac{\partial z_{-1}}{\partial x} &= z_{xx} + bz_x + b_x z, \\ \frac{\partial z_1}{\partial y} &= z_{yy} + az_y + a_y z, & \frac{\partial z_{-1}}{\partial y} &= -\,az_x + (b_y - c)\,z. \end{aligned} \tag{7}$$

The homogenous coordinates of an arbitrary point on the line $Z_1\,Z_{-1}$ will be obtained from λZ_1 and μZ_{-1}, and this expression, on account of (7), assumes the form

$$\begin{gathered} \{\lambda\,[a + (a_x - c)\,\delta x + (a_y + q)\,\delta y] + \mu\,[b + b_x\delta x + (b_y - c)\,\delta y]\}\,z \\ + \{\lambda n\delta y + \mu\,(1 + b\delta x - a\delta y)\}\,z_x + \lambda\,\{1 - b\delta x + (a + p)\,\delta y\}\,z_y \\ + (\mu\delta x + \lambda m\delta y)\,z_{xx}. \end{gathered} \tag{8}$$

In order that such a point may also be on the line $P_1\,P_{-1}$, i.e., in order that these two lines may intersect, (8) must reduce to a linear homogeneous combination of z_1 and z_{-1}. This will be so, if and only if λ, μ, δx, and δy can be determined subject to the two conditions

$$\lambda m \delta y + \mu \delta x = 0, \tag{9}$$
$$\lambda \left[h + (a_y + q - bn - a^2 - ap)\, \delta y\right] + \mu \left[(b_x - b^2)\, \delta x + k \delta y\right] = 0.$$

If we eliminate the ratio $\lambda : \mu$ from (9), we obtain the differential equation of the developables of the congruence, viz.:

$$h \delta x^2 + \left[a_y + q - bn - a^2 - ap - m\,(b_x - b^2)\right] \delta x \delta y - mk \delta y^2 = 0. \tag{10}$$

This equation may also be regarded as the differential equation of the curves which the developables of the congruence determine on S. The asymptotic curves of S are determined by

$$D \delta x^2 + D'' \delta y^2 = 0. \tag{11}$$

Therefore the curves (10) form a conjugate system on S if and only if the simultaneous invariant of (10) and (11) is equal to zero, i.e., if and only if $hD'' - mkD = 0$. But, according to (5), this reduces to $h = k$.

We have, therefore, obtained the following theorem. *Consider a net of conjugate curves on a non-developable surface S. Let P be any point of this net and let* P_1 *and* P_{-1} *be the corresponding points of the nets obtained from the given one by the 1st and −1st Laplace transformations. Consider further the congruence of all of the lines such as* $P_1\,P_{-1}$. *The curves on S, which correspond to the developables of this congruence, will form a conjugate system, if and only if the original net of conjugate curves has equal Laplace-Darboux invariants.*

I wish to add one further remark. Darboux[1] has given a geometric interpretation of the condition $h = k$, different from mine, by means of a certain conic in the plane of P, P_1, P_{-1}.

I have found it advisable to introduce two such conics, which coincide with each other and with that of Darboux in the case $h = k$ and only in that case. By using these conics I have been able, quite recently, to interpret geometrically the condition which Bianchi expresses by saying that a conjugate system is *isothermally conjugate.* These systems have made their appearance in so many problems of differential geometry that such a geometrical interpretation seems to me to be a matter of very great interest. I shall, however, reserve the details of this interpretation for a place, in its appropriate setting, in a paper on the general theory of congruences which I hope to present for publication before long.

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[1] *Leçons sur la Théorie générale des Surfaces*, vol. 4, p. 38.

TRANSFORMATION OF SURFACES Ω

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Presented to the Academy, December 28, 1914.

When any surface S is referred to a conjugate system with equal point invariants, its cartesian coördinates x, y, z, are solutions of an equation of the form

$$\frac{\partial^2 \theta}{\partial u \partial v} + \frac{\partial \log \sqrt{\rho}}{\partial v} \frac{\partial \theta}{\partial u} + \frac{\partial \log \sqrt{\rho}}{\partial u} \frac{\partial \theta}{\partial v} = 0. \tag{1}$$

If θ_i is any solution of this equation, linearly independent of x, y and z, the surface S_i whose cartesian coördinates, x_i, y_i, z_i, are given by equations of the form

$$\frac{\partial}{\partial u}(\lambda_i x_i) = -\rho\left(x\frac{\partial \theta_i}{\partial u} - \theta_i \frac{\partial x}{\partial u}\right), \quad \frac{\partial}{\partial v}(\lambda_i x_i) = \rho\left(x\frac{\partial \theta_i}{\partial v} - \theta_i \frac{\partial x}{\partial v}\right), \tag{2}$$

where λ_i is given by the quadratures

$$\frac{\partial \lambda_i}{\partial u} = -\rho\frac{\partial \theta_i}{\partial u}, \quad \frac{\partial \lambda_i}{\partial v} = \rho\frac{\partial \theta_i}{\partial v}, \tag{3}$$

is referred to a conjugate system with equal point invariants, and corresponding points M and M_i on S and S_i are harmonic with respect to the focal points of the line MM_i for the congruence of these lines. We say that S_i is obtained from S by a *transformation K*. We have studied these transformations at length in a recent memoir.[1] In the present note we consider the case where the lines MM_i form a normal congruence. In this case there exists a solution t of equation (1) such that $x^2 + y^2 + z^2 - t^2$ also is a solution. Thus the parametric conjugate system is 2 O in the sense of Guichard, and t is the *complementary function.* The surface S_i has the same properties.

By definition a *surface C* is one possessing a conjugate system 2 O with equal point invariants. When this system is parametric, the first fundamental coefficients of C have the form

$$E = \left(\frac{\partial t}{\partial u}\right)^2 + \frac{1}{\rho}, \quad F = \frac{\partial t}{\partial u}\frac{\partial t}{\partial v}, \quad G = \left(\frac{\partial t}{\partial v}\right)^2 + \frac{1}{\rho}, \tag{4}$$

and this property is characteristic.

It can be shown that *when a surface C is referred to the system 2 O with equal point invariants, there can be found without quadratures two sur-*

faces of the same kind, say C_0 *and* C_0'*, such that the congruences of lines* MM_0 *and* MM_0' *are normal. Furthermore, the spheres of radius t with their centers on C are enveloped by two surfaces, each of which is orthogonal to one of the congruences.* These orthogonal surfaces are *surfaces* Ω as defined by Demoulin,[2] who showed that they are characterized by the property that their fundamental coefficients $\overline{E}$, $\overline{G}$, $\overline{D}$, $\overline{D}''$, when the lines of curvature are parametric, satisfy the condition

$$\frac{\partial}{\partial u}\left(\frac{\dfrac{\partial}{\partial v}\left(\dfrac{\overline{D}''}{\overline{G}}\right)\sqrt{\overline{G}}\;\overline{U}}{\left(\dfrac{\overline{D}}{\overline{E}}-\dfrac{\overline{D}''}{\overline{G}}\right)\sqrt{\overline{E}}\;\overline{V}}\right)+\frac{\partial}{\partial v}\left(\frac{\dfrac{\partial}{\partial u}\left(\dfrac{\overline{D}}{\overline{E}}\right)\sqrt{\overline{E}}\;\overline{V}}{\left(\dfrac{\overline{D}}{\overline{E}}-\dfrac{\overline{D}''}{\overline{G}}\right)\sqrt{\overline{G}}\;\overline{U}}\right)=0, \tag{5}$$

where $\overline{U}$ and $\overline{V}$ are functions of u and v respectively.

If $X, Y, Z; X_1, Y_1, Z_1; X_2, Y_2, Z_2$ denote the direction-cosines of the normal to a surface C, and of the bisectors of the angles between the tangents to the parametric curves, we may write the equations of a transformation K in the form (cf. *Transactions*, loc. cit.)

$$x_i - x = \frac{1}{m_i\lambda_i}(a_iX_1 + b_iX_2 + \omega_iX), \tag{6}$$

where m_i is a constant and a_i, b_i, ω_i are functions satisfying the completely integrable system of equations

$$\begin{aligned}
\frac{\partial a_i}{\partial u} &= -m_i(\lambda_i - \rho\theta_i)\sqrt{E}\cos\omega + b_iA + \frac{w_iD}{2\sqrt{E}\cos\omega},\\
\frac{\partial a_i}{\partial v} &= -m_i(\lambda_i + \rho\theta_i)\sqrt{G}\cos\omega - b_iB + \frac{w_iD''}{2\sqrt{G}\cos\omega},\\
\frac{\partial b_i}{\partial u} &= m_i(\lambda_i - \rho\theta_i)\sqrt{E}\sin\omega - a_iA = \frac{w_iD}{2\sqrt{E}\sin\omega},\\
\frac{\partial b_i}{\partial v} &= -m_i(\lambda_i + \rho\theta_i)\sqrt{G}\sin\omega + a_iB + \frac{w_iD''}{2\sqrt{E}\sin\omega},\\
\frac{\partial w_i}{du} &= -\frac{D}{2\sqrt{E}}\left(\frac{a_i}{\cos\omega} - \frac{b_i}{\sin\omega}\right), \quad \frac{\partial w_i}{dv} = -\frac{D''}{2\sqrt{G}}\left(\frac{a_i}{\cos\omega} + \frac{b_i}{\sin\omega}\right),
\end{aligned} \tag{7}$$

where 2ω is the angle between the parametric lines on C, D and D'' are the second fundamental coefficients of C, and

$$A = \sqrt{\frac{E}{G}}\,\frac{\partial\log\sqrt{\rho}}{\partial v}\sin 2\omega - \frac{\partial\omega}{\partial u}, \quad B = \sqrt{\frac{G}{E}}\,\frac{\partial\log\sqrt{\rho}}{\partial u}\sin 2\omega - \frac{\partial\omega}{\partial v}.$$

For the sake of brevity we put

$$T_i^2 = a_i^2 + b_i^2 + w_i^2, \quad H = \sqrt{EG - F^2}. \tag{8}$$

The functions $a_0, b_0, w_0, \lambda_0, \theta_0$ which determine C_0 are given by

$$\begin{aligned} a_0 &= - \sin\omega \cdot w_0 \rho \left(\sqrt{G}\frac{\partial t}{\partial u} + \sqrt{E}\frac{\partial t}{\partial v} \right), \\ b_0 &= \cos\omega \cdot w_0 \rho \left(\sqrt{G}\frac{\partial t}{\partial u} - \sqrt{E}\frac{\partial t}{\partial v} \right), \end{aligned}$$

$$\frac{\partial \log w_0}{\partial u} = \frac{D}{H}\frac{\partial t}{\partial u}, \quad \frac{\partial \log w_0}{\partial v} = \frac{D''}{H}\frac{\partial t}{\partial v}, \quad T_0 = H\rho w_0, \tag{9}$$

$$\frac{m_0}{T_0}(\lambda_0 - \rho\theta_0) = \frac{D}{H} + \frac{1}{\frac{\partial t}{\partial u}}\frac{\partial}{\partial u}\log H\rho, \quad \frac{m_0}{T_0}(\lambda_0 + \rho\theta_0) = \frac{D''}{H} + \frac{1}{\frac{\partial t}{\partial v}}\frac{\partial}{\partial v}\log H\rho.$$

The complementary function t_0 for C_0 is given by

$$t_0 = t - \frac{T_0}{\lambda_0 m_0}. \tag{10}$$

The functions for the surface C_0' are analogous to the above.

Ordinarily the surfaces S_i derived from a surface C by transformations K are not surfaces C. However, the equations

$$\begin{aligned} &\frac{\partial \theta_1}{\partial u} + \sqrt{E}\left[\cos\omega a_1 - \sin\omega b_1 + (t_1 - t)\frac{m_1\lambda_1}{T_0}(\cos\omega a_0 - \sin\omega b_0)\right] \\ &\frac{\partial \theta_1}{\partial v} + \sqrt{G}\left[\cos\omega a_1 + \sin\omega b_1 + (t_1 - t)\frac{m_1\lambda_1}{T_1}(\cos\omega a_0 + \sin\omega b_0)\right] \end{aligned} \tag{11}$$

are consistent with equations (7) for $i = 1$, the function θ_1 so defined is a solution of equation (1), and the new surface S_1, given by (6), is a surface C, say C_1. In particular we remark that the function t_1 given by (2), when x_i and x are replaced by t_1 and t respectively, is the complementary function for C_1. Furthermore, if x_i and x in equations (2) are replaced by $x_1^2 + y_1^2 + z_1^2 - t_1^2$ and $x^2 + y^2 + z^2 - t_1^2$ the resulting equations are satisfied.

With the aid of the theorem of permutability of general transformations K (cf. *Transactions*, loc. cit., p. 406) we show that *if* C, C_0 *and* C_1 *are three surfaces in the relation indicated above, a fourth surface* C_{10} *can be found without quadratures such that the lines joining corresponding points* M_1, M_{10}, *on* C_1 *and* C_{10} *is a normal congruence.*

Likewise it is found that *the surfaces* Ω *and* Ω_1 *normal to the congruences of the lines* MM_0 *and* $M_1 M_{10}$ *at the distances* t *and* t_1 *from* C *and* C_1 *respectively envelope a two-parameter family of spheres, and the lines of* *curvature on Ω and Ω_1 correspond.* Thus equations (7) and (11) define transformations of surfaces Ω of the Ribaucour type. We call them *transformations A*. When in particular the surfaces C and C_1 are *associate* surfaces, which is a special case of transformations K, the surfaces C_0 and C_{10} are likewise associate, and the surfaces Ω and Ω_1, as defined in the preceding theorem, have the same spherical representation of their lines of curvature.

By means of a generalized theorem of permutability for transformations K in general we prove the following theorem of permutability for transformations A: *If Ω_1 and Ω_2 are two surfaces obtained from a surface Ω by transformations A, there exists a surface Ω' which is in the relations of transformations A with Ω_1 and Ω_2, and Ω' can be found without quadratures.*

Isothermic surfaces are surfaces C for which $t = 0$. In this case the transformations A are equivalent to the transformations D_m of isothermic surfaces, discovered by Darboux and studied at length by Bianchi.[3]

Surfaces with isothermal representation of their lines of curvature are surfaces Ω in the sense that the surface C is the locus of the point midway between the centers of principal curvature of Ω, and C_0 is at infinity. This case requires special study, but the results are analogous to those of the general case. However, the transformations A are now the same as the transformations of these surfaces established from another point of view by me.[4]

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[1] Eisenhart, *Trans. Amer. Math. Soc.* 15, 397–430 (1914).

[2] Demoulin, *Paris, C. R. Acad. Sci.*, 153, 703 (1911).

[3] Bianchi, *Annali Mat. Milano*, Ser. 3, 11, 93–158 (1905).

[4] Eisenhart, *Trans. Amer. Math. Soc.*, 9, 149–177 (1908).

POTASSIUM AMMONO ARGENATE, BARATE, CALCIATE, AND SODATE

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Presented to the Academy, January 9, 1915

It has been shown by me that reactions strictly analogous to those which accompany the solution of the hydroxides of zinc, lead, and aluminium in aqueous solutions of potassium hydroxide take place when the amides of certain metals are treated with liquid-ammonia solutions of potassium amide. Thus, just as zinc hydroxide is known to dissolve in an aqueous solution of potassium hydroxide to form potassium aquozincate, so potassium amide in liquid ammonia reacts with zinc amide to form potassium ammonozincate.[1] These analogous reactions are represented by the equations:

$$Zn(OH)_2 + 2KOH = Zn(OK)_2 + H_2O,$$
$$Zn(NH_2)_2 + 2KNH_2 = Zn(NHK)_2 + 2NH_3.$$

In view of the many close analogies which have been shown to exist between the derivatives of water and the derivatives of ammonia the formation of this potassium ammonozincate and of potassium ammonostannate[2] and potassium ammonoplumbite[3] was not unexpected. Further work has, however, shown that elements which do not give rise to similar salts in water form these compounds in ammonia. Thus in previous articles this was shown to be true of copper[4] and thallium;[5] and researches have now been completed which show it to be true of silver, cadmium, and nickel. The work on silver will be here briefly described; that on cadmium and nickel, which was carried out by G. S. Bohart, will be later reported upon.

Some years ago while studying the action of potassium amide on liquid-ammonia solutions of silver salts[6] it was observed that silver amide dissolves readily in an excess of potassium amide solution. This solubility recalled the amphoteric properties of the hydroxides of aluminium, zinc, and lead, and suggested that in all probability the solution resulting from the action of a solution of potassium amide on silver amide would be found to contain an ammonoargentate of potassium.

It was found in fact that a crop of beautiful crystals of the composition represented by the formula $AgNHK.NH_3$ or $AgNH_2.KNH_2$ is obtained when a liquid-ammonia solution prepared by dissolving silver amide in an excess of potassium amide is first adjusted to a proper concentration and then cooled in a bath of liquid ammonia. After two or three recrystallizations the salt is pure. The equation expressing its formation is $AgNH_2 + KNH_2 = AgNHK + NH_3$. Because of the relationship of the compound to the familiar (aquo) zincates and aluminates it is appropriately called potassium ammonoargentate.

More surprising was the result of a later investigation[7] in which the amide of such a strongly electropositive element as magnesium was found to react with potassium amide to form potassium ammonomagnesate, a compound of the formula $Mg(NHK)_2 \cdot 2NH_3$. This made it seem probable that potassium amide possesses the property of forming similar compounds with the amides of all the metals. It was therefore deemed advisable to study the action of solutions of potassium amide on the amides of the strongly electropositive barium, strontium, and calcium, and on the amide of sodium, in order to determine whether potassium ammonobarate, ammonostrontiate, and ammonocalciate, and possibly even potassium ammonosodate, can be prepared.

When a barium salt in solution in liquid ammonia is added to a solution of potassium amide a microcrystalline precipitate is obtained which has been found to have the composition represented by the formula, $BaNK.2NH_3$.

When solutions of strontium and calcium salts in liquid ammonia are poured into an excess of a solution of potassium amide there are formed apparently noncrystalline products, which are represented respectively by the formulas, $SrNK.2NH_3$ and $CaNK.2NH_3$. The calciate was also prepared by the action of potassium amide on metallic calcium.

When sodium amide is treated with a liquid-ammonia solution of potassium amide, or when a solution of a soluble salt of sodium is added to an excess of potassium amide in solution in liquid ammonia beautiful crystals of a product whose composition is represented by the formula $NaNK_2.2NH_3$, separate from the solution.

For the reason that these compounds have been prepared by methods entirely similar to those whereby potassium ammonomagnesate and the other salts of the same class mentioned above are formed they have received the names given in the title of this paper and are formulated as salts with ammonia of crystallization. It is possible, however, to formulate them as molecular compounds of potassium amide with the respective amides of the less positive metals, as follows, $Ba(NH_2)_2.KNH_2$, $Sr(NH_2)_2.KNH_2$, $Ca(NH_2)_2.2NH_3$, $NaNH_2.2NHK_2$; thus representing them as ammonia analogues of Werner's hydroxo salts.

It is hoped that transference measurements which are to be undertaken in this laboratory may show for such of these compounds as are sufficiently soluble whether or not the less positive metal travels together with the nitrogen toward the anode during electrolysis.

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[1] Fitzgerald, *J. Amer. Chem. Soc.*, 29, 657; Franklin, Ibid., 29, 1274 (1907).

[2] Fitzgerald, *J. Amer. Chem. Soc.*, 29, 1693 (1907).

[3] Franklin, *J. Phys. Chem.*, 15, 509 (1911).

[4] Franklin, *J. Amer. Chem., Soc.*, 34, 1501 (1912).

[5] Franklin, *J. Phys. Chem.*, 16, 682 (1912).

[6] Franklin, *J. Amer. Chem. Soc.*, 27, 835 (1905).

[7] Franklin, *J. Amer. Chem. Soc.*, 34, 1455 (1913).

METALLIC SALTS OF AMMONO ACIDS

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DEPARTMENT OF CHEMISTRY, STANFORD UNIVERSITY

Presented to the Academy, January 9, 1915

The Action of Liquid-Ammonia Solutions of Ammono Acids on Metallic Amides, Imides, and Nitrides. The acid amides and imides, and the metallic derivatives of the acid amides and imides are the acids, bases, and salts respectively of an ammonia system of acids, bases, and salts.[1]

Guided by the relationships implied in the above statement Franklin and Stafford were able to prepare potassium derivatives of a considerable number of acid amides by the action of potassium amide on certain acid amides in solution in liquid ammonia. That is to say, an ammono base, potassium amide, was found to react with ammono acids in liquid ammonia to form ammono salts just as the aquo base, potassium hydroxide, acts upon aquo acids in water solution to form aquo salts. Choosing, for example, benzamide and benzoic acid as representative acids of the two systems, the analogous reactions taking place respectively in liquid ammonia and water are represented by the equations:

$$C_6H_5CONH_2+KNH_2 = C_6H_5CONHK + NH_3.$$
$$C_6H_5CONH_2 + 2KNH_2 = C_6H_5CONK_2 + 2NH_3.$$
$$C_6H_5COOH + KOH = C_6H_5COOK + H_2O.$$

The ammono acid, since it is dibasic, reacts with either one or two molecules of potassium amide to form an acid and a neutral salt.

Having thus demonstrated the possibility of preparing ammono salts of potassium by the interaction of potassium amide and acid amides in liquid ammonia solution, it was further found that ammono salts of the heavy metals may be prepared by the action of liquid ammonia solutions of ammono acids on insoluble metallic amides, imides, and nitrides—that is, by reactions which are analogous to the formation of aquo salts in water by the action of potassium hydroxide on insoluble metallic hydroxides and oxides.

In the present investigation the action of acetamide, benzenesulfonamide, and paratoluenesulfonamide on the amide of silver, on the imides of copper and lead, and on the nitride of thallium has been studied.

The method for the preparation of a typical compound may be briefly described. Pure silver amide, prepared in a two-legged reaction-tube[2] by thoroughly washing the precipitate resulting from the interaction of potassium amide and silver nitrate in liquid-ammonia solution, is dissolved in a liquid-ammonia solution of benzenesulfonamide. After adjusting the concentration of the solution and cooling in a bath of liquid ammonia a crop of beautiful crystals of the composition represented by the formula, $C_6H_5SO_2NHAg.2NH_3$, separates from the solution. On standing in a vacuum at 20° the salt loses one molecule of ammonia of crystallization. Heated at 200° it loses the second molecule.

The action of the acid on the base is shown by the equation:

$$C_6H_5SO_2NH_2 + AgNH_2 = C_6H_5SO_2NHAg + NH_3.$$

In a similar way the following salts have been obtained:

Thallous ammonobenzenesulfonate, $C_6H_5SO_2NHTl$, $C_6H_5SO_2NHTl . NH_3$, and $. 2NH_3$.
Cuprous ammonobenzenesulfonate, $C_6H_5SO_2NHCu$ and $C_6H_5SO_2NHCu . 2NH_3$.
Thallous ammonoparatoluenesulfonate, $CH_3C_6H_4SO_2NHTl$, and $CH_3C_6H_4SO_2NHTl . 2NH_3$.
Cuprous ammonoparatoluenesulfonate, $CH_3C_6H_4SO_2NHCu$, and $CH_3C_6H_4SO_2NHCu . 2NH_3$.
Silver ammonoacetate, $CH_3CONHAg$ and $CH_3CONHAg . 2NH_3$.
Thallous ammonoacetate, $CH_3CONHTl$ and $CH_3CONHTl . 2NH_3$.

The Action of Liquid-Ammonia Solutions of Ammono Acids on Metallic Hydroxides and Oxides. Reactions between some of the stronger ammono acids and metallic hydroxides are known to take place in water solutions. In order to ascertain whether metallic hydroxides and oxides would dissolve in liquid ammonia solutions of acid amides to form ammono salts, cupric hydroxide and silver oxide were submitted to the action of liquid-ammonia solutions of benzenesulfonamide with the result that reactions took place as represented by the equations:

$$Cu(OH)_2 + 2C_6H_5SO_2NH_2 = 2C_6H_5SO_2NHCu + 2H_2O$$
$$Ag_2O + C_6H_5SO_2NH_2 = C_6H_5SO_2NHAg + H_2O,$$

Dried *in vacuo* at 20° cupric ammonobenzenesulfonate retains four molecules of ammonia of crystallization. The silver salt is identical with the silver ammonobenzenesulfonate described above.

The Action of Liquid-Ammonia Solutions of Acid Ammono Esters on Ammono Bases. Such compounds as acetanilide and other aryl and alkyl derivatives of the acid amides are acid ammono esters which by virtue of their acid properties should be capable of reacting in liquid-ammonia solutions with metallic amides to form compounds which are at the same time ammono esters and ammono salts.

Four such compounds have been prepared; namely, silver phenyl ammonoacetate, $CH_3CONAgC_6H_5$ and $CH_3CONAgC_6H_5.NH_3$, by the action of a solution of acetanilide on silver amide; thallous phenyl ammonoacetate, $CH_3CONTlC_6H_5$, by the action of acetanilide on thallous nitride; potassium benzyl ammonoacetate, $CH_3CONKCH_2C_6H_5.NH_3$, by the interaction of benzylacetamide and potassium amide; and potassium phenetol ammonoacetate, $CH_3CONKC_6H_4OC_2H_5$, by the interaction of phenetolacetamide and potassium amide.

The nature of these reactions will be clear from the equation,

$$(C_6H_5)(CH_3CO)NH + AgNH_2 = (C_6H_5)(CH_3CO)NAg + NH_3,$$

which represents the action of acetanilide on silver amide, or in other words, the action of an acid ammono ester on an ammono base.

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[1] Franklin, *Amer. Chem. J.*, 47, 285 (1912).

[2] Franklin, *J. Amer. Chem. Soc.*, 27, 820 (1905).

AMMONOBASIC IODIDES OF ALUMINIUM

By Edward C. Franklin

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Presented to the Academy, January 9, 1915

While investigating the action of potassium amide on salts of aluminium in liquid-ammonia solution it was noted that a considerable amount of potassium amide could be added to aluminium iodide solutions without the formation of a permanent precipitate. Under analogous conditions in aqueous solutions aquobasic salts are formed.

The attempt was therefore made to isolate an ammonobasic aluminium iodide from a liquid-ammonia solution of aluminium iodide to which potassium amide had been added in an amount not quite sufficient to produce a permanent precipitate. When a solution so prepared is carefully concentrated a crop of well formed crystals of a compound is obtained which, after several recrystallizations, gave analytical results sharply in accord with the formula, $AlI_3 . Al(NH_2)_3 . 6NH_3$. The equation expressing its formation is $2AlI_3 + 3KNH_2 = AlI_3 . Al(NH_2)_3 + 3KI$.

The compound is obviously related to ammonia as the ordinary basic salts are related to water. It is therefore an ammonobasic salt.

When sufficient potassium amide is added to a liquid-ammonia solution of aluminium iodide to produce a permanent precipitate a second ammonobasic salt is formed the composition of which is represented by the formula $Al(NH_2)_3 . Al(NH_2)_2I . NH_3$.

A REVISION OF THE ATOMIC WEIGHT OF LEAD: THE ANALYSIS OF LEAD BROMIDE AND CHLORIDE

By Gregory Paul Baxter, Fred Leslie Grover and Thorbergur Thorvaldson
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Presented to the Academy, January 3, 1914

In a recent paper by Baxter and Wilson[1] the atomic weight of lead was found from analyses of fused lead chloride, by comparison with pure silver and by weighing the silver chloride produced. The result of this investigation, Pb = 207.09 (Ag = 107.880), is considerably higher than most earlier determinations. For this reason and because the work of Baxter and Wilson was essentially preliminary in its nature it was obviously desirable to test the matter further, and ever since the publication of the above paper the atomic weight of lead has been under almost continuous investigation in the Harvard Laboratory. Because of the advantages offered by the greater insolubility of silver bromide, the analysis of lead bromide was next undertaken; and later lead chloride was investigated anew. The outcome of all the work is that the atomic weight of lead is very close to 207.20, a value even higher than that previously found by Baxter and Wilson.

The method of experimentation was the usual one employed with metallic halides. Carefully purified lead bromide, which had been crystallized from concentrated hydrobromic acid solution, was dried by fusion in an atmosphere containing a large proportion of hydrobromic acid gas. The acid was synthesized from hydrogen and bromine, and during a fusion was mixed with either nitrogen, hydrogen, or bromine. After being weighed the salt was dissolved in hot water containing a very small amount of nitric acid, and at low concentration the solution was precipitated with a very dilute solution of a carefully weighed, very nearly equivalent quantity of pure silver. The solution was allowed to stand for some days with occasional shaking, and then was tested in a nephelometer for excess of silver or bromide. If a deficiency of either was found, this deficiency was made up with very dilute standard silver or bromide solution. Then the solution was again tested, and the operations were repeated if necessary. As soon as the exact end-point had been found, a moderate excess of silver nitrate was added, and eventually the silver bromide was collected and weighed. The usual correction for moisture retained by the dried silver bromide was determined and applied.

In order to test the purity of the lead salt, metallic electrodes were prepared from the purified material, and the ultraviolet spark spectrum of the metal was photographed by means of a Féry quartz spectrograph. The only lines which could be detected were those commonly ascribed to lead. In fact the original material was fairly pure, containing only traces of silver, copper, and iron, which rapidly disappeared in the course of the purification. Incidentally we found that 0.004% of copper in lead yields fairly strong copper lines and that 0.0001% of silver could readily be detected. Our material was further tested in a sensitive gold leaf electroscope, but showed no evidence of radioactivity.

Several difficulties were met which deserve special attention. Lead bromide, when dissolved in water, hydrolyses to give an insoluble basic bromide. This phenomenon was first met with material which had been fused in an atmosphere containing a large proportion of hydrobromic acid gas, but it was also found that even unfused salt which has been crystallized from concentrated hydrobromic acid solution, and which probably contains a small amount of the latter substance, hydrolyses in the same way, and, apparently to about the same extent. Thus it was evident that at any rate the greater part of the hydrolysis occurred during the solution of the salt and not during the fusion. The evidence that *no* hydrolysis with loss of bromine occurred during the fusion is that the different samples of fused salt, in spite of the widely varied atmospheres in which they were fused, were essentially identical in composition and that the atomic weight of lead found from the bromide agrees very closely with that found later in this research by analysis of the chloride.

In order to prevent the basic salt from forming during the solution of the bromide, a small quantity of nitric acid was added to the hot water in which the salt was dissolved. It was proved experimentally that the nitric acid added was far too small in amount to cause loss of either bromine or hydrobromic acid from the solution.

A second difficulty was caused by the presence of a small amount of black insoluble material in the fused salt. Although puzzling at first, this was eventually found to consist partly of carbon, partly of silica. The greatest care in the purification of the material failed to eliminate the trouble entirely, although the proportion of insoluble residue was reduced on an average to 0.004%. Error from this source was avoided by filtering the lead bromide solution through a weighed platinum-sponge crucible in each analysis before precipitation of the silver bromide, and weighing the residue.

The weights given in the tables are converted to the vacuum standard by applying the following corrections for each apparent gram of substance: $PbBr_2$, +0.000036; $PbCl_2$, +0.000060, Ag, −0.000031; AgBr, +0.000041; AgCl, +0.000071.

The analyses in the following table were made by F. L. Grover. Two preliminary series, one by T. Thorvaldson and one by Grover, gave essentially the same result, but since the insoluble residue was not determined in these experiments, the details are not given.

THE ATOMIC WEIGHT OF LEAD

$PbBr_2 : 2Ag : 2AgBr$

Ag = 107.880 Br = 79.916

FUSION ATMOSPHERE	GRAMS OF $PbBr_2$	GRAMS OF Ag	RATIO $PbBr_2:2Ag$	ATOMIC WEIGHT OF Pb	GRAMS OF AgBr	RATIO $PbBr_2 : 2AgBr$	ATOMIC WEIGHT OF Pb
$HBr+H_2$	5.27845	3.10271	1.70124	207.23			
$HBr+H_2$	2.65094	1.55822	1.70126	207.23			
$HBr+N_2$	4.08410	2.40104	1.70097	207.17			
$HBr+N_2$	4.97468	2.92473	1.70090	207.15			
$HBr+N_2$	4.05541	2.38398	1.70111	207.20	4.15017	0.97717	207.18
$HBr+N_2$	3.44139	2.02288	1.70123	207.23	3.52224	0.97705	207.14
$HBr+N_2$	5.17387	3.04158	1.70105	207.19	5.29498	0.97713	207.17
$HBr+N_2$	3.84497	2.26022	1.70115	207.21	3.93446	0.97726	207.22
$HBr+H_2$	4.30513	2.53086	1.70105	207.19	4.40616	0.97707	207.15
$HBr+H_2$	4.53445	2.66549	1.70117	207.21	4.64048	0.97715	207.18
$HBr+N_2$	5.78437	3.40044	1.70107	207.19	5.91976	0.97713	207.17
$HBr+N_2$	4.87079	2.86337	1.70107	207.19	4.98467	0.97715	207.18
$HBr+N_2$	6.28446	3.69447	1.70104	207.19	6.43124	0.97718	207.19
$HBr+N_2$	4.74639	2.79011	1.70115	207.21	4.85708	0.97721	207.20
$HBr+N_2$	6.82424	4.01148	1.70118	207.21	6.98380	0.97715	207.18
$HBr+N_2$	6.53689	3.84274	1.70110	207.20			
$HBr+Br_2$	4.10098	2.41088	1.70103	207.18	4.19791	0.97691[1]	207.09[1]
$HBr+Br_2$	2.64256	1.55352	1.70101	207.18			
$HBr+Br_2$	6.30683	3.70718	1.70125	207.23			
		Average	1.70111	207.20		0.97715	207.18

[1] This result is omitted in computing the average.

Since the result of these analyses of lead bromide is even higher than that obtained by Baxter and Wilson, lead chloride also was investigated anew by essentially the same method as was used in the work with the bromide. The chloride was finally dried by fusion in a current of hydrochloric acid gas.

In preliminary experiments to our surprise a basic salt precipitated during the solution of the lead chloride in hot water, as soon as a very considerable proportion of the salt was dissolved, although a similar phenomenon was never observed in the earlier research upon the chloride.

One marked difference, however, exists between the two cases. In the experiments by Baxter and Wilson from 4 to 5 grams of salt were dissolved in a liter of water, whereas in our first experiments 8 to 9 grams of salt were dissolved in the same volume. In other words the basic salt precipitates only at high concentrations

As in the case of the bromide, uncertainty exists as to whether the basic salt is formed during the fusion of the slightly damp material, even in the presence of a high hydrochloric acid concentration, or whether it is due to hydrolysis during solution. Since, however, we found that lead chloride which has been crystallized from dilute hydrochloric acid solution, when dissolved in enough hot water to yield a solution nearly saturated at room temperature (about 10 g. per liter), yields a similar insoluble compound which dissolves readily in dilute acid and contains chlorine, the latter explanation seems the more probable one.

More direct evidence that the fused chloride is neutral was obtained as follows: Chloride which had been crystallized from very dilute hydrochloric acid solution was dried in a vacuum at room temperature. Weighed portions were then fused in a current of hydrochloric acid gas and the loss in weight due to water was found. The fused portions were compared with silver in the usual way, and at the same time other weighed portions of the vacuum-dried material were similarly analyzed. After correction for the moisture retained by the unfused material, both fused and unfused salt were found to possess the same composition.

Further evidence that the fused salt is free from basic chloride, is afforded by the agreement of the chloride analyses among themselves, as well as with the results of the bromide analyses.

In dissolving the fused salt for analysis the precipitation of the basic salt was prevented as with the bromide by adding a small amount of nitric acid to the hot water in which the chloride was dissolved. It was proved experimentally that no chlorine is lost in this way.

Furthermore, as with the bromide, the fused chloride usually contained a trace of black insoluble matter, in spite of the utmost care in the purification of the salt and in the preparation of the hydrochloric acid gas in which the salt was fused. Since, however, this residue was collected upon a weighed platinum-sponge crucible, and since, after it had been weighed, a correction was applied, no appreciable error from this source could have occurred. The average per cent of the black residue found in the chloride was essentially the same as that found in the bromide.

The source of the lead material used in the preparation of the lead bromide and a portion of the lead chloride is unknown to us. Both commercial metal and commercial lead nitrate were employed. Because the atomic weight of radioactive lead has recently been found to differ from that of ordinary lead[2] we also undertook the examination of inactive lead from widely different geographical sources. Lead nitrate was prepared from the various minerals, and after crystallization was converted to lead chloride. The latter in turn was several times crystallized from dilute hydrochloric acid solution, fused and analyzed by comparison with silver. Each specimen of lead nitrate was carefully examined spectroscopically as previously described, and not the slightest difference could be detected between the different specimens. Furthermore the radioactivity of all the minerals was tested in a sensitive gold-leaf electroscope and no one of them was found to increase the normal leak of the instrument by a measurable amount.

In the experiments with lead chloride no attempt was made to collect the silver chloride obtained in the comparisons with silver. Instead, in separate experiments with new portions of the chloride, silver chloride was precipitated with an excess of silver nitrate and the precipitate was collected and weighed.

The analyses in the following table were made by Grover.

THE ATOMIC WEIGHT OF LEAD

	$PbCl_2 : 2Ag$ $Ag = 107.880$				$PbCl_2 : 2AgCl$ $Cl = 35.457$		
SOURCE OF MATERIAL	GRAMS OF $PbCl_2$	GRAMS OF Ag	RATIO $PbCl_2$: 2Ag	ATOMIC WEIGHT OF Pb	GRAMS OF AgCl	RATIO $PbCl_2$: 2AgCl	ATOMIC WEIGHT OF Pb
Commercial Nitrate	5.63567	4.37200	1.28904	207.21			
	5.58730	4.33427	1.28910	207.22			
	6.86319	5.32402	1.28910	207.22			
Average......			1.28908	207.22			
Joplin, Mo, U. S. A.[1]	4.70770	3.65223	1.28899	207.20			
	4.20222	3.25968	1.28915	207.23			
Average			1.28907	207.22			
Wallace, Id. U. S. A.[2]	7.04688	5.46691	1.28901	207.20			
	5.88935	4.56868	1.28907	207.22			
Average			1.28904	207.21			
Wallace, Id. U. S. A.[2]	6.96370				7.17754	0.97021	207.22
	6.89046				7.10231	0.97017	207.21
Average						0.97019	207.21
Tucson, Ariz., U. S. A.[3]	4.90083	3.80171	1.28911	207.22			
	5.79300	4.49404	1.28904	207.21			

[1] Galena; [2] Cerussite; [3] Vanadinite and Wulfenite.

THE ATOMIC WEIGHT OF LEAD—Continued

SOURCE OF MATERIAL	GRAMS OF $PbCl_2$	GRAMS OF Ag	RATIO $PbCl_2$: 2Ag	ATOMIC WEIGHT OF Pb	GRAMS OF AgCl	RATIO $PbCl_2$: 2AgCl	ATOMIC WEIGHT OF Pb
Average			1.28908	207.22			
Metalline Falls, Wash, U. S. A.[1]	5.43965 5.74504	4.21992 4.45674	1.28908 1.28907	207.21 207.22			
Average			1.28906	207.21			
Nassau, Germany[1]	6.57216 5.66330	5.09849 4.39340	1.28904 1.28905	207.21 207.21			
Average			1.28905	207.21			
Eifel Mts., Germany[2]	5.73434 4.17445	4.44857 3.23862	1.28903 1.28896	207.21 207.19			
Average			1.28900	207.20			
New South Wales[2]	6.25884 5.25882	4.85584 4.07933	1.28893 1.28914	207.19 207.23			
Average			1.28904	207.21			
New South Wales[2]	7.33227 6.51699				7.55732 6.71690	0.97022 0.97024	207.22 207.23
Average						0.97023	207.23
Average of all experiments			1.28905	207.21		0.97021	207.22

[1] Galena; [2] Cerussite.

SUMMARY OF ATOMIC WEIGHT VALUES

$PbBr_2$: 2Ag	207.20
$PbBr_2$: 2AgBr	207.18
$PbCl_2$: Ag	207.21
$PbCl_2$: 2AgCl	207.22
Average	207.20

The close agreement of the chloride analyses with those of the bromide is very reassuring; for it is difficult to believe that any constant error, such as hydrolysis of the salts during fusion, could have affected both series of results to the same extent. Furthermore it is obvious that the different specimens of lead chloride are identical within the limit of experimental error, and that ordinary lead, if composed wholly or in part of isotopes, is astonishingly constant in composition.

The results of this investigation indicate (1) that lead bromide and chloride after fusion in atmospheres containing hydrobromic acid and hydrochloric acid respectively are neutral, (2) that no difference exists between specimens of lead of different geographical origin, and (3) that the atomic weight of lead referred to silver as 107.880 is very nearly 207.20.

We are very greatly indebted to the Carnegie Institution of Washington for generous assistance in the pursuit of this investigation.[3]

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[1] *Proc. Amer. Acad.*, **43**, 365 (1907); *J. Amer. Chem. Soc.*, **30**, 187; *Zeit. anorg. Chem.*, **57**, 174.

[2] See, Richards and Lembert, *J. Amer. Chem. Soc.*, **36**, 1329 (1914); M. Curie, *C. R. Paris Acad. Sci.*, **158**, 1676 (1914); Hönigschmid and St. Horowitz, Ibid., **158**, 1796 (1914); Soddy and Hyman, *J. Amer. Chem. Soc.*, **105**, 1402 (1914).

[3] This work will be described in detail before long in *Proc. Amer. Acad. Arts Sci., Boston,* and in *J. Amer. Chem. Soc.*

A REVISION OF THE ATOMIC WEIGHT OF PRASEODYMIUM

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Presented to the Academy, December 16, 1914

Some years ago the atomic weight of neodymium was investigated in the Harvard Chemical Laboratory by the analysis of the chloride;[1] and the success met in obtaining pure material and in the method used, led us to investigate in a similar fashion the atomic weight of the twin element praseodymium. The International Committee upon Atomic Weights has chosen the value 140.6 for this constant, yet some of the recent investigations, notably those of Brauner, have yielded considerably higher results, in the neighborhood of 141.

Naturally, the first problem in the investigation was to prepare pure praseodymium salt. Through the great kindness of Dr. H. S. Miner of the Welsbach Light Company we were fortunate enough to secure as a starting point about 10 kg. of crude praseodymium ammonium nitrate, containing about 50% of the corresponding lanthanum and cerium salts as well as a small amount of neodymium. Since one of the most rapid and effective methods of freeing praseodymium salts from the closely related elements, lanthanum, cerium, neodymium, and samarium is the fractional crystallization of the double ammonium nitrate, we chose this method of purification. In this crystallization the bases separate in the order lanthanum, cerium, praseodymium, neodymium, samarium, and yttrium earths. The usual method of fractional crystallization was followed;[2] that is, the mother liquor of a less soluble fraction was used for dissolving the crystals of the next more soluble fraction. The crystallization proceeded with the occasional rejection of fairly large fractions, chiefly lanthanum and cerium, at the less soluble end of each series, and the rejection of much smaller fractions, partly neodymium, at the more soluble end, until 174 series of crystallizations had been made. The number of fractions in each series rapidly increased to a fairly constant number between 20 and 25. In series 174 an examination of the absorption spectra of the more soluble fractions showed that the extreme fraction contained a very small quantity of neodymium, not more than 0.05%, and that samarium was absent. At this point the seventh fraction from the more soluble end, 3474, was removed for analysis, since it seemed likely that this fraction was as pure as any in the series, and the more soluble fractions were rejected Fractional crystallization was then continued for 41 more series, without, however, rejecting any material at the more soluble end, although the least soluble fraction was occasionally discarded. The most soluble fraction of the 215th series, 4383, upon careful examination of the absorption spectrum, was still found to be free from neodymium, although it was found possible by the absorption spectrum to detect less than 0.05% of the latter element. This result indicates that the other fractions also were free from both neodymium and samarium. The least soluble fraction, 4368, was analyzed for cerium by precipitating the hydroxide and treatment with chlorine, and was found to contain 0.4%. The atomic weight of cerium is so nearly like that of praseodymium that even this proportion would lower the observed atomic weight of praseodymium less than 0.003 unit. Fraction 4371 was found to contain only 0.1% of cerium.

The quantitative examination of selected fractions of the last series was then undertaken. First, the double nitrate was converted into chloride. This was done by precipitating praseodymium oxalate, igniting the washed and dried product to oxide, dissolving the oxide in nitric acid, reprecipitating and igniting the oxalate, dissolving the oxide in hydrochloric acid in a quartz dish, and crystallizing the product from concentrated hydrochloric acid solution. The salt was then superficially dried in a vacuum desiccator

In order to dry the salt for weighing, it was carefully dehydrated by efflorescence at gradually increasing temperatures in a current of dry hydrochloric acid gas. The salt was contained in a weighed platinum boat, and this was placed in a quartz tube forming part of a Richards 'bottling apparatus.'[3] When essentially all the water had been expelled the temperature was raised rapidly until the salt was fused. After the salt had been allowed to cool the hydrochloric acid was displaced by dry nitrogen and this in turn by dry air. Then the boat was transferred to the weighing bottle without exposure to moisture, and it was weighed.

Upon treating salt thus prepared with pure water, sometimes a very small proportion of insoluble matter is found. The proportion of this material depends upon the care with which the salt is first dehydrated previous to fusion. Careful analysis of this material showed it to be praseodymium oxychloride. By exercising every precaution in the drying of the salt and by fusing the salt as rapidly as possible, formation of the oxychloride can either be avoided completely or kept within a limit so small as to produce no important effect upon the final result. These precautions were taken in drying all the specimens of chloride.

The solution of the praseodymium chloride was precipitated in dilute solution with a solution of a weighed amount of the purest silver very nearly equivalent to the chloride. By examination of the mother-liquor with a nephelometer it was determined whether the amount of silver used was exactly equivalent to the chloride, and an initial deficiency or excess was made up with very dilute standard solutions. Finally the silver chloride was collected and weighed. Corrections were applied for moisture retained by the silver chloride and for the silver chloride dissolved in the washings.

The following table gives a comparison of the results obtained with the various fractions of the last series, as well as with the one selected from series 174, Fraction 3474. Of the others, Fraction 4383 is the mos 4368 the least soluble. As a rule 5 to 6 gram portions of the chloride were used. Vacuum corrections were applied to the apparent weights of praseodymium chloride, silver, and silver chloride.

It is apparent that the variations among the different fractions are chiefly fortuitous; for the extreme fractions examined show a difference which is hardly beyond the experimental error of analysis. The atomic weight of praseodymium seems to be very nearly 140.92, if silver is taken

THE ATOMIC WEIGHT OF PRASEODYMIUM

Ag = 107.880 Cl = 35.457

FRACTION	$PrCl_3$: 3Ag	$PrCl_3$: 3AgCl	AVERAGE VALUES
3474		140.913	
3474		140.939	
3474	140.943	140.923	
3474	140.924		
3474	140.914	140.941	
Average	140.927	140.929	140.928
4383	140.910	140.909	
4383	140.914	140.902	
Average	140.912	140.906	140.909
4381	140.908	140.937	
4381	140.920		
4381	140.915	140.925	
4381	140.921		

The Atomic Weight of Praseodymium—Continued

FRACTION	$PrCl_3$: 3Ag	$PrCl_3$: 3AgCl	AVERAGE VALUES
Average	140.916	140.931	140.921
4379	140.910	140.913	
4379	140.915	140.928	
Average	140.913	140.921	140.917
4377	140.917		
4377	140.917	140.951	
Average	140.917		140.928
4374	140.916	140.965	
4374	140.916	140.943	
4374	140.959	140.952	
4374	140.926	140.911	
4374	140.911	140.916	
4374	140.911		
Average	140.923	140.937	140.930
4371	140.915	140.926	
4371	140.922	140.924	
Average	140.919	140.925	140.923*
4368	140.916	140.935	
4368	140.922	140.919	
Average	140.919	140.927	140.925†
Final Average			140.923
Average of all Individual Analyses			140.924

* Increased by + 0.001 units to correct for cerium-content.
† Increased by + 0.002 units to correct for cerium-content.

at 107.880. This result is not far from the value obtained by Brauner in both his researches, but is over three-tenths of a unit higher than the choice of the International Committee on Atomic Weights.

We are indebted particularly to the Carnegie Institution of Washington for very generous assistance in carrying out this investigation,[4] as well as to Dr. H. S. Miner of the Welsbach Light Company for the praseodymium material.

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[1] Baxter and Chapin, *Proc. Amer. Acad.*, 46, 213 (1911).

[2] The crystallization was begun by Mr. W. H. Whitcomb, continued by Messrs. B. W. Grimes and C. C. Wallace, and completed by O. J. Stewart.

[3] Richards, *Proc. Amer. Acad.*, 32, 59 (1896).

[4] This work will be described in detail before long in *Proc. Amer. Acad. Arts Sci., Boston*, and in *J. Amer. Chem. Soc.*

ON THE EARTH CONSIDERED AS A HEAT ENGINE

By George F. Becker

UNITED STATES GEOLOGICAL SURVEY, WASHINGTON

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Imagine that the earth solidified in such a way that fluid equilibrium were perfectly preserved and that the exterior were perfectly smooth, presenting an ideal equipotential surface of uniform temperature. Suppose that the only differences between different portions of this surface were in the diffusivity (that there are large variations in the diffusivities of different rocks, the published determinations clearly show). For simplicity's sake, suppose that a certain square area had a uniform diffusivity smaller than that of the surrounding surface. It is evident that this square would cool more slowly than the adjacent portions, and by reason of the slower escape of heat it would develop a slight relative elevation—provided indeed that the material of the globe contracted in cooling as almost all substances actually do. Furthermore, the relative contraction of the surrounding mass would bring to bear a pressure on all four sides of the square, and this pressure would extend downward as far as the difference of temperature was sensible. Such a pressure might even suffice to rupture the rock within the square. Indeed the order of magnitude of the difference in temperature needful to induce rupture is easily estimated.

Let M be the Young's modulus of the rock composing the square, P the lateral pressure on any face and e the elongation. Then by definition $Me = P$ or $e = P/M$. The crushing load for the best granite is about 1000 kilograms per square centimeter, and the average Young's modulus found by H. Nagaoka[1] for 5 granites is 25×10^{10} dynes or 254800 kg. per square centimeter, only about half as great as that given by Everett for glass. This would indicate at the breaking point $e = 0.004$, very nearly. Now the linear expansion between 0°C. and 100°C. of mineral substances such as glass, quartz, topaz, etc., is not far from 0.08, or some 20 times the value for e just found. Hence a difference of temperature between the square and the surrounding area of the order of 5°C. would be enough to produce rupture in the square column imagined.

Thus a difference in diffusivity would bring about an intumescence on the surface of the globe followed by the formation of four systems of joints which in the simple case supposed would cross one another at right angles at the surface and dip at about 45° in four directions.[2] These joints are equivalent to voids, which as I have recently shown may in extreme cases approach 6.73 percent of the volume.[3] Thus a further very considerable intumescence or uplift would result and an additional diminution in the diffusivity, because joints interfere with the conduction of heat. This diminution would further increase the depth to which sensible differences in temperature on the same horizontal plane would extend. Besides systematic rupture it is evident that the lateral pressures on the square column would or might bring about deformations and the crumpling of layers originally plane.

It would seem then that the initial difference in diffusivity need not be great eventually to ensure a considerable uplift within the hypothetical square since so soon as it sufficed to establish a temperature-difference of a few degrees the process of uplift would be increased by the effects of rupture.

Supposing no water to exist upon the earth, the square column under discussion might attain a notable elevation. The average land surface now stands about 4 km. or two and a half miles above the average sea bottom and if the material from which the salt of the ocean has been derived represents eroded continents of present area, the average land surface might have stood some 2.5 km. higher than it now does. Whether so lofty a mass could sustain its own weight need not be discussed here, the present object being merely to bring out a particular feature of the whole problem.

The mere configuration of the present continents standing at a mean elevation of nearly 13,000 feet above sea bottom manifestly represents a vast amount of energy of position, or potential energy, and to this must be added that of the total mass which has been eroded from the continents, something like half as much as now remains. (Thus, according to the computations made by F. W. Clarke in his *Data of Geochemistry*. the sodium of the ocean represents a spherical shell 2050 feet in thickness. If this is supposed piled onto continents of an area one-fourth that of the globe, its thickness would be 8200 feet, or somewhat more than one half of 13,000 feet.)

The internal temperature of the continental mass likewise calls for consideration. At the present day the mean depth of the ocean is about 3496 m., the means elevation of the land is about 440 m. and the mean thermometric gradient 1° in 38 m. or possibly as high as 1° in 32 m. The temperature at the bottom of the sea is not far from zero, while at the same level beneath the continents it is over 100°. Thus the mean temperature of the actual continents down to the level of sea bottom is more than 50°C. above the temperature of the sea-bottom itself, while for a long distance below this level the sub-continental masses must be hotter than the sub-oceanic layers. (Even if no energy were dissipated within the continental plateaus, the rate at which heat would be conducted from the subterrestrial to the suboceanic regions would be exceedingly low.)

Some of this temperature-excess is probably a remnant of the original temperature of the globe equivalent to that which has failed to escape because of the low diffusivity of the continental rocks. Another portion, probably considerable, represents kinetic energy dissipated in the crushing and plication of the continental mass. Alas that we cannot evaluate the proportion!

On the hypothesis under discussion all of this temperature-excess and all of the energy of position represents heat which would have been radiated into space had all areas of the earth's surface been endowed with equal diffusivity.

The outer shell of the earth down to a depth of perhaps 70 or 80 miles at which the primeval temperature still prevails without sensible diminution may thus be regarded as an imperfect heat engine receiving heat energy at an absolute temperature approaching 2000° and emitting it at less than 300°. The difference is proportional to the energy which would be available were this engine perfect. Though far from perfect, it has sufficed, it seems to me, to supply what has been expended in maintaining in part the relatively high temperature of the sub-continental masses, and also in epeirogenic and orogenic upheavals, in the shattering and crumpling of rocks, and in earthquakes and volcanoes. (It is to be expected that the dissipation of energy would be peculiarly intense near the surface dividing the rising continental columns from the oceanic basins. It is in such positions that most of the volcanoes are found.)

So far the ocean has been practically ignored, but only a few years can have elapsed after the consistentior status before the sea came into being. Even a very small difference in diffusivity acting for a very short time would have served to outline depressions into which the incipient ocean would gather while, after a time at any rate, the presence of the ocean with its convective circulation would tend further to increase the difference in temperature between the areas of relatively great and relatively small diffusivity, which would then become oceanic basins and continental plateaus.

The moment an ocean formed, or rather a moment before it began to form, erosion commenced and introduced a new factor into the world system. Were the globe completely covered by the sea, evaporation and precipitation would furnish no energy of geological significance. The energy absorbed in evaporation would be liberated on precipitation and the molecules of water would return to their original level. But water falling on a continent and carrying sediment to the sea depotentializes or sets free the energy of position acquired at the expense of the heat stored in the earth.

At great depths we know that rocks are deformed and undergo solid flow; and it is well known that under an appropriate system of stresses any solid must flow.[4] At the surface, so far as rocks are concerned, such a system of stresses does not exist, and the rocks do not flow. But erosive action lends them a mobility almost equivalent to fluidity so that the net result is in some respects analogous to that which would ensue if the solid surface of the globe were replaced by a mass of hyperviscous liquid some column or columns of which had a higher temperature than the surrounding matter. These columns would rise above the general surface because of the diminished density and the mounds thus formed would overflow or run down because they lack rigidity. The outflowing portions would cool, and sinking into general mass, would establish a convective circulation.

Not just in the same way, but similarly, erosion effects the flow of the continental surface matter to or beyond the edge of the continental plateaus overweighting the ocean floor and bringing about a corresponding subsidence.

In an asphalt lake like that of Trinidad, convection due to lack of temperature equilibrium would be attended by an undertow. Material rising from any particular depth would diminish the horizontal pressure which it had previously exerted on surrounding portions of the hyperviscous mass and these would press inwards to fill the partial void. In a solid earth there must be an analogous action, excepting that the partial pressure needful to produce lateral flow or undertow must exceed that which would strain the solid rock to its elastic limit.

The analogy of an asphalt lake must not be applied without caution. In such a lake it is easy to conceive of convective circulation indefinitely continued. Not so in the solid earth. If the whole rock mass from which the oceanic salt has been derived was really once piled on the continents and if the ocean is 100×10^6 years old then the total uplift of about $6\frac{1}{2}$ km. has only been effected at the rate of 1 mm. in 15 years or 1 inch in 380 years. Thus the process might be compared with *incipient* convection in an asphalt lake.

None the less, so far as it has gone, the undertow has tended to contract the area of incipient continents, to increase the crumpling and to exaggerate the elevation to which they would have attained had there been no convective tendency.

On the hypothesis that the origin of continents is due to the inferior diffusivity of certain areas of the earth's surface the condition of the ocean's bottom is very noteworthy. As is well known this floor is relatively featureless, consisting of vast plains, low ridges, and a few deeps, seven-eighths of its area lying at a depth of more than a kilometer below sea level. There are very few indications on the ocean floor of continental topography, and yet if a continent were to be submerged to a depth of a hundred fathoms or more, that is below the reach of wave action, it is difficult to see how any process of base-leveling could reduce its accentuation. Neither on the hypothesis under discussion is it easy to see how a continent could be submerged though it is barely possible that a thin layer of rock of small diffusivity might be removed by erosion leaving exposed masses of diffusivity so high as to undergo rapid contraction. Judging from the bathymetrical maps there are no important cases of this description, it would seem that, as the elder Dana so ably maintained, the oceanic areas have been persistent; and if so the subsidences which have occurred and recurred have been subordinate features of movements the net result of which in each case was uplift. This is in line with the results of Hayford, Helmert, and their associates. Since they have compelled us to concede that the earth is even now in a condition of approximate isostatic equilibrium, it seems impossible to believe that it has not been so in the past. Erosion has been in progress during every era from the Algonkian upward and there must have been a persistent and prevailing tendency to upheaval. Of a complete drowning of the continents, such as would occur during a prolonged era in which subsidence prevailed, there is now no trace.

Two reasons have been suggested above for the high level at which the continents stand relatively to the ocean floor, viz., superior temperature and the existence of voids. The difference in level is 3.936 km. or 0.032 of 122 km. If this difference were entirely due to excess of temperature, and if the linear expansion of average rock is 0.0008 per degree, the whole elevation of the continental columns would indicate a mean temperature difference of 40°C. If this elevation were due entirely to the existence of voids, these would amount to about half the maximum interstitial space found for me by Mr. Melcher in experiments on the crushing of sulphur in sealed brass tubes. It is evident that the two causes in combination might bring about elevations not only corresponding to the mean height of the continents but also to those of lofty mountain ranges.

Until erosion began the terrestrial mechanism must be regarded as a heat engine of the irreversible type. It could potentialize energy and do mechanical work, but the cycle was incomplete. When erosion supervened and conferred upon the superficies of the continents a certain mobility and kinetic energy, the cycle was completed and the stage answering the adiabatic expansion was supplied.

It is true that the efficiency of this engine must be very small, but the store of energy upon which it draws—the available boiler capacity—is enormous. The mechanism thus appears competent to bring about all of the dynamical effects with which geology has to deal.

[1] H. Nagaoka, *Phil. Mag.*, **50**, 53 (1900).

[2] Simultaneous joints. *Proc. Wash. Acad.* **7**, 267 (1905).

[3] Becker, *J. Wash. Acad.*, **4**, 429 (1914).

[4] Ibbetson, *Math. Theory of perfectly elastic solids*, 1887, p. 174.

A PHYLOGENETIC STUDY OF CYCADS

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In any phylogenetic study it is desirable to compare related forms which are widely separated in time and space. In case of the cycads, such comparisons are becoming possible. The general plan of an investigation, begun more than ten years ago, is to study numerous forms in the field; study life histories in all parts of the family; and then, by comparative study, to gain some idea in regard to relationships and also in regard to the behavior of various structures during great periods of time.

A typical cycad, with its unbranched trunk and crown of pinnate leaves, looks like a tree fern, an aspect which dates back as far as anything is known about the phylum.

In the Carboniferous, two great lines of seed plants were abundant and widely distributed. One of them is represented today by such familiar forms as the pines, junipers, araucarias and taxads. They are mostly evergreen, are widely distributed, and form immense forests. Being easily available, they have been thoroughly studied, especially the north-hemisphere genera. The other line is represented by a single living family, the Cycadaceae, or cycads, as they are commonly called. The Paleozoic ancestors of the cycads had a world-wide distribution and made such a display that botanists have called the Paleozoic the Age of Ferns, since the leaves were very naturally mistaken for fern leaves. These fern-like seed plants undoubtedly came from an even earlier fern ancestry and, in the Carboniferous, existed side by side with true ferns; but, whether a given specimen should be assigned to the ferns or to these primitive seed plants, cannot be determined by the leaves alone. The only decisive character is the seed.

From splendidly preserved material, English botanists have been making a vigorous study of this group, so that their results are available for comparison. Their researches have covered the external form and also the internal structure of the stem, leaf, root, microsporangium and megasporangium; but, as yet, almost nothing is known about the gametophytes and embryo.

A Mesozoic order—known as the Bennettitales—which may have been the predecessor of the cycads, was also fern-like in appearance. These forms still retained the unbranched trunk and crown of pinnate leaves, so characteristic of ferns and Paleozoic seed plants; but only pinnae of the simplest type persisted and the trunk was short and stocky, so that in general appearance they bore more resemblance to the modern cycads than to their Paleozoic ancestors. The microsporangia were borne on a whorl of leaves, reduced considerably, but still retaining the pinnate character of the foliage leaves. The seed bearing structures, however, were collected into a compact cone, in which no one but a morphologist would recognize the homologues of leaves. Both the pollen producing and the ovule bearing leaves were in the same strobilus, so that the bisporangiate strobilus is a striking feature of the group, or at least, of its upper Mesozoic members. These strobili were small and numerous and were borne in the axils of foliage leaves. The material which has been under investigation has come from Idaho, Dakota, Maryland, Mexico, Europe and India; but this does not mean that the group was confined to the northern hemisphere, for these places are all in the university belt or are easily reached from universities. Forms which may belong here, or may belong to the true cycads, occur in Cretaceous deposits in South Africa, and vast regions remain to be explored.

G. R. Wieland has made the most extensive contributions in this field, but, as in case of the Paleozoic members of the phylum, scarcely anything is known of the gametophytes and not much is known about even the sporophyte structures of Triassic forms.

While I have studied, with great interest, the best material of the Paleozoic and Mesozoic forms, my own research has been confined to the only living family of the phylum, the Cycadaceae, or cycads.

The geological history of the cycads is still very indefinite, largely because stem and leaf characters are hardly sufficient to distinguish true cycads from the Mesozoic Bennettitales. The confident determinations which one sees in museums bring to mind the so-called ferns of the Paleozoic. However, some of the living genera are distinguishable in the Tertiary and it is possible that the family goes back even to the Triassic. The resemblance of the stems and leaves of the living cycads to those of the Bennettitales is too striking to be accidental; on the other hand, there is nothing in the seed-bearing cones of any of the Bennettitales yet described which could have given rise to the seed-bearing cones of the living cycads. Further investigation, especially of Triassic forms, must be made before one can say with any confidence whether the modern cycads have come from the Bennettitales, or both have come from the Paleozoic Cycadofilicales.

The geographic distribution of the living cycads is very restricted. All are tropical or subtropical. Four of the nine genera belong to the western hemisphere and five to the eastern, there being no genus common to both hemispheres. With two exceptions, the genera are very local in their distribution. In the western hemisphere, the thirty-odd species of *Zamia* range from Florida to Chili; while, in the eastern hemisphere, the dozen species of *Cycas* range from Japan to Australia. Two genera are found only in Mexico; one only in Cuba; two only in Australia, one of these being confined to Queensland; and the other two are found only in South Africa. So, with the exception of the two genera which cross the equator, the western genera are in the northern hemisphere and the eastern genera are in the southern. Even in the places mentioned, the plants occur singly or in scattered groups. There is no such things as a cycad forest.

Perhaps, later, it may be possible to give some reason for this peculiar distribution and for the scarcity of individuals.

The nine genera of living cycads are so sharply defined that there is no difficulty in recognizing them. In herbarium specimens, this rigidity extends to species, and, before I began to study cycads in the field, I supposed that the species were rather sharply marked. By the time I had made four excursions into the Mexican tropics, it was evident that species could vary, even in *Dioon* and *Ceratozamia*. However, I was not prepared for the bewildering maze of forms which I found as I tramped the *Macrozamia* regions of Australia. Three species, with erect cylindrical trunks, are easily distinguishable, but I was almost tempted to say, "Call the rest *Macrozamia spiralis*, and let them go." Taxonomists describe several species of the South African genus, *Stangeria*, but, after wandering over nearly the entire range of the genus, from Zululand to Port Elizabeth, I should not be surprised if all of them should be raised from the seeds of a single cone. Some species of the African genus, *Encephalartos*, seem comfortably distinct, while others are so variable that one could identify herbarium specimens with more assurance than he could plants in the field.

An experimental study of variation in cycads is not attractive. *Dioon edule* probably reaches an age of fifty years before it produces cones; *Macrozamia Moorei* has been known to cone at twenty years; some species of *Zamia* may cone at ten years, or even less; but the succession of generations is too slow for any experimental work involving the whole life history. Seedlings of all the genera and many of the species are growing at the University of Chicago and it is evident that there is considerable variation in the seedlings from a single cone. Some work in hybridization is being attempted and one might reasonably hope to see results if any should appear in the first generation.

In comparing Paleozoic, Mesozoic and living members of the phylum, it is evident that the Mesozoic forms are larger than the Paleozoic, and that the living forms are still larger than the Mesozoic. Some of the seeds of the Paleozoic forms were very small, but some were larger than any yet known in the Mesozoic, yet none even approached the large seeds of some of the living cycads. Cones of the Bennettitales can be carried in the pocket, but some of the cones of living cycads reach a weight of ninety pounds. While the sperms of the fossil forms have never been identified with certainty, the structure of the ovules makes it certain that they could not have been nearly so large as those of the living cycads. On the whole, there has been an increase in size as we pass from the Paleozoic to the living forms.

In all the living Gymnosperms, the development of the female gametophyte begins with a series of free nuclear divisions which is followed by cell formation. Undoubtedly, the lowest seed plants came from heterosporous Pteridophytes, and it is more than probable that these heterosporous Pteridophytes of the Paleozoic had female gemetophytes beginning with free nuclear division, although the earliest heterosporous forms, just emerging from the homosporous condition, probably had female gametophytes in which all nuclear division was accompanied by the formation of walls. No homosporous form, either living or fossil, shows free nuclear division at this stage in the life history. Free nuclear division came with the increase in the size of the spore, and the extent of the free nuclear period is more or less correllated with the size of the spore. In the large megaspores of *Dioon* there may be thousands of free nuclei before wall formation begins; in the smaller megaspores of *Zamia*, only hundreds.

That the theory, just expressed, is correct, is indicated by the early development of the sporophyte in Gymnosperms. The eggs are large, and—with two or three exceptions—fertilization is followed by a period of free nuclear division before walls begin to appear. In the two or three exceptions, the egg is small, and the first division of its nucleus is followed by the formation of a wall. In all Angiosperms, the egg is small and a wall follows the first division of its nucleus.

Thus we have a series beginning with the homosporous Pteridophytes in which a wall always followed the first division of the spore, free nuclear division appearing somewhere, as heterospory was attained and the megaspore increased in size; the free nuclear period becoming more prolonged in the larger megaspores of the Gymnosperms, then culminating and beginning to decline in the cycads. The series is even more striking in case of the embryo, since there is the same increase, culmination and decline of the free nuclear period and a few Gymnosperms, together with all Angiosperms have come back to the original condition in which a wall follows the first nuclear division of the egg.

Some of the microscopic details are remarkably constant, e.g., in the pollen tube of all the genera the prothallial cell presses deeply into the body cell. The cycads can be distinguished from all other living seed plants by this cytological character. In all the genera, two blepharoplasts appear in the body cell and, in the two sperms derived from it, develop into coiled bands bearing thousands of cilia by means of which the sperm becomes a vigorous swimmer. Since these details are so uniform, they must antedate the differentiation into the modern genera. On the other hand, there are characteristic differences in the pollen tubes and their contents, so that the pollen tube structures will not only distinguish the family from other seed plants, but will distinguish the genera from each other.

The foregoing paragraphs are intended to indicate the drift of a few phases of an investigation which has been in progress for more than ten years. So far, only scattered descriptive sketches have been published, but all the genera and many of the species have been studied in the field and material has been collected for detailed studies of practically all phases of the life histories. Besides, obliging correspondents in Cuba, Mexico, Australia and Africa are constantly sending field notes, photographs and material, so that, in time, the interrelationships of the genera and the origin of the family may be cleared up and some opinion may be ventured in regard to phylogenetic characters and their gradual modification.

DIMORPHISM IN SIZE OF SPERMATOZOA AND ITS RELATION TO THE CHROMOSOMES

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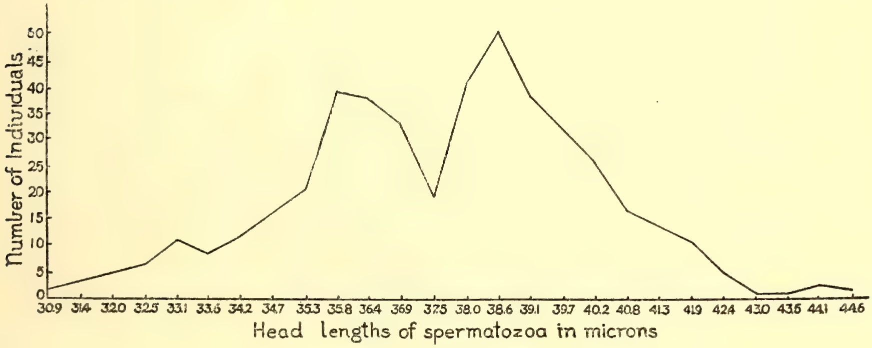
Cytological studies have shown that in the maturation divisions of a large number of species of animals the cells which are to give rise to the spermatozoa are of two kinds, differing from each other in the nature of their chromatin content. The resulting spermatozoa therefore are assumed to be likewise of two kinds. A large mass of indirect evidence makes it highly probable that eggs fertilized by one of these produce only males and those fertilized by the other only females.



Explanation of Figure

Frequency distribution of head-lengths of 444 spermatozoa from right testis of Musca domestica.

Value in microns	30.9	31.4	32.0	32.5	33.1	33.6	34.2	34.7	35.3	35.8	36.4	36.9	37.5
Frequency	1	3	5	6	11	9	12	16	20	39	38	33	20
	38.0	38.6	19.1	39.7	40.2	40.8	41.3	41.9	42.4	43.0	43.5	44.1	44.6
	40	50	39	33	27	17	14	11	5	1	1	2	1

Direct experimental test of this hypothesis has not been made because recognition of the difference has been confined to the immature stages. It is therefore of importance to see if the two kinds can be made out among the completed functional spermatozoa. The present studies were planned with a view to such a determination. If there is a difference in the amount of chromatin received by the two kinds of spermatids it is natural to expect a corresponding difference of size in the heads of the completed spermatozoa since these are made up almost wholly of chromatin material.

The head-lengths of spermatozoa from fifteen species of animals were measured. The number of measurements for a single determination ranged from 322 to 1008 with an average of 523, including in each case only the spermatozoa from a single testis. A considerable variation in length was present and when the frequency distributions were plotted the resultant curves were found to be distinctly bimodal in fourteen of the fifteen species. One of these curves is shown in the accompanying figure. The complete data are summarized in a table. The inference is drawn that there are two groups as regards size among the functional spermatozoa of these species, the group with the larger individuals in each case consisting of those which the studies in spermatogenesis have shown to have the larger chromosomal volumes.

The species for which these determinations have been made belong to several animal groups:

Diptera:	Musca domestica, three determinations.
Heteroptera:	Lygaeus palmii, two determinations.
	Alydus pilosulus, two determinations.
	Anasa tristis, ten determinations.
Coleoptera:	Trirhabda tomentosa, one determination.
	Phytonomus punctatus, two determinations.
Orthoptera:	Melanoplus femur-rubrum, one determination.
	Melanoplus differentialis, three determinations.
	Gryllus abbreviatus, one determination.
Odonata:	Aeshna canadensis, one determination.
Amphibia:	Rana pipiens, one determination.
Reptilia:	Pseudemys troosti, two determinations.
Mammalia:	Ovis aries, one determination.
	Bos taurus, one determination.
	Canis familiaris, one determination.

In all but one of these species two size-groups were recognized. The exception is the one determination of Gryllus abbreviatus. Chromosomal dimorphism has been described for all of the groups except the Amphibia, in most of the cases for the particular species here studied.

A further proof of the close relation between the chromosomal dimorphism of the spermatids and the dimorphism in size of the adult spermatozoa was sought in a determination of the volume of chromosomal material in each of the two kinds of spermatids. From the volumes obtained the expected ratio between the head-lengths of the resultant spermatozoa was calculated under the assumption that the volume of the heads is directly proportional to the amount of the chromatin received and that the shape is the same in the different sizes.

For three of the species studied, Musca domestica, Alydus pilosulus and Anasa tristis, there are good published figures of the chromosomes in the spermatocyte divisions. From these figures it is possible to make a rough estimate of the total chromosomal volume in each of the two kinds of cells. The expected ratio between the head-lengths of the two kinds of resulting spermatozoa was calculated from these data. A close agreement was discovered between the calculated ratios and the observed ratios. For Musca domestica the calculated ratio is 1.00 : 1.08 and the average of the observed ratios 1.00 : 1.07. For Alydus pilosulus these ratios are 1.00 : 1.06 and 1.00 : 1.055. For Anasa tristis the calculated ratio is 1.00 : 1.11 and the average of the observed ratios 1.00 : 1.09 with six of the nine determinations located between 1.00 : 1.10 and 1.00 : 1.12

While there undoubtedly is a considerable error in making minute measurements like those here described and while there may be doubt as to the significance of some of the individual determinations yet the data when taken as a whole must be interpreted as showing the very general existence of dimorphism in size among the completed spermatozoa. The evidence further makes it very probable that this dimorphism is the result of the chromosomal dimorphism present in the spermatids. If this conclusion is a correct one the hypothesis that the chromosomal dimorphism is related to sex determination may be subjected to direct test as soon as a method can be devised for separating the different sizes of living spermatozoa.

TABULATION OF DATA ON HEAD-LENGTHS OF SPERMATOZOA

The observed ratio is the ratio between the two modal values. The expected ratio is calculated from the chromosomal difference between the two kinds of spermatids.

(*over*)

NAME	NUMBER OF SPERMATOZOA	HEAD-LENGTH LOWER MODE IN MICRONS	HEAD-LENGTH UPPER MODE IN MICRONS	OBSERVED RATIO	EXPECTED RATIO
1. Musca domestica	444	35.8	38.6	1.00 : 1.08	1.00 : 1.08
2. Musca domestica	465	35.8	38.6	1.00 : 1.08	1.00 : 1.08
3. Musca domestica	769	36.4	38.0	1.00 : 1.04	1.00 : 1.08
4. Lygaeus turcicus					1.00 : 1.02
5. Lygaeus palmii	493	36.8	38.3	1.00 : 1.04	
6. Lygaeus palmii	501	36.8	37.8	1.00 : 1.03	
7. Alydus pilosulus	429	31.9	33.9	1.00 : 1.06	1.00 : 1.06
8. Alydus pilosulus	469	32.25	33.75	1.00 : 1.05	1.00 : 1.06
9. Anasa tristis	653	28.35	30.2	1.00 : 1.07	1.00 : 1.11
10. Anasa tristis	391	17.5	30.2	1.00 : 1.10	1.00 : 1.11
11. Anasa tristis	370	28.1	30.8	1.00 : 1.10	1.00 : 1.11
12. Anasa tristis	443	17.5	30.8	1.00 : 1.12	1.00 : 1.11
13. Anasa tristis	404	28.1	30.8	1.00 : 1.10	1.00 : 1.11
14. Anasa tristis	384	28.6	30.8	1.00 : 1.08	1.00 : 1.11
15. Anasa tristis	394	28.1	30.8	1.00 : 1.10	1.00 : 1.11
16. Anasa tristis	322	30.8	31.8	1.00 : 1.03	1.00 : 1.11
17. Anasa tristis	993		31.5		1.00 : 1.11
18. Anasa tristis	444	28.4	31.5	1.00 : 0.11	1.00 : 1.11
19. Trirhabda virgata					1.00 : 1.06
20. Trirhabda tomentosa	481	17.02	17.78	1.00 : 1.045	
21. Phytonomus punctatus	506	33.0	34.0	1.00 : 1.03	
22. Phytonomus punctatus	507	33.3	35.3	1.00 : 1.06	
23. Melanoplus femur-rubrum	491	80.5	83.0	1.00 : 1.03	
24. Melanoplus differentialis	481	88.0	90.5	1.00 : 1.03	
25. Melanoplus differentialis	1008	88.8	90.5	1.00 : 1.02	
26. Melanoplus differentialis	734	88.5	90.9	1.00 : 1.03	
27. Gryllus abbreviatus	552				
28. Anax junius					1.00 : 1.07
29. Aeshna canadensis	496	50.2	50.6	1.00 : 1.03	
30. Rana pipiens	494	9.74	10.6	1.00 : 1.09	
31. Pseudemys troosti	501	10.43	10.91	1.00 : 1.05	
32. Pseudemys troosti	487	10.50	11.36	1.00 : 1.08	
33. Ovis aries	498	5.94	6.37	1.00 : 1.07	
34. Bos taurus	606	8.05	8.33	1.00 : 1.035	
35. Canis familiaris	540	5.17	5.55	1.00 : 1.07	

AN INTERPRETATION OF SELF-STERILITY

By E. M. East

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In certain hermaphroditic animals and plants, self-fertilization is often impossible. This gametic incompatibility has been called self-sterility. In the vegetable kingdom it is known to be comparatively widespread; in the animal kingdom, though it may be found later to be characteristic of many species, as yet only the Ascidian *Ciona intestinalis* has furnished material for study of the problem. (See Morgan,[1] Adkins, in Morgan,[2] and Fuchs.[3])

Ciona is not perfectly self-sterile. Individuals appear to vary in degree of self-sterility, though no case has yet been found where self-fertility is equal to cross-fertility. Morgan believes that there is a great difference in the compatibility of ova to sperm from other individuals, though Fuchs maintains that 100% of segmenting eggs can be obtained in every cross with normal ova if a sufficiently concentrated sperm suspension is used.

Fuchs has shown a chemical basis for the phenomenon by the difference in ease of cross-fertilization after contact of ova with sperm from the same animal and by the variation in ease of self-fertilization after certain artificial changes in the chemical equilibrium of the medium surrounding the ova, and by this work has brought the matter of self-sterility in Ciona in line with that in Angiosperms as worked out by Jost.[4]

Jost has shown that in the plants with which he worked only short tubes were formed after pollination with pollen from the same plant, though the necessary length of pollen-tube was easily developed after cross-fertilization. He saw as cause of these phenomena a chemotropism due to the presence of 'individueller Stoffe.' Pollen was indifferent to 'Individualstoff' from the same plant, but was stimulated by that from other plants.

To Correns[5] such an explanation of self-sterility seemed too general. He believed that a simple interpretation would account for the results he had obtained from *Cardamine pratensis*. Two plants B and G were crossed reciprocally and sixty of the offspring tested by pollinating from the parents, on the parents and from sisters. The back crosses of (B × G) or (G × B) with B and with G seemed to him to indicate four equal-sized classes with reference to gametic compatibility: (1) plants fertile with both B and G; (2) plants fertile with B but not with G; (3) plants fertile with G but not with B; (4) plants fertile with neither B nor G.

These facts were interpreted by assuming the existence of two independently inherited factors that inhibit the growth of pollen-tubes. Representing these factors by the letters B and G, the original plants must be supposed to have had the formulae Bb and Gg respectively, since it is clear that type BB and GG could never be formed. When Bb is crossed with Gg the four types BG, Bg, bG and bg should result, of which the first three should be self-sterile. Plants BG should be fertile with plants bg, plants Bg should be fertile with bG and bg, plants bG should be fertile with Bg and bg, while plants bg should be self-fertile as well as cross-fertile with the other three classes. Attractive as this theory is, it is not clearly in accord with the facts. Plants of the type bg—inherently self-fertile—were not found, and the other classes showed many discrepancies.

Morgan[2] has offered another hypothesis that fits the data from both plants and animals. If I have not misunderstood the meaning of his rather general statement of the proposition, my own theory is only an extension of it, laid down perhaps a little more specifically. He says:

> The failure to self-fertilize, which is the main problem, would seem to be due to the similarity in the hereditary factors carried by the eggs and sperm; but in the sperm, at least, reduction division has taken place prior to fertilization, and therefore unless each animal was homozygous (which from the nature of the case cannot be assumed possible) the failure to fertilize cannot be due to homozygosity. But both sperm and eggs have developed under the influence of the total or duplex number of hereditary factors: hence they are alike; i.e., their protoplasmic substance has been under the same influence. In this sense, the case is like that of stock that has long been inbred, and has come to have nearly the same hereditary complex. If this similarity decreases the chances of combination between sperm and eggs we can interpret the results.

My own work has been done with the descendants of a cross between *Nicotiana forgetiana* (Hort) Sand., a small red-flowered species, and *Nicotiana alata* Lk. and Otto. var. *grandiflora* Comes, the large white-flowered sort commonly known as *Nicotiana affinis*. Both parents were undoubtedly self-sterile as over 500 plants of the F_1, F_2, F_3, and F_4 generations have been found to be self-sterile by careful tests.

Several experiments were made in which selfing, crossing *inter se*, and back crossing were done on a large scale, using plants of the F_2,F_3 and F_4 generations which had segregated markedly in size and were of at least eight different shades of color. In the F_2 generation, twenty plants coming from two crosses between F_1 plants were selected for experiment. Each was selfed many times and in addition 131 inter-crosses were attempted, from four to twelve flowers being used in each trial. All attempts at selfing failed, while only two attempts at crossing were unsuccessful. Of the 129 successful inter-crosses, all but 4 produced full capsules, and it is probable that even this variability in ease of cross-fertilization was caused by attending conditions. One hundred and twenty other inter-crosses were made in the F_2 generation, with three failures.

In the F_3 generation, about one hundred inter-crosses were made between twelve plants which were the progeny of two sister F_2 plants. Six of these attempts failed.

In the F_4 generation, fifty-eight inter-crosses were made between ten plants that were the daughters of two F_3 plants. Fifty-three of these cross-fertilizations were successful.

Back crosses also were made in considerable numbers, though not to the extent one might desire. Plants A, B, C and D were combined in four different ways and among the plants resulting from these combinations eighty-five back crosses were attempted, of which five failed.

These facts will not fit any simple Mendelian formula similar to that proposed by Correns; furthermore, data from an experiment of a different kind appear to support Jost's idea of 'Individualstoffe' rather than Correns' idea of inhibitors. Pairs of plants were provided to furnish series of selfed and crossed flowers. The pistils of these flowers were fixed at regular periods after pollination, stained, sectioned and the pollen-tubes examined. Since the flowers on each plant had about the same length of pistils, curves of pollen-tube development for both crossing and selfing could be constructed. The pollen grains germinated perfectly on stigmas from the same plant, from 1200 to 2000 tubes having been counted in sections of single pistils. The difference between the development of the tubes in the selfed and the crossed styles was wholly one of rate of growth. The tubes in the selfed pistils developed steadily at a rate of about 3 mm. per twenty-four hours, with even a slight acceleration of this rate as the tubes progressed. If the flowers were sufficiently long-lived, one could hardly doubt but that the tubes would ultimately reach the ovules, though this would not necessarily mean that fertilization must occur. Since the maximum life of the flower is about 11 days, however, the tubes never traverse over one-half of the distance to the ovary. On the other hand, the tubes in the crossed pistils, though starting to grow at the same rate as the others, pass down the style faster and faster, until they reach the ovary in four days or less.

From these facts it seems reasonable to conclude that the secretions in the style stimulate the pollen-tubes from other plants instead of inhibiting the tubes from the same plant.

The whole question, therefore, becomes a mathematical one, that of satisfying conditions whereby no stimulus is offered to pollen-tubes from the same plant, but a positive stimulus is offered to tubes from *nearly* every other plant.

The nearly constant rate of growth of pollen-tubes in the pistils of selfed flowers, compared with the regular acceleration of growth of the tubes from the pollen of other plants, undoubtedly shows the presence of stimulants of great specificity akin to the 'Individualstoffe' of Jost, though I believe their action to be indirect. Experiments by several botanists, which I have been able partially to corroborate, point to a single sugar, probably of the hexose group, as the direct stimulant. The specific 'Individualstoffe' I believe to reside in the pollen grains and to be in the nature of enzymes of slightly different character, all of which except the one produced by the plant itself for the use of its own pollen, or by another individual of the same genotype, can call forth secretion of the sugar that gives the direct stimulus. At least this idea links together logically the fact of the single direct stimulus and the need of 'Individualstoffe' to account for the results of the crossing and selfing experiments But whether or not this be the correct physiological inference, the crossing and selfing experiments call for an hypothesis that will account for no stimulation being offered the tubes from the plant's own pollen while at the same time great stimulation will be given the tubes from the pollen of *nearly* every other plant. This is a straight mathematical problem, and it is hardly necessary to say that it is insoluble by a strict Mendelian notation such as Correns sought to give. This is obvious to anyone familiar with the basic mathematics of Mendelism. On the other hand a near Mendelian interpretation satisfies every fact.

Let us assume that different hereditary complexes stimulate pollen-tube growth and in all likelihood promote fertilization, and that like hereditary complexes are without such effect. One may then imagine any degree of heterozygosis in a mother plant and therefore any degree of dissimilarity between the gametes it produces, without there being the possibility of a single gamete having anything in its constitution not possessed by the somatic tissues of the mother plant. From the chromosome standpoint of heredity the cells of the mother plant are duplex in their organization: they contain N pairs. The cells of the gametes contain N chromosomes, one coming from each pair of the mother cell; but they are all parts of the mother cell and contain nothing that that cell did not contain. These gametic cells cannot reach the ovaries of flowers on the same plant because they cannot provoke the secretion of the direct stimulant from the somatic cells of that plant.

All gametes having in their hereditary constitution something different from that of the cells of a mother plant, however, can provoke the proper secretion to stimulate pollen-tube growth, reach the ovary before the flower wilts, and produce seeds. Such differences would be very numerous in a self-sterile species where cross-fertilization must take place; nevertheless like hereditary complexes in different plants should be found, and this should account for the small percentage of cross-sterility actually obtained. It must be granted that this hypothesis satisfies the facts, but that is not all. It is admittedly a perfectly formal interpretation, but from a mathematical standpoint—granting the generality of Mendelian inheritance—it is the only hypothesis possible that can satisfy the facts.

In conclusion it should be mentioned that the cross-pollinated pistils show a considerable variation in the rate of growth of individual pollen-tubes, though our curves of growth have been made by taking the average rate of elongation. Is this variation a result of chance altogether or must one assume a differential rate of growth increasing directly with the constitutional differences existing between the somatic cells and the various gametes? If it is assumed that any constitutional difference between the two calls forth the secretion of the direct stimulus to growth, chance fertilization by gametes of every type different from that of the mother plant will ensue; if there is a differential rate, selective fertilization will occur. This question cannot be decided definitely at present, but two different lines of evidence point toward chance fertilization:

1. Flowers from a single plant pollinated by different males show no decided difference in rate of fertilization.

2. Color differences are transmitted to expected ratios.

Further, it will be recalled that beginning with the F_2 generation sister plants crossed together have given us our F_3 and F_4 populations, and that these F_3 and F_4 populations apparently have given a constantly increasing percentage of cross-sterility. This is what should be expected under the theory that a small difference in germ plasm constitution is as active as a great difference in causing the active stimulation to pollen tube growth. Breeding sister plants together in succeeding generations causes an automatic increase of homozygosity as is well known. This being a fact, cross-sterility should increase. Such an increase in cross-sterility has been observed in the F_3 and the F_4 generations, but it would not be wise to maintain dogmatically that it is significant.

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[3] Fuchs, H. M., On the conditions of self-fertilization in Ciona. *Archiv. f. Entwickl. d. Org.*, **40**, 157-204 (1914); The action of egg-secretions on the fertilizing power of sperm. *Archiv. f. Entwickl. d. Org.*, 40, 205-252 (1914).

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THE BASAL CALORIC OUTPUT OF VEGETARIANS AS COMPARED WITH THAT OF NON-VEGETARIANS OF LIKE WEIGHT AND HEIGHT

By Francis G. Benedict and Paul Roth

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Among the numerous, more or less scientifically attested, facts concerning the influence of a vegetarian diet are statements relative to an observed increase in endurance and the belief that vegetarians live upon a somewhat lower metabolic plane than do flesh eaters, who, it is asserted, are unduly stimulated by the protein in their food. The measure of the basal gaseous metabolism, which may be considered as the carbon-dioxide production and oxygen consumption during complete muscular repose and in the post absorptive condition, that is, at least 12 hours after the last meal, gives an admirable index of the metabolic activity.

We have been able to make observations on the basal gaseous metabolism of 11 men and 11 women, who had subsisted upon a vegetarian diet for a considerable period of time, extending, on the average, over several years. With the subjects in the post absorptive condition and lying quietly upon a comfortable couch, the total carbon-dioxide production and oxygen consumption of each of these individuals were measured on several days by means of the universal respiration apparatus. As a rule the values obtained agree well with each other and the averages may be looked upon as reasonably accurate indices of the actual basal metabolism from which the energy output may be computed.

In order to study particularly the influence of a vegetarian diet, it was necessary to compare these values for vegetarians with those obtained on non-vegetarians of the same height and weight. A sufficiently large number of observations on non-vegetarians, including both men and women, was available for a satisfactory comparison of this kind. The body weights of the men ranged from 75 kg. to 49.3 kg. and of the women from 93.6 kg. to 40.0 kg. A comparison of the heat production per 24 hours as computed from the gaseous exchange showed that the men vegetarians produced 25.5 calories per kilogram and the non-vegetarian men of like height and weight 26.4 calories. On the commonly used, yet questionable, basis of the heat per square meter of body surface per 24 hours the vegetarians showed 798 calories as compared with 828 calories for the non-vegetarians. With the women the corresponding values per kilogram of body weight were 24.6 calories for the vegetarians and 25.0 calories for the non-vegetarians; and per square meter of body surface 753 calories for the vegetarians compared with 766 calories for the non-vegetarians.

Believing that the relatively large proportion of carbohydrate supposedly eaten in the vegetarian diets might tend to a larger storage of body glycogen, thus giving available carbohydrate material to be drawn upon in the endurance and similar tests of muscular efficiency, a computation was made in all cases of the respiratory quotient, i.e., the relationship between the carbon-dioxide production and oxygen consumption. When the katabolism is exclusively from carbohydrate this quotient is 1.0; with pure fat it is 0.70. For the 22 vegetarians (11 men and 11 women) the average quotient was found to be 0.83 while the average for the 132 non-vegetarians (77 men and 55 women) it was 0.81. The mathematical difference between these average respiratory quotients is too slight to be taken as evidence of a larger glycogen storage.

The results show, therefore, that the gaseous metabolic processes of the vegetarians are qualitatively and quantitatively essentially those of non-vegetarians of similar height and weight with whom they are compared.

The detailed report of this study has been transmitted to the *Journal of Biological Chemistry*.

THE INFLUENCE OF ATHLETIC TRAINING UPON BASAL METABOLISM

By Francis G. Benedict and H. Monmouth Smith

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Presented to the Academy, January 13, 1915

Two fundamental conceptions exist with regard to the heat production of the body. One is that the heat production is proportional to the active mass of protoplasmic tissue; the other, and more generally accepted view, is that the heat production is proportional to the surface area of the body. To throw light upon this question we have made an investigation upon the metabolism of a number of trained college athletes in whom the proportion of active protoplasmic tissue would undoubtedly be greater than that in the body of a normal, untrained individual. In considering the results of this study, the differences in surface area have been eliminated by making the comparisons only between groups of individuals with like height and weight.

Fifteen athletes, varying in weight from 88.5 kg. to 56.3 kg., were studied by means of the universal respiration apparatus, the subjects being in a condition of complete muscular repose and in the post absorptive state, i.e., at least 12 hours after the last meal. Only the average values of several closely agreeing tests with each subject are used in the comparisons. From an extended series of metabolism experiments on normal individuals a sufficient number of suitable non-athletes of similar height and weight are selected for a series of comparisons which are made in ten groups. Of these ten groups the first four are given in the table herewith, thus serving to indicate the method of comparison as well as the actual values found in the four groups. In the complete comparison we found that in the ten groups the heat production per kilogram of body weight was greatest with the athletes in all but two groups, and in these two there was no difference between the athletes and non-athletes. The heat production per kilogram per 24 hours averaged for the athletes 26.0 calories and for the comparable non-athletes, 24.4 calories.

Per square meter of body surface the athletes again exceeded the metabolism of the non-athletes in all groups with but slight increases in 3 groups. The average heat production per square meter per 24 hours was 863 calories with the athletes and 807 calories with the non-athletes.

In general there was a distinct tendency for the athletes to have a measurably larger basal resting metabolism (about 7%) than the non-

COMPARISON OF THE HEAT PRODUCTION OF ATHLETES AND NORMAL NON-ATHLETIC MEN.

Group and subject	Nude weight	Height	HEAT PRODUCTION PER 24 HOURS (computed) Total	Per kilogram	Per square meter
	kilos	*cms.*	*cals.*	*cals.*	*cals.*
Group I					
Athlete					
W. S.	88.5	165	2017	22.8	823
Non-athlete					
O. F. M.	85.8	171	1827	21.3	761
Group II					
Athletes					
J. H. R.	82.2	187	1978	24.1	849
D. H. W.	82.1	186	2034	24.8	873
M. H. K.	79.0	188	1944	24.6	856
E. G.	78.9	184	2126	27.0	940
Non-athletes					
F. G. B.	83.1	183	1802	21.7	770
W. A. M	78.0	183	1816	23.3	807
Group III					
Athlete					
F. G. R.	74.0	179	1914	25.9	882
Non-athletes					
W. J. T.	74.2	183	1770	23.9	816
C. B. S.	71.1	179	1700	23.9	806
Group IV					
Athletes					
C. D. R.	74.0	173	1908	25.8	879
H. R. W.	73.9	175	1842	24.9	848
Non-athletes					
Dr. M.	75.9	175	1877	24.7	849
J. P. C.	73.7	169	1526	20.7	706
H. W. E.	73.0	168	1559	21.4	725

athletes with whom they were compared. Since in these comparisons we considered only individuals of similar height and weight, it is clear that these differences were not due to changes in body surface, but from what is known with regard to athletic training we may maintain that the increased katabolism is directly due to the larger proportion of active protoplasmic tissue in the body of the athletes, thus suggesting that the katabolism of the body is proportional not to the surface area, but to the mass of active protoplasmic tissue.

The detailed report of the investigation has been transmitted to the *Journal of Biological Chemistry*.

A COMPARISON OF THE BASAL METABOLISM OF NORMAL MEN AND WOMEN

By Francis G. Benedict and L. E. Emmes
NUTRITION LABORATORY, CARNEGIE INSTITUTION OF WASHINGTON

Presented to the Academy, January 13, 1915

From the earliest attempts to adjust food intake to the energy requirement it has been recognized that the dietetic needs of men as a class are somewhat greater than those of women. This increase has been commonly ascribed in large part to the variations in the muscular activity and yet there has been a definite belief that the basal energy requirement for women may be materially different from that for men. In connection with observations made on a large number of normal men and women, primarily for the purpose of comparing them with pathological subjects, we have accumulated the results of observations on 89 men and 68 women, all of whom were in 'presumably good health.' The experiments were made with essentially the same technique and with the subject in the same condition of muscular repose and the post-absorptive state, i.e., 12 hours after the last meal. Under these conditions, differences due to muscular activity are entirely eliminated and we obtain the basal normal caloric output of the individuals studied.

The total heat production, computed on the 24-hour basis, was greater with the men, i.e., 1638 calories as against 1355 calories with the women. Of special interest is the fact that the oxygen consumption per kilogram per minute—a unit of measurement commonly employed by physiologists—was remarkably uniform with both sexes, being 3.65 cc. with the men and 3.58 cc. with the women. On the basis of heat-production per 24 hours, the men showed 25.5 calories per kilogram of body weight and the women 24.9 calories. On the debatable unit of apportionment of heat-production per square meter of body surface the men had 832 calories per 24 hours against 772 calories for the women.

It should be borne in mind, however, that the 89 men had an average body-weight, without clothing, of 64.3 kg. and the 68 women a body-weight of but 54.5 kg.; furthermore, the average height of the men was 172 cm. and that of the women 162 cm. A direct, unqualified comparison of the total metabolism of these two classes, or of the metabolism per kilogram of body weight or per square meter of body surface is, therefore, not permissible for it is not logical to compare two groups of individuals with unlike weight and height.

From the groups of men and women it was possible to make selections of a number of individuals who had approximately the same height and weight; on this basis alone can an intelligent comparison be made. Eleven groups, with body weights varying from 67 kg. to 45 kg., were compared and it was found that the average heat-production of all eleven groups, computed on the basis of per kilogram of body weight, was 26.5 calories with men and 25.0 calories with women. The heat output of the men was greater in all but three of the eleven groups.

Using again the computation on the basis of the heat production per square meter of body surface we found that the average for the eleven groups showed 819 calories per 24 hours for the men as compared with 770 calories for the women. Of the eleven groups only two had lower values for the men than for the women.

On the basis of the foregoing paper, giving the results of a study of athletes, we rigorously excluded athletes from these comparisons, and hence we are dealing here with non-athletic men and women of the same height and the same weight. It is thus reasonable to suppose that the actual body surface of the different groups must have been very nearly the same, and it is not logical to assume that the larger heat production noted with the men was due to a disproportion between the body measurements and the body surface. We believe that these data show a basal metabolism for men some 5 or 6% greater than for women of similar height and weight, and that this increase is due to the fact that in all probability the women, particularly in those groups with the greater body weight, had a much larger proportion of subcutaneous fat than did the men, thus indicating a consequent smaller proportion of active protoplasmic tissue.

The detailed report of this investigation has been transmitted to the *Journal of Biological Chemistry.*

THE FACTORS AFFECTING NORMAL BASAL METABOLISM

By Francis G. Benedict

NUTRITION LABORATORY, CARNEGIE INSTITUTION OF WASHINGTON

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Although it is well known that the basal normal metabolism of individuals varies considerably, the exact cause for these variations is not known. For many years physiologists have believed that the heat production of a living body is proportional to the surface area. This most helpful hypothesis served to bring order out of chaos in the earlier study of energy requirements and transformations in the living organism, but as experimental evidence accumulated, the validity of this so-called 'law of surface area' has been frequently called into question. An extended series of observations on 89 men and 68 women, all of them normal individuals in presumably good health, together with observations upon a group of athletes, a large number of new-born infants, normal infants under one year, and atrophic infants, and particularly a recent 31-day study of a fasting man, have given us data which warrant a reconsideration of the factors affecting basal metabolism.

When all of the observations on the normal men and women are plotted on charts, it is seen that there is no direct relationship between body weight and the total metabolism other than that, in general, people of large body weight have a greater metabolism than smaller individuals. However, there are so large a number of exceptions to this general rule that nothing approximating a physiological law can be derived from these observations. The same is true with regard to the computations of the heat production per kilogram of body weight, for the widest variations are found with our so-called normal people, with no tendency towards uniformity. Furthermore, it has been shown in a foregoing article that the total metabolism and the metabolism per kilogram of body weight are distinctly larger with athletes than with normal individuals. In all of these comparisons the differences between men and women have been recognized; hence separate plots for men and women have been made.

It has been practically impossible to make direct surface measurements of the men and women that we have studied, and we have relied, as have all physiologists hitherto, upon the commonly accepted formula of Meeh, in which the body surface is considered to be the cube root of the square of the weight multiplied by the constant 12.312. For infants the slightly smaller constant of 10.3, determined by Lissauer, has been accepted as the best available. Using Meeh's formula, we find that the metabolism of normal men, as computed from the body surface and expressed in calories per square meter of body surface per 24 hours, ranged from 693 to 958 calories. Since, according to the currently accepted belief in the constancy of heat production per square meter of body surface, we would expect to find constant values with all individuals, irrespective of size, it can be seen that this variation is extraordinarily large. It should furthermore be stated that both of these extremes were found with non-athletic individuals. With women the range was from 633 calories per square meter per 24 hours to 906 calories.

In neither of these comparisons is there any tendency to regularity or to a grouping of the plots. In other words, the evidence all points towards distinct individuality with no relationship between the computed body surface and the heat production.

With infants the variations in the metabolism on this basis are even greater. Thus it was found with normal and atrophic infants that the 24-hour heat production per square meter ranged from 554 calories to 1334 calories, while with the strictly normal infants the range was from 554 to 991 calories. It is obvious that any basis of comparisons which involves possible variations of 40% with men, 43% with women and 80% with normal infants cannot be considered as a physiological law.

On the other hand, an examination of the available material shows that there is a relationship between the heat production and the body composition, that is, that any physiological data which imply a difference in the proportion of active protoplasmic tissue are invariably accompanied by a difference in the basal metabolism. Thus the trained athletes showed a distinctly greater metabolism than did the non-athletic individuals. Furthermore, men with a smaller amount of subcutaneous fat and a correspondingly greater proportion of active protoplasmic tissue have been found to have a greater metabolism than women of the same height and weight. The study of normal and atrophic infants showed that with two infants of the same height and weight the elder, who would naturally be somewhat atrophic, invariably had a higher basal metabolism than the normal, well-nourished infant. Even with normal adults it can be maintained that of two individuals having the same weight but different heights, the taller individual will, in general, have the greater proportion of active protoplasmic tissue and the comparison of the heat production of normal men of like age and weight but different heights shows that there is almost invariably a distinctly greater metabolism with the taller individual. It should be remembered that in all of these comparisons only the basal metabolism, measured in complete muscular repose and in the post absorptive condition, is used.

A factor that has heretofore been neglected in considering basal metabolism is the possibility that the mass of active protoplasmic tissue may functionate with varying degrees of intensity. In comparing the metabolism of normal individuals we find that frequently individuals with approximately the same weight had very great variations in the heat production. This is true of a group of eight men weighing over 80 kgm. and likewise with a group of five men weighing 50 kgm. or under.

This, of itself, points strongly towards distinct differences in the intensity of cellular activity.

While the larger proportion of individuals studied were in youth, i.e., 20 to 30 years of age, a few were under 17 and a number over 40. On studying the plots showing the variations with normal individuals, both for men and women, a distinct tendency may be noted for the older people of both sexes to have a somewhat decreased metabolism. The evidence is equally as clear that in youth the metabolism is considerably increased, thus pointing towards an increased cellular activity in early youth and a decreased cellular activity or possible atrophy of active protoplasmic tissue with increasing age.

One of the most noticeable factors influencing metabolism is that of sleep. It has been commonly assumed that sleep *per se* does not affect metabolism, but it has been found in a long study of a fasting man that there were numerous metabolic planes during the day, showing that the stimulus to cellular activity must have varied considerably. With this fasting man neither the mass of protoplasmic tissue nor the surface area of the body could have altered materially in the course of 24 hours, yet we find that if we give a value of 100 to the basal metabolism during the night, when the subject was sound asleep, the value when the subject was lying awake in the morning, with complete muscular repose, would be 114, and in the late afternoon under the same conditions the value would be 122.

Our laboratory data include experiments with a considerable number of normal men covering several months, and in some cases, several years. A study of these data shows that the metabolism of 35 subjects on whom experiments were made 5 days or more apart and, on the average, several months apart, varied not far from 14%, although in all cases the subject was in complete muscular repose and in the post absorptive condition. Since during this period there was no material alteration in the body weight and consequently in the body surface, the variation in the metabolism must be ascribed to a difference in the stimulus to cellular activity.

In the fasting experiment not only was there a difference in the metabolic level noted at different times of the day due to the condition of being awake and asleep, and in the late afternoon after the experimental program of the day, but, as the 31-day fast progressed, the heat production per square meter of body surface varied from 859 calories on the first fasting day to 668 calories on the twenty-third fasting day. Although the loss in body weight was material, apparently the skin shrank in proportion and there was no obvious disproportion between body weight and surface area. During the fourth week of the fast there was a distinct tendency for the metabolism to rise, accompanied by a measurable increase in the pulse rate. Thus the tendency for the prolonged fasting to depress the metabolism was actually overcome by some unknown stimulus increasing the cellular activity of the remaining body substance, ultimately resulting in a positive increase in the basal metabolism during the last week of fasting.

Two other factors have been experimentally demonstrated in this laboratory as influencing the metabolism. One of these is the character of the preceding diet, it having been shown that when a carbohydrate-free diet is eaten an acidosis is developed which distinctly increases the cellular activity and results in a very noticeable increase in the basal metabolism. The second factor is the marked after-effect of severe muscular work, such work causing the pulse rate and the basal metabolism to remain at an increased level for many hours after the cessation of the muscular activity. This may logically be ascribed to a stimulus to the protoplasmic tissue, resulting in a higher metabolic plane.

From the evidence cited, therefore, it may be concluded that there is no direct relationship between total body weight and total heat production, that the metabolism or heat output of the human body, even at rest, does not depend upon Newton's law of cooling, and is therefore independent of the surface area. The observations on athletes, the comparison of normal men and women, and of atrophic and normal infants indicate that the proportion of active protoplasmic tissue plays an important rôle. Perhaps the most striking factors causing variations in the stimulus to cellular activity are age, sleep, prolonged fasting, character of the diet, and the after-effect of severe muscular work.

From the evidence gathered with the various subjects studied it is clear that the basal metabolism of an individual is a function, first, of the total mass of active protoplasmic tissue, and, second, of the stimulus to cellular activity existing at the time the measurement of the metabolism was made.

The detailed report of this investigation has been transmitted to the *Journal of Biological Chemistry.*

ON THE NATURE OF THE NERVE IMPULSE

By Shiro Tashiro

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Read before the Academy, December 9, 1914. Received January 13, 1915

The nerve fiber, as is well known, shows not only the highest degree of irritability, but also possesses power to transmit a state of excitation in the most efficient manner. Our interest in the study of the nerve impulse lies not only in the fact that the nervous system is one of the most important factors in all organic activities in the higher animals, but also, in the fact that it constitutes the center of a general problem that deals with the mechanism by which a living substance adjusts itself to its environment.

In order to study the nature of the nerve impulse, many physiologists have investigated what takes place in the nerve fiber during the conduction of the impulse. The results were very peculiar: there were no structural changes of any kind, no heat production, and there appeared to be no production of carbon dioxide, nor did they find any change in the distribution of the organic materials in the tissue. When electrical changes were discovered in the stimulated nerve, the problem was considered to be settled once for all. The idea that the nerve impulse is of an electrical nature was, however, soon doubted, when Helmholtz observed that the velocity of the nerve impulse is of an entirely different magnitude from that of an electrical current. One thing seemed to be certain: that the nerve impulse can pass through the fiber without consuming any material. The fact that we can not detect fatigability in the nerve after continuous stimulation supported the belief that certain quickly reversible physical conditions exist in the nerve, and that changes in these conditions must determine the phenomena of excitability and conductivity in the nerve. Such changes were thought to be either of the colloidal state, of the surface tension, or of the permeability of the nerve.

With the new apparatus which measures as little carbon dioxide as one ten-millionth gram, we have demonstrated that an isolated nerve fiber gives off a definite amount of carbon dioxide and that when it is stimulated this carbon dioxide production is more than doubled. These facts not only indicate that there is another kind of functional change beside the electrical, but also show that the chemical changes in the nerve fiber are of the same magnitude as those of other organs.

To some physiologists, however, our results do not seem to be conclusive enough to show that the carbon dioxide given off by an isolated nerve is produced by life processes. Experimental conditions, to their minds, might have caused the tissue to decompose in a manner quite different from its normal behavior.

Further experiments show the following facts:

1. If the sciatic nerve is removed from a frog, it shows an electrical response for many hours. Since electrical changes are characteristic only of living nerves, we consider that the isolated nerve does not die for many hours. If measurements are made on an isolated nerve at successive time intervals, we find that carbon dioxide production steadily decreases as the nerve approaches death The point of minimum carbon dioxide production corresponds nearly to the point where electrical response ceases. The dead nerve gives almost no carbon dioxide.

2. Although the nerve remains active for some time without oxygen, it is a known fact that absence of oxygen diminishes the excitability of the nerve. The carbon dioxide production of a nerve fiber is much less in hydrogen than in atmospheric air. We should expect that there would be no difference under these two conditions if the carbon dioxide production of an isolated nerve fiber were due entirely to the death process.

3. The normal, uncut fiber in the body is very susceptible to many chemical reagents. A weak concentration of an anesthetic, for instance, renders the nerve nore excitable, or stimulates it, while a higher concentration makes it unexcitable. These reagents show similar effects on the carbon dioxide production of the cut nerve.

These facts, to my mind, indicate that the production of carbon dioxide from an isolated nerve fiber is at least a correct expression of what is going on in the normal nerve in the body, and indicate clearly that the normal nerve must have a chemical activity which is accelerated when the nerve is stimulated.

If the chemical activity is so vigorous as our results indicate, one naturally asks how we can explain the fact that the nerve impulse can pass continuously for hours without any measurable sign of fatigue. Apparent lack of fatigability in the nerve is a remarkable fact, but I wonder whether we should be willing to ignore the presence of metabolic activity in the contracting wings of insects which can fly continuously for hours at a rate of as high as three hundred vibrations per second. The ordinary induction coil we use for fatigue experiments, by the way, maintains not more than one hundred vibrations per second, at the maximum.

Granting that the absence of fatigability in the nerve, as measured by ordinary methods, may not be a question of absence of metabolism, but merely of the speed at which breaking and repairing processes of the tissue come to equilibrium, one may still ask how we explain the lack of heat production. Snyder reported very recently[1] that a smooth muscle failed to show any sign of heat formation during contraction, which, no one doubts, increases carbon dioxide production. When we dip a zinc rod into copper sulphate solution we observe heat formation, but that with proper arrangements, as in a Daniell cell, the reaction goes on practically isothermally, all the chemical energy being converted into electrical energy. I do not think that we should ignore the increase of carbon dioxide production in the nerve on stimulation because of the fact that we cannot detect heat formation.

There are some physiologists who admit that the living nerve should be chemically active to maintain a 'normal' condition, like any other living tissue, but who hold that the increase in carbon dioxide production on stimulation must be a secondary effect due to primary physical changes. We will consider this question in a quite different way.

The state of a nerve fiber depends upon three conditions: the degree of irritability, the direction of the impulse and the rate of the impulse. The relation of these conditions to chemical activity in the nerve is analyzed in the following way:

1. Degree of irritability and carbon dioxide production.

We have already cited the fact that chemical reagents which modify the degree of excitability invariably modify the rate of carbon dioxide production in the same proportion. The chemical activity in the nerve fiber seems to determine the state of nerve excitability.

2. The direction of nerve impulse and carbon dioxide production.

If one takes nerve bundles containing only sensory fibers, which conduct the normal nerve impulse in a central direction, the portion of the nerve nearer the natural source of the nerve impulse (i.e., nearer the end organ) gives more carbon dioxide than the portion away from it. There is a gradient of carbon dioxide production in the unstimulated nerve. This gradient of chemical condition seems to determine the direction of nerve impulse. Many experiments made on various kinds of pure nerve fibers enable us to generalize this by saying that the normal nerve impulse passes toward a point of lower carbon dioxide production.

3. Rate of the nerve impulse and carbon dioxide production.

There seems to exist a close relation between the rate of nerve impulse and carbon dioxide production in the resting nerve, if one compares the corresponding nerves of different animals. The data for such a generalization must necessarily be cumulative. The limited data we have secured indicate that the nerves which give off more carbon dioxide in the resting state conduct the nerve impulse more quickly.*

I may add here that conditions which influence the speed of the nerve impulse modify the chemical activity of the resting nerve. It has been known for a long time that the temperature coefficient of velocity of the nerve impulse is greater than that of most purely physical processes. We find that the temperature coefficient of carbon dioxide production of the non-stimulated nerve is of about the same magnitude as that of the velocity of the nerve impulse.

Basing our conclusions on the foregoing experimental facts, we consider the nature of the nerve impulse as follows:

There are two chemical conditions necessary to enable the nerve to conduct the nerve impulse.

The first condition is the maintenance of normal chemical activity, i.e., the presence of certain chemically unstable substances. Just what these substances are, we do not know. This condition constitutes, to my mind, the so-called state of excitability. The nerve must be in this condition to be capable of response to a stimulus. The instability of these substances enables the nerve to undergo greater chemical changes when stimulated; that is, stimulation is accompanied by an increase in chemical activity. This is true not only for nerve tissue, but also for all other living tissues, including living seeds. The second condition is, if this increase in chemical change at the point of stimulus is sufficiently greater than that in its neighborhood, the impulse will go in that direction. The normal nerve impulse, therefore, will go only in one definite direction. The possibility of nerve conduction in two directions is obvious. It must depend on the gradient of chemical activity along the fiber, and the amount of increase of this activity at the point of stimulation.

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* In this connection it is interesting to note that carbon dioxide production from medullated nerves is not always greater than from non-medullated ones, in spite of the fact that medullated nerves invariably carry the nerve impulse much more quickly than non medullated ones. This is rather important in view of the fact that the conducting mass in the medullated fiber is known to be much less than that in the non-medullated, if we compare the total nerve fibers gram for gram. It is very likely that if we could compare the carbon dioxide production from the reacting masses of the two nerve fibers, the relative rate of metabolism of the axis cylinder of the medullated fiber would be greater than that of the non-medullated. Since such a comparison is practically impossible at present, the only way to test the correctness of our hypothesis will be to make a series of carbon dioxide measurements on various medullated fibers and compare them with their corresponding rates of nerve impulse. If this relation proves to be general, then not only our current notion that the function of the medullary sheath is to supply nutrition to the conducting medium will be proved correct, but also we may easily understand the morphological development of the medullary sheath in relation to the functional activity of the nerve fiber.

I believe the nerve impulse is a propagation of chemical change—the propagation being due to a restoration of an equilibrium disturbed by the increase of metabolism at the point of stimulus. This propagation is always toward the point where there is less chemical activity, as measured by carbon dioxide production.

[1] *Amer. J. Physiol.*, 35, 340.

A POINT SCALE FOR MEASURING MENTAL ABILITY

By Robert M. Yerkes

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Presented to the Academy, December 4, 1914

Alfred Binet, in 1905, devised a method of roughly estimating the intellectual capacity, or degree of mental development, of the child in terms of age. The method depended upon the application of series of single tests or measures, each series being especially arranged for a particular year of age. If a subject could satisfactorily meet the requirements for his age, he was considered up to the standard. Obviously he might measure either a certain number of years above or below the expected intellectual age.

The Binet measuring scale of intelligence has been revised, perfected, and adapted by various individuals, and now after nearly ten years of practical application, it stands as the only convenient and reasonably expeditious method of classifying children with respect to intelligence. It possesses, however, many serious defects which may not now be enumerated, since the purpose of this abstract is to present a brief description of a new method which is based, on the one hand upon the work of Binet and his associates, and on the other upon a suggestion made by the late E. B. Huey. We may call this new method the point scale for measuring mental capacity. It has been developed at the Psychopathic Hospital, Boston, as one result of the demand for reasonably detailed and reliable information concerning the mental characteristics of individuals both immature and mature. The scale consists of a single series of measurements to be made on all subjects. Each measurement is evaluated according to a graded scale, and the maximum credit obtainable in an examination is one hundred points. The various measurements in question are distributed among the most important intellectual functions (affectivity being omitted), so that the several aspects—memory, imagination, perception, association, suggestion, judgment, ideation—may be measured.

The value of the point scale depends upon reliable norms. Two boys, from poor surroundings, each of the age seven years, six months, subjected to examination, obtain, for example, the one a credit of twenty-two points and the other a credit of fifty-nine points. By reference to our norms, we discover that boys of the age in question born to English-speaking parents of poor or medium circumstances, should obtain a score of 36—while those born to parents of good or superior circumstances should obtain 45. Since these norms, as is evident, take account of sex, age, language, and social status, it is clear that the first individual is nearly 40 per cent below our reasonable expectation in intellectual ability, whereas the second individual is instead about 37 per cent above what we should expect.

It thus is clear that instead of stating that an individual is a certain number of years and months above or below age, as in the Binet method, we can definitely state the relation of a given individual examined to a more or less inclusive group in which he belongs. This value we call the coefficient of mental ability. It is obtained by dividing the individual's score by the average or norm for his group. We may also, if it is desirable, state the exact frequency of occurrence in the group of the degree of intelligence indicated, as for example, one in one hundred, with respect to inferiority or superiority, as the case may be.

A single measure chosen from our scale must serve to indicate its chief features. We choose the test for the span of auditory memory. The subject is required to repeat, after the experimenter, groups of digits ranging in number from three to seven. These are presented by the experimenter orally, clearly and distinctly, at the rate of two per second. If the subject fails to repeat correctly the first group of a given number, he is given a second trial with another group containing the same number. If he succeeds with that, the experimenter passes on to a group containing one additional digit, and so on. One point credit is allowed for each of the five groups correctly reproduced. If a subject fails in both trials for a given number of digits, the observation is terminated. The test for memory-span is:

	1st trial	*2nd trial*	*Credit*
(a)..........	374	581	(1)
(b)..........	2947	6135	(1)
(c)..........	35871	92736	(1)
(d)..........	491572	516283	(1)
(e)..........	2749385	6195847	(1)

The preliminary scale, which we have thoroughly tested by applying it to about eight hundred normal children and adults and to more than three hundred abnormal individuals, has proved so serviceable that we are now attempting to develop a more highly perfected and inclusive series of measurements which shall be universally applicable and shall take account of the affective as well as the intellectual functions.

Since the object of our work was the development of a practically serviceable method of measuring mental ability for use in hospitals, clinics, schools, reformatories, prisons, and wherever a rough estimate of mental status is demanded, it does not seem worth while to present results in this article.

The principles involved in the universal point scale are as follows:

1. A single series of measurements to be made on all subjects examined.

2. Gradation of each member or part of the scale with respect to difficulty so that measurement may be made, with equal facility, of the capacity of the child of three and the adult.

3. Partial credits according to the extent and nature of the response.

4. Distribution of the several measurements equally among the chief groups of mental processes: for example, according to the following four categories of processes, one-fourth of the measurements being devoted to the processes under each. (*a*) Sensibility, perceptivity, discrimination, association (receptivity); (*b*) Memory, in several of its aspects, and imagination (imagination); (*c*) Simple feeling, emotion, sentiment, volition, and suggestibility (affectivity); (*d*) Ideation, judgment, reasoning (thought).

5. Arrangement of the several measurements of the scale, probably twenty in all, in the four groups suggested above, namely, (*a*) receptivity; (*b*) imagination; (*c*) affectivity; (*d*) thought. So that, one-fourth of the scale being devoted to each group of processes, the credits achieved by an individual may conveniently be represented by a simple formula. Assuming that the maximum number of credits obtainable is two hundred and that individual *X* achieves one hundred and fifty-three points, his mental formula might be written thus:

$$X = \text{R.}43 + \text{I.}48 + \text{A.}22 + \text{T.}40 = 153.$$

Such a formula would indicate to the examiner that X is especially deficient or peculiar in affective characteristics.

6. The measuring scale shall be arranged on four pages, those measurements dealing with one of the four categories of mentality occupying a page. On each sheet, the several measurements shall be arranged in order of increasing difficulty, and the same shall hold of the order of arrangement within any given part of the series, that is, any one of the twenty types of measurement.

7. The measurements shall be chosen, so far as possible, with a view to simplicity of materials and ease and uniformity of observation.

8. The scale shall be dependent for its value upon safely determined norms.

COLOR VISION IN THE RING-DOVE (Tutur risorius)

By Robert M. Yerkes

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Presented to the Academy, December 4, 1914

The psychophysiological literature, both naturalistic and experiental, on color vision in infra-human animals, is surprisingly extensive. But even more surprising is the extreme uncriticalness of the methods which have been employed. A realization of this condition of affairs within the past decade led simultaneously to the development, by C. Hess, in Europe, and by R. M. Yerkes in association with J. B. Watson, and more recently by G. H. Parker, in America, of spectral methods for the comparative study of color vision. These methods enable the experimenter to measure and control his stimuli in their various aspects and to observe with reasonable accuracy organic response to specific stimuli. The method now in use in this country, developed by Watson and me, may be named from the nature of the stimulus and the form of reaction demanded 'the method of discriminating spectral stimuli.' It involves the use of a special form of prism spectrometer with devices for selecting, spacing, reversing, and displaying any two portions of the spectrum, with means of controlling the selected stimuli qualitatively and intensively, of measuring them in photometric and energy units, and of so presenting them to the reacting animal that it may, if capable of so doing, recognize them and react appropriately.[1]

Until very recently, it has been the prevalent opinion even among scientific persons that many, if not most of the vertebrates, possess fairly highly developed color vision, which in many instances is closely similar to the human. That this opinion is erroneous and demands correction is proved by the results obtained by the use of spectral stimuli. It is now definitely known, for example, that, among the rodents, the mouse, rat, and rabbit well-nigh lack the ability to distinguish colors. For them, long wave-lengths are of surprisingly low stimulating value. The spectral range of certain day-birds appears to be similar to that of man. Among the higher animals, critical observation indicates that the cat and dog possess slight power to respond differentially to different wave-lengths. The observations at present available do not justify dogmatic statements, but they conclusively prove that all earlier accounts of color vision in these animals are misleading.

During the past ten years, I have devoted a large amount of time to the development of methods and the accumulation of facts. My study of vision in the dancing mouse,[2] which went far toward proving the absence of color discrimination in that organism, led me to abandon the use of stimuli obtained from colored papers, cloths, or the use of ray-filters, and to depend wholly upon spectral light.

At present I have undertaken the study of color vision in the ring-dove,[3] using the Watson-Yerkes spectral light apparatus with a type of reaction-box which was developed in connection with the study of the dancing mouse. The method which is employed in connection with this apparatus has been called 'the discrimination method.' Watson has recently suggested the name 'sensory habit method.' It involves the simultaneous presentation to an organism of two stimuli which differ definitely and measurably in some respect or respects. It demands of the animal that it react differently to the two stimuli; to the one of them positively, to the other negatively. Positive reaction is encouraged by the reward of food and inappropriate response is discouraged by disagreeable electrical stimulation. The essential features of the apparatus are, first, a home compartment in which the animal receives stimulation from two adjacent sources. If, in a certain required manner, it approaches the stimulus which has been defined as the 'positive,' it is permitted to escape thence by way of a narrow passage to a food compartment where it receives its reward. If, on the contrary, it approaches the stimulus designated as the 'negative,' it is punished by electrical stimulation, is forced to retreat, and then again has the opportunity to react to the 'positive' stimulus.

The sensory habit apparatus, even in its most improved form, is a fairly simple mechanism, and the sensory habit method, with the employment of reward and punishment as conditions for careful discrimination and habit formation, has proved remarkably satisfactory and has yielded valuable results in numerous investigations.

It has thus far been demonstrated that the ring-dove may be trained, although rather laboriously, to appropriate forms of achromatic discrimination, and that similarly it may be used by the sensory habit method for studies of chromatic discrimination. Experiments with two individuals demonstrate the existence of the Purkinje phenomenon in this organism. This was done by use of a red of 630 $\mu\mu$ and green of 505 $\mu\mu$, approximately equal in energy. In general illumination, the two stimuli seemed to be of approximately equal stimulating value for the female, whereas for the male, the red was the more stimulating. A moderate degree of adaptation to darkness reversed this relation, the green then being much more stimulating than the red for the female, whereas the two were approximately equal for the male.

This observation indicates at once changing sensitiveness with adaptation and an individual, or more likely, a sex, difference in the stimulating value of the colors in question. Apparently the male ring-dove is markedly more sensitive to red than is the female.

This report of work which is still in progress may be concluded with a word concerning the significance of temperamental differences. In the doves in question, these were very striking indeed and are responsible for certain important features of the results obtained. The female was tame and timid; the male, on the contrary, somewhat wild and bold. In view of these facts, it is not surprising that the male proved an excellent subject for the sensory habit method, whereas the female was much less satisfactory. The suggestion from the work is that temperament should be analyzed by the student of animal behavior and its various important components separately studied and accurately measured so that our animal subjects may be described with respect to the same, even although the investigation especially in point be one on color vision or some other aspect of reaction or experience which might at first seem unrelated to 'temperament.'

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[1] See Yerkes, Robert M. and Watson, John B. Methods of studying vision in animals. *Behavior Monographs*, vol. 1, no. 2. (1911).

[2] Robert M. Yerkes. *The dancing mouse.* New York, 1907.

[3] This work was made possible by a grant from the Bache Fund of the National Academy, for which the writer takes this opportunity to express his hearty thanks.

THE FEEBLY INHIBITED, II. NOMADISM OR THE WANDERING IMPULSE, WITH SPECIAL REFERENCE TO HEREDITY

By C. B. Davenport

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Presented to the Academy, January 12, 1915

Some persons are always satisfied to remain at home and dislike the thought of traveling; at another extreme are the tramps and nomadic gypsies; at still another extreme are those who, capable of steady and effective work, periodically, often in a more or less dazed condition, run away from their homes. The term nomadism is here adopted for this trait in all of its varied manifestations, the racial connotation of the term being advantageous rather than otherwise.

Nomadism has been widely studied by psychiatrists who have seen in the various periodic disorders with which it is often associated the causes of the different forms that it takes. Thus are distinguished demented, melancholic, epileptic, hysteric and other "fuges." The present study starts with the inquiry: Are the similar symptoms that are associated with such different mental states wholly independent, or have they a common cause?

A tendency to wander in some degree belongs to all locomotor animals. That such a tendency is a fundamental instinct in man also is indicated by four lines of evidence: (1) that the anthropoid apes (representing the primate stock from which man sprang) are nomads; (2) that primitive peoples (Fugeians, Australians, Bushmen, Hottentots) are nomads and this trait is widespread among other, less primitive, tribes; (3) that the tendency to wander is nearly universal among young children who have only recently learned to walk; (4) and that, at adolescence, the instinct (no doubt associated with the mating impulses) becomes keen again.

To get further light on nomadism one hundred family histories, deposited at the Eugenics Record Office, have been analyzed and tabulated according to the nature of the matings. The results of this tabulation is given on the following page.

This table shows that far more males than females are nomadic, 171 males to 15 females. This suggests the hypothesis that nomadism is a sex-linked trait. If this hypothesis be true, in a mating of a normal man and a woman who, though normal, comes of nomadic stock, half of the sons and none of the daughters are nomadic. Combining matings 1, 6, 7, and 8, which are the matings in question, we get the following distribution of tendencies in the offspring. Males: W, 133; not W, 109; females: W, 4; not W, 118. The shortage of not-W males is easily accounted for by the circumstances that only fraternities of offspring showing nomadics are included in our table; any fraternity which by chance, through small size or otherwise, shows no nomads is excluded. Of the four female nomads (where none is expected) it is to be said that all are daughters of three men who are really quite insufficiently known and ought not to be considered in this connection. The details are set forth in the full paper. There is no clear case of a nomadic daughter whose father is known to be non-nomadic.

Table of Distribution of Nomadism (W) in Offspring of Various Matings

PARENTS	OFFSPRING						
	TOTAL	x	† yg	♂		♀	
				W	NOT W	W	NOT W
1. Father not W, of not-W strain; Mother not W, of W strain	208	13	29	71	43	1	52
2. Father not W, of not-W strain; Mother, W	9	1	1	5±	(3±)	0	0
3. Father and Mother both W	13	0	2	6	1	4	0
4. Father, W; Mother not W, of W strain	35	6	7	8	5	2	7
5. Father W; Mother not W, W relatives unknown	73	17	6	16	16	5	13
6. Father not W, of W strain; Mother not W, of W strain	3	0	0	1	0	0	2
7. Father not W, of W strain; Mother not W, W relatives unknown	55	8	6	13	16	1	11
8. Relatives of father and mother (and frequently the parents themselves) little known	218	33	28	51	47	2	53

Abbreviations: W, wandering impulse, nomadism; x, unknown; † yg, died young.

The criterion that all sons of a nomadic mother are nomadic is tested by reference to matings 2 and 3. All but four sons are nomadic. Of the exceptions three are recorded with great doubt and one with much lack of detail.

The next criterion, that all daughters of two nomadic parents are nomadic is realized, in mating 3, unfortunately in too small numbers.

The criterion that half of the daughters and half of the sons of nomad-bearing fraternities derived from nomadic fathers are nomadic may be tested in matings 4 and 5. The expected equality is, indeed, found in the sons but, probably on account of small numbers, inexactly in the daughters. In general our Histories show that nomadic fathers may have no nomadic sons, but there is no case of a nomadic mother of more than two children none of whom is nomadic. Thus our data support the hypothesis that the paralysis of the control of the momadic impulse is a sex-linked trait.

Other hypotheses, such as (1) that nomadism is, like the beard in man, essentially a male characteristic, and (2) that nomadism is less common among women than among men solely because it is less feasible for women to live a nomadic life, are not at all supported by a full consideration of the facts.

Finally, the frequent association of nomadic impulses with psychoses both periodic (depressions, migraine, hysteria, sprees) and constitutional (feeblemindedness and dementia-precox) is explained as follows. The wandering instinct is, in man, fundamental, basic. In the more intellectual part of the population, under the influence of the *mores*, this impulse is more or less satisfactorily repressed where there is good reason why it should be, except in families of *periodics* where the inhibitions are temporarily paralyzed and the person 'breaks out.' Among constitutional mental inferiors on the other hand, the inhibitions may be slightly developed and such persons show a constant roving tendency—as ne'er-do-wells, some tramps, the gypsies and other nomadic tribes. All classes of nomadism can thus be ascribed to one fundamental cause, the nomadic instinct. In addition, since the more intelligent representatives of the nomadic race are able, in a measure, under ordinary circumstances, where it appears desirable, to inhibit their impulses, we find that, with such people, the unstable, wandering impulse is apt to be associated with a periodic disturbance that renders inoperative the inhibitory machinery; these disturbances are not the fundamental cause of the nomadic impulse but merely permit it to show itself. The capacity for such periodic disturbance is sex-linked.

The complete paper will probably appear in the *Journal of Nervous and Mental Disease* and in the *Bulletin of the Eugenics Record Office.*

ANNOUNCEMENTS

WILLIAM ELLERY HALE LECTURES

The third course of lectures on the William Ellery Hale Foundation will be given by Professor Thomas Chrowder Chamberlin, of the University of Chicago, at the next annual meeting of the Academy, on "The Evolution of the Earth."

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Volume 1 MARCH 15, 1915 Number 3

AN ATTEMPT TO MEASURE THE FREE ELECTRICITY IN THE SUN'S ATMOSPHERE

By George E. Hale and Harold D. Babcock
MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Presented to the Academy, January 16, 1915

We have the strongest of reasons to infer the existence of free electricity in the sun. At high temperatures in the laboratory carbon and other elements emit great numbers of negatively charged particles, while friction, collision and exposure to ultra-violet light may be mentioned among various other agencies capable of producing free electrons under solar conditions. On the other hand, the presence of intense magnetic fields in sun-spots points to a marked preponderance of negative electrons whirling in the spot vortex. Thus, without adducing further evidence, we find that both laboratory results and solar phenomena indicate the presence of free electricity in various parts of the solar atmosphere.[1]

The greater mobility of negative electrons and the influence of light pressure must coöperate to transfer negative electricity toward the upper levels of the atmosphere, thus leaving the body of the sun with a positive charge. However, hot vapors and gases are such good conductors that any considerable potential difference would tend to be compensated by a flow of current. In the presence of complex and uncertain conditions, such as the possible influence of radio-active substances, definite and unimpeachable knowledge of solar electric phenomena must be based upon a direct method of observation, which has recently become available through Stark's capital discovery of the effect of an electric field on radiation. If adequately applied, this method may ultimately furnish as reliable information of solar electricity as the Zeeman effect has already afforded of solar magnetism.

The Stark effect may be briefly described. Positively charged particles (canal rays), emitted from the anode of a vacuum tube, pass through perforations in the kathode into an intense electric field. When viewed across the lines of electric force, the series lines of such an element as hydrogen are split into two sets of components, polarized in planes at right angles to one another. When seen along the lines of force, one set of components disappears, while the others are present but unpolarized.[2] Thus, while the phenomenon resembles the Zeeman effect, a fundamental distinction lies in the fact that the components observed along the lines of force of a spectrum line resolved by a magnetic field are circularly polarized in opposite directions.

Our studies of the magnetic phenomena of sun-spots and of the sun as a whole have been based mainly upon this circular polarization, thus eliminating any possibility of attributing the observed effects to electric rather than magnetic fields. Many other criteria, such as differences in the number of components and the variation of separation with wave-lengths are also available to remove possible doubts, which may enter when the observations are made at right angles to the lines of force.

The $H\alpha$ line of hydrogen, when observed by Stark with moderate dispersion in an electric field of 28,500 volts per centimeter, was resolved into three components. The two outer components are polarized parallel to the field while the central line (which is double under higher dispersion) is polarized in the opposite plane. Hence the total width of the resolved line may be greatly varied by rotating the Nicol prism mounted above the spectroscope slit, since in one position the two outer components will be transmitted, while if the Nicol is turned ninety degrees these will be cut off and the central line transmitted.

In the sun the only known cases of line resolution (other than unpolarized reversals) are those found in the spectra of sun-spots. Immediately after the announcement of Stark's discovery, we examined our photographs of spot spectra to determine whether any anomalous cases of widened or resolved lines might be attributable to an electric rather than a magnetic field. In general, however, it was found that the outer components of spot triplets were sharply and completely cut off by the Nicol and quarter-wave plate under favorable conditions of observation, and are thus circularly polarized. Even in the case of spots near the middle of the sun, the central line of these triplets is usually present, apparently indicating that the lines of magnetic force are not exactly radial. But this component is very narrow, and if the Stark effect, not yet determined for the elements represented by these triplets, even approaches in order of magnitude the values indicated by the hydrogen and helium lines, the electric field at the level in question must be of very low intensity.[3] This point will be followed up as soon as the Stark effect can be observed for iron, chromium, nickel, titanium, manganese, vanadium and other elements whose lines are resolved in sun-spots.

The hydrogen lines are shown by our photographs to be weakened and narrowed in spot spectra. Thus they offer no indication of an electric field, but they will be carefully studied for possible traces of polarization phenomena.

As the magnetic fields in sun-spots seem to point so plainly to the existence of electric fields, the negative evidence of the Stark effect so far found in this quarter is not promising for researches in other parts of the sun. But it must not be forgotten that the establishment of a definite upper limit of intensity for electric fields at many different levels in the solar atmosphere is of the utmost importance, and this can be secured even if no positive evidence of the Stark effect can be detected. In sun-spots the lines of force of the electric field would presumably be tangential to the surface, making the center of the sun the best point for studies of the Stark effect. For the sun as a whole, on the contrary, the lines of electric force would be radial, so that evidence of a general electric field should be sought in the behavior of lines near the limb. These are well known to be unresolved and the only effect to be anticipated is a very slight widening of the lines, which should have plane polarized edges.

The marked widening and displacement of solar lines near the limb, found here some years ago to be a general phenomenon of much importance, is not due to the Stark effect, if we may judge from the fact that the lines cannot be reduced by a Nicol prism to their normal width at the center of the sun. We must determine, however, whether there may remain a very minute effect of widening, such as can be detected only with the most refined methods of observation. Three requirements must be met:

1. High resolving power and linear dispersion in the spectrograph, supplied in our experiments by a large Michelson grating, giving theoretically perfect resolution. As used in the 75-foot spectrograph of the 150-foot tower telescope the linear dispersion in the second order is about 3 mm. to the angstrom unit, or slightly greater than in Rowland's map of the solar spectrum, which was enlarged from negatives of

about one-quarter this scale. The third order, where the linear dispersion is nearly 5 mm. to the angstrom, has been used in some of our work.

2. A strictly differential method of observation, involving the determination of the width of the same line on contiguous strips of spectra photographed in a single exposure with apparatus transmitting light polarized in planes at right angles to one another. A long Nicol prism mounted over the slit, with a compound half-wave plate above it, made of mica strips 2 mm. in width, fully met this requirement. In order to eliminate possible absorption effects in individual strips, the half-wave plate was made by combining the compound quarter-wave plate used in our investigations of the general magnetic field of the sun with a long piece of quarter-wave mica. By inverting this between exposures, a given strip can be made to transmit light polarized in either plane.

3. A measuring machine capable of exhibiting the smallest variations in the width of the lines on the odd and even strips. A Koch registering micro-photometer, recently constructed in our instrument shop, served admirably for this purpose.

A series of photographs of the $H\alpha$ and $H\beta$ lines of hydrogen, made with the slit set about 3 mm. within the limb (parallel to a tangent) of the large solar image of the 150-foot tower telescope, furnished the required observational material. As employed for this work, the Koch machine gave photographic curves (reducible to intensity curves) of the $H\alpha$ and $H\beta$ lines on a scale fifty times that of the original negatives. Combining measures of curves made for several sets of odd and even strips, the probable error of the average width of one group of curves is ± 2.7 mm. Thus a difference in mean width of 5 mm., corresponding in the second order to 0.034 angstrom, should certainly be discernible by our method. Assuming this least appreciable difference in width to be of the same order of magnitude as the Stark separation of the components, we may at once determine the maximum electric field present from Stark's published results for $H\alpha$. For a field of 28,500 volts per centimeter he obtained a difference in separation of 6.4 angstroms between the outer and inner components (polarized parallel and normal to the field). As the total separation is directly proportional to the field-strength, and as the $H\alpha$ line shows no appreciable difference in width on the odd and even strips, it follows that the intensity of the solar electric field at the point of observation cannot exceed 150 volts per centimeter. A similar determination for the $H\beta$ line in the third order gives a corresponding value of 100 volts per centimeter. It therefore seems safe to say that the electric field-strength at the level in question is less than 200 volts per centimeter.[4]

Salet and Millochau, using lower dispersion, had previously found a maximum value of 7000 volts per centimeter for $H\gamma$ in the chromosphere.[5] Our much lower value indicates that in order of magnitude the electrical potential differences in the solar atmosphere may not greatly exceed those in the lower atmosphere of the earth, where they average about 1 volt per centimeter. In thunderstorms, of course, enormously greater differences occur, and it remains to be seen whether appreciable electric fields can be detected in solar eruptions, where the conditions for their production appear to be more favorable than in the quiet regions of the atmosphere.

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[1] For a summary of the views of Goldstein, Bigelow, Deslandres, Arrhenius, and others on the electrical condition of the solar atmosphere see Bosler, *Les théories modern du soleil;* also recent papers in the *Comptes Rendus Paris Acad. Sci.*

[2] See Stark, *Elektrische Spectralanalyse chemischer Atome*, Hirzel, Leipzig, 1914.

[3] Hale, Solar Magnetic Phenomena, *Proc. Amer. Phil. Soc.*, April 24, 1914, p. 254.

[4] Under high dispersion, Stark has resolved $H8$ into many components (loc. cit., Plate III). A variation in the relative intensities of these components under solar conditions, which is not improbable, might introduce an error into a determination of the maximum intensity of the electric field. It is likely, however, that the results here given are of the true order of magnitude.

[5] Salet and Millochau, *C. R. Paris Acad. Sci.*, **158**, 1000 (1914).

RESULTS OF AN INVESTIGATION OF THE FLASH SPECTRUM WITHOUT AN ECLIPSE

REGION λ 4800 TO λ 6600

By Walter S. Adams and Cora G. Burwell

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Presented to the Academy, January 18, 1915

The study of the bright line spectrum given by the comparatively thin layer of gases which constitutes the sun's atmosphere has usually been limited to the brief periods of total solar eclipses. During the few minutes that the dark body of the moon covers the sun's image the spectrum of the shell of radiating gases surrounding the sun may be observed without difficulty, and photographs of this spectrum, known to astronomers as the spectrum of the flash, have formed one of the most important products of recent eclipse expeditions.

Although admirable results have been secured in this way the short duration of eclipses has necessitated the use of spectroscopes of moderate power, and as a consequence the determination of the bright line wave-lengths has by no means reached a degree of accuracy comparable with that of the dark lines of the solar spectrum. Accordingly such questions as relate to small differences of wave-length between solar and flash spectrum lines, as well as some which depend upon the structure of the lines have of necessity remained unsettled.

It was shown by Young, Lockyer and others that the flash spectrum might be observed without an eclipse under conditions of good definition and high transparency, and Young was enabled to observe a considerable number of bright lines visually. Through the use of photographic methods this work was extended greatly by Hale at the Kenwood Observatory and later at the Yerkes Observatory with the 40-inch refracting telescope. With this instrument he observed visually the green carbon fluting in the flash spectrum which lies in a very thin stratum close to the sun's visible edge.

The photography of the flash spectrum was one of the investigations undertaken with the 60-foot tower telescope of the Mount Wilson Observatory and the 30-foot spectrograph used in conjunction with it. A preliminary account of some of the results obtained was published in 1909 by Hale and Adams. The results of a more detailed study of the photographs obtained at that time are indicated in this communication.

The total number of lines measured upon the negatives is slightly larger than that obtained from eclipse negatives in the same region of the spectrum (λ 4800 to λ 6600). Thus a comparison with Mitchell's recent determinations from plates obtained at the eclipse of 1905 shows a total of 901 lines for the eclipse photographs and 1027 for those taken without an eclipse. The chief interest in such a comparison, however, lies in the difference in the lines observed. The eclipse photographs show as strong bright lines nearly all of the strong dark lines of the solar spectrum. On plates taken without an eclipse many of these lines remain dark, or at most show faint bright fringes on either side. The proportion, however, of faint lines of the solar spectrum represented as bright lines in the flash spectrum is much greater than on the eclipse photographs. It seems very probable that this difference in behavior is due to the difference in the level of the observations. The eclipse plates refer to a high level in the solar atmosphere, and, as St. John has shown, the strong lines of the solar spectrum originate at a higher effective level than do the fainter lines. A comparatively low level for the observations made without an eclipse is also indicated by the great strength of the green carbon fluting, and by the appearance of a large number of double reversals among the bright lines observed.

This phenomenon of double reversal appears to be essentially a universal characteristic of the lines in the flash spectrum. In appearance the reversals resemble closely those of the hydrogen lines and of the *H* and *K* lines of calcium, and like them they are always approximately symmetrical. There seems to be no sufficient reason for concluding that these reversals are not due, as in the laboratory, to the radiation of a dense mass of gas through a cooler outer layer of lower density. On this basis the presence of numerous double reversals in the flash spectrum would indicate the existence of comparatively dense gas at the point under observation, and hence a low level in the solar atmosphere.

The wave-lengths of the bright lines in the spectrum of the flash taken without an eclipse have been determined with reference to those of the dark lines of the limb spectrum which are present on the same photographs. Accordingly it is possible to judge of the existence of any systematic displacement of the bright lines, a result which cannot be obtained from eclipse spectra which contain bright lines alone and no standards of reference. A comparison of the wave-lengths of 512 identified lines gives a value: *bright lines – dark lines at limb* = – *0.002* angstrom. The preponderance of the negative sign in the case of the individual elements makes it fairly probable that this small quantity is to be regarded as real. If so, it finds a ready explanation in the slightly higher level of the point under observation in the case of the flash spectrum. At this higher level the cause which produces the displacements of the solar lines at the limb toward the red, which is probably a pressure effect, in part, at least, would act less strongly, and the bright lines would have a position to the violet of the dark lines.

A comparison of these measures with those by Mitchell on his eclipse spectra indicates a marked gain in accuracy for the Mount Wilson results. The average deviation of all of the lines common to the two sets of observations between the limits λ 4800 and λ 5880 is as follows: *Eclipse Results*, 0.030 angstrom; *Mount Wilson Results*, 0.012 angstrom. In this comparison blends and double reversals have been omitted. It appears, accordingly, that the greater linear scale of the Mount Wilson photographs contributes notably to the accuracy of the measurements.

It is well known that the lines of certain of the elements of high atomic weight are extraordinarily prominent in the flash spectrum.

If we compare the intensities of these lines on photographs taken with and without an eclipse with the intensities of lines due to such elements as iron, we find the lines of the heavy elements relatively stronger on the plates taken without an eclipse. The heavier elements, therefore, are at a comparatively low level in the solar atmosphere, a result in harmony with that found from investigations of solar rotation and of radial motion in sun-spots.

The close agreement in wave-length between the bright lines of the flash spectrum and the dark lines of the limb spectrum is difficult of explanation on the basis of anomalous refraction in the solar atmosphere. If hypothetical irregular density gradients are used to account for the displacements of the dark lines at the sun's limb as has been done by Julius, it would seem necessary to conclude that they would have a marked effect upon the wave-lengths of the bright lines in the flash spectrum for which the path of the light is very similar. No such effect is found. Neither the regular density gradient nor the presence of irregular density gradients appears to be capable of explaining simultaneously the displacements at the sun's limb and the close correspondence in wave-length of bright and dark lines without the introduction of additional assumptions. The symmetry of the double reversals of the flash spectrum lines is also opposed in general to a theory which ascribes their origin to anomalous refraction and dispersion in the solar atmosphere.

Some peculiar characteristics of the dark line spectrum of the sun's limb are seen on the photographs of the flash spectrum. In some regions, especially near λ 5050, the intensities of the solar lines are so greatly modified as to render identifications difficult. A number of lines of considerable intensity make their appearance, which are not seen at all in the solar spectrum, and conversely, strong lines in the solar spectrum are weakened very greatly at the sun's limb. The identification of the elements to which these lines belong should prove to be a matter of decided interest.

The full details of this investigation with tables showing the results for 1027 bright lines will appear as *Contributions from the Mount Wilson Observatory* No. 95, in the current volume of the *Astrophysical Journal.*

VARIABILITY OF SPECTRUM LINES IN THE IRON ARC

By C. E. St. John and H. D. Babcock

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Presented to the Academy, January 18, 1915

Recent progress in the application of spectroscopy to physical and astronomical problems has depended in large measure upon the determination of very minute changes in spectrum lines, involving not only their position in the spectrum, but their variations of intensity and shape and their alteration by magnetic and electric fields. In the attempt to interpret observations upon the spectra of heavenly bodies, which in general are produced under the most complex conditions of radiation, it is therefore of prime importance to possess detailed information regarding the peculiarities of behavior of spectrum lines individually.

These considerations make clear the need for standard lines whose wavelength is accurately known when they are produced under known conditions, and whose response to definite variations in the physical conditions surrounding the source is well established. For the general purposes of precision spectroscopy, the necessity for the fixing of a complete system of working standards of wavelength extending throughout the spectrum has led to the adoption of the electric arc between iron terminals under certain specified conditions as a standard source of radiation. However, the determinations by different observers of the wavelengths of lines selected as standards do not show the accordance which would be expected, if the errors were solely those inherent in the method of observation. The suspicion thus aroused as to the constancy of these wavelengths, when the arc is employed under the specified conditions, coupled with the need for extensions to our knowledge of the character of the lines under different conditions of radiation has made necessary a careful study of the iron arc spectrum, some results of which form the substance of this paper.

A visual observation of the arc made with the help of various colored screens brings out the fact that there are considerable differences in brightness in different parts of this source. At the negative pole the vapor is in general far brighter than at the positive pole, while there is an equatorial region which is less bright than either. With a deep red screen the outer envelope of the arc appears to surround a comparatively dark core which contains two small bright brushes, one emanating from each pole. A blue screen, on the other hand, causes the outer parts to appear much fainter than the part which appeared dark in red light.

These facts and others associated with them led us to compare the spectra of different parts of the arc with apparatus of high power, and a method was developed for making studies of this kind under conditions favorable to the detection of very small variations in the spectrum. Even with very stable apparatus, considerable difficulty is found with high dispersion spectrographs in eliminating minute instrumental disturbances sufficiently large to interfere seriously with observations of quantities whose order of magnitude is a few thousandths of an angstrom, and it was not until we made our exposures to the two parts of the source simultaneous that we obtained satisfactory consistency in the results. This was accomplished by means of a pair of small total reflecting prisms arranged over the slit of the spectrograph in a 5-fold enlarged image of the arc, with a rotating sectored disk interposed in the path of the light from the brighter portion of the source. The latter adjunct serves to equalize the effective intensities so that the corresponding spectral images are practically identical in blackness—a necessary precaution when their positions on the photographic plate are to be determined by visual observations with a filar micrometer. The same auxilliary apparatus was employed for comparing the spectra of two separate arcs operated with different current strengths, and it may be applied to many comparisons requiring the highest precision. By the aid of it we have obtained photographs of the entire visible spectrum of iron and part of the ultraviolet, the spectrum of the light from near the negative pole appearing as a narrow strip extending along the center of the plate, with contiguous spectra on each side taken from the midpoint of the arc. Upon these plates we have measured for about 1600 lines the wavelength at the negative pole using the same line at the center of the arc as a reference standard. Of the lines examined about 1300 show no determinable difference in wavelength, 249 are displaced toward the red and 64 toward the violet, the larger shifts amounting to about +0.025 angstrom and −0.030 angstrom respectively. The presence upon the same photograph of lines belonging to all three of these classes establishes the fact that many lines in the spectrum are modified in some way at the pole of the arc to an extent which must be taken into account when they are involved in precise measurements. It becomes of importance, therefore, to enumerate and classify these lines and to determine under what conditions, if any, they may be used as standards of reference. Also, the question may be raised as to whether the shifts we observe are actual displacements of the maxima of the lines or are due merely to unsymmetrical widening. Concerning this latter point it should be said that especial attention was given it both in collecting and discussing our data. The equalization of the brightness of the two parts of the source by the rotating disk referred to above was always nearly complete except when the light from the pole was purposely made fainter than from the center. Under these latter conditions it was found that the displacements persisted, although if due to unsymmetrical widening they should disappear. Furthermore, a set of typical lines showing displacements in both directions were observed with two different forms of microphotometer, both of which confirmed the conclusion that the maximum of the line is actually displaced at the pole as compared to its position at the center of the arc. With either of these instruments the location of the maximum of a line is independent of any bias on the part of the observer. It is true that the shifts are often accompanied by considerable dissymmetry, though there are some exceptions.

An examination of our data brings out a partial correspondence between the effects observed at the negative pole and those known to be due to increase of pressure around the source. But we find a large number of lines whose wavelengths are unaffected at the negative pole, which should show an easily measurable increase in wavelength if there were a general increase in pressure near the pole sufficient to account for the observed displacements. This would suggest the probability that some other agency than pressure is effective, but the possibility remains of a local increase in pressure affecting only the innermost portion of the vapor in the vicinity of the pole. To determine the rôle played by pressure a fuller knowledge is required of its effect upon the wavelengths of these affected lines, a subject now under investigation at this observatory. A more complete discussion of this question and of the possible effect of density as distinguished from pressure, together with complete lists of affected lines, will appear in our more extended paper soon to be published in the *Astrophysical Journal.*

In addition to the observations discussed above, we have measured the relative intensities in different parts of the arc of a few typical lines in the green part of the iron spectrum. For this purpose still higher dispersion and resolving power were employed in the spectrograph, and the slit was placed parallel to the axis of the arc. As precautions against the numerous sources of error in such observations, may be mentioned the impression of a photometric scale upon each photographic plate used, the avoidance of overexposure, etc. The enlarged image of the arc and the long slit of the spectrograph permit an excellent analysis of the image from one pole of the arc to the other, although each line image represents an integration of all the light in the line of sight. These photographs display the differences between different classes of lines in a striking manner, showing the variations in width, intensity, dissymmetry, etc., from point to point along the arc axis. For deriving from these plates curves showing the variation of intensity *across the spectrum line* at any desired point, a registering microphotometer was employed. This instrument automatically draws a curve which can be transformed by the aid of the photometric scale into an intensity curve. By comparing these intensity curves for a given line taken at a series of points along the axis of the arc, an idea is had of the actual shape of the line in different parts of the arc. Figure 1 shows typical intensity curves taken from near the negative pole and the center of the arc respectively for two lines. The dotted curves refer to λ 5424, which shows a large shift toward the violet at the negative pole, while the other curves belong to λ 5434, whose wavelength is unchanged. Both lines are seen to be more intense at the negative pole than at the center, but the difference is greater for λ 5424. On account of the greater widening and dissymmetry shown by this line in the vicinity of the pole, it is important to compare the total energy of radiation at center and pole for each line. This has been accomplished by taking intensity curves at short intervals along the line from one pole to the other, integrating these curves and plot-

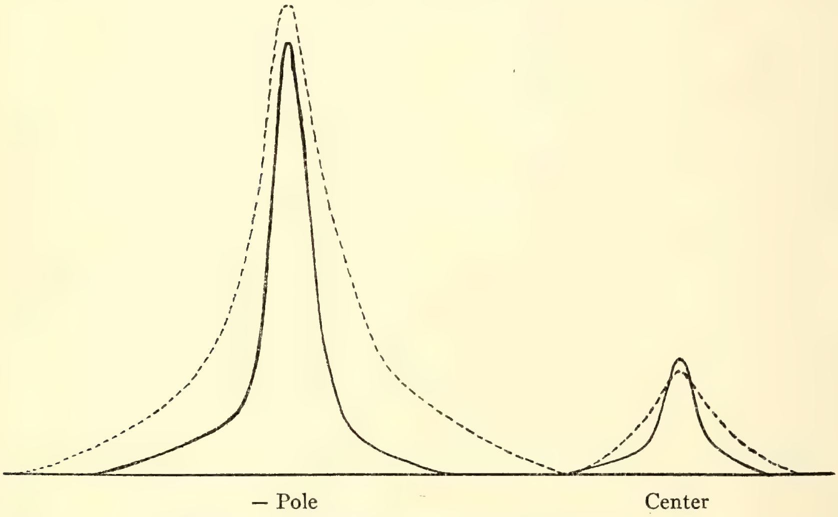

Fig. 1. Comparative intensities at pole and center.
......λ 5424 ———λ 5434

ting their areas as ordinates with abscissae corresponding to the points at which the intensity curves were taken. Figure 2 shows the energy curves thus obtained for the same lines. The apparent decrease in energy close to the negative pole is undoubtedly fictitious, being introduced by several causes operating during the exposure of the photographic plate. It is seen from the curves of Figure 2 that the energy of emission of λ 5424 is 6 times as great near the negative pole as it is at the center, while for λ 5434 the ratio is 3.2. Since the former line occurs chiefly in the core of the arc and the latter is strong in both inner and outer parts, viz., throughout a much greater volume of arc vapor, it is possible that the effects observed at the negative pole for lines like λ 5424 are due in part to the greater proximity of the separate radiating centers; in other words, to an effect of density of radiating vapor as distinguished from pressure. It is by no means certain that the true pressure effect depends entirely upon increased nearness of the centers of emission and not upon changes in the surrounding medium.

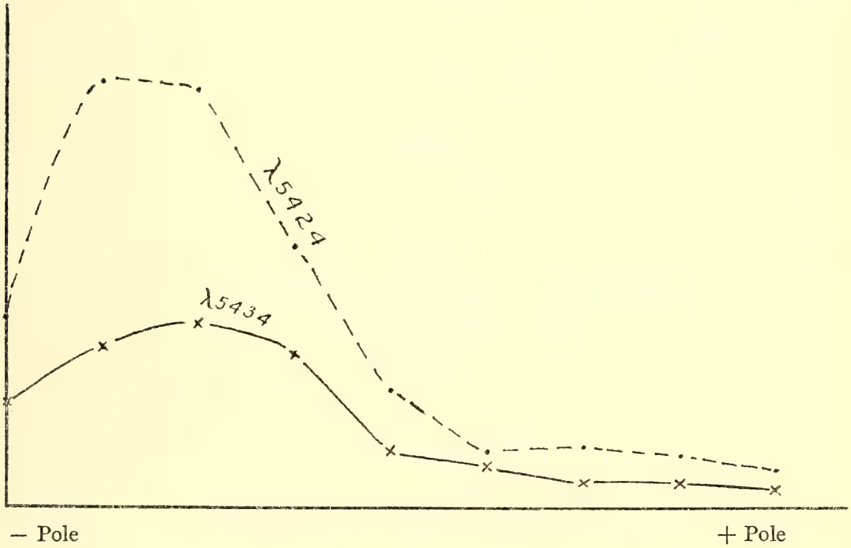

Fig. 2. Distribution of energy along the axis of the arc.

From an examination of the variations of wavelength when the light is taken from points between the center and the negative pole, as well as from a comparison of spectra taken from the centers of two arcs carrying currents of different strength, we find that the most sensitive lines exhibit measurable displacements at considerable distance from the pole and are affected by changes of 40% in the current even when observed at the center. Attention is thus called to the difficulty of securing reproducible values of the wavelength for such lines when the spectrograph slit is placed parallel to the axis of the arc, especially if an astigmatic spectrograph is used. With the slit at right angles to the arc at its middle point, on the other hand, it is easy to obtain reproducible results.

Our conclusions may be summarized as follows:

1. It has been shown that the wavelengths of many lines in the iron arc spectrum depend upon the portion of the source used.
2. These variations in wavelength appear not to be due to a general increase in pressure in the vicinty of the negative pole, but the questions of a local increase in pressure and the possible effect of density are still under investigation.
3. The energy distribution in the arc has been shown for two types of lines.
4. Some working conditions whose observance favors the obtaining of reproducible values of wavelength have been quantitatively determined.

AN EXPERIMENTAL STUDY OF LIPOLYTIC ACTIONS

By K. George Falk

HARRIMAN RESEARCH LABORATORY, ROOSEVELT HOSPITAL, NEW YORK

Presented to the Academy December 31, 1914

Purpose of this Investigation. The chemical changes which occur in animal and vegetable growth have focused attention in recent years upon a group of catalytic agents, the enzymes, which are capable of accelerating these changes. The study of the chemical nature and behavior of enzymes is, however, extremely difficult because of the complexity of the substances which occur in living matter and which constitute in most cases the material upon which the enzymes act. Thus most enzymatic reactions involve changes in substances such as proteins or starches, which are themselves of unknown chemical structure. There are, however, some which produce changes in simpler substances. Among these are the lipases, or the ester-hydrolyzing (including the fat-hydrolyzing) enzymes. In this case the composition and structure of the initial and final substances involved in the reaction are definitely known, and the uncertain factors due to the chemical nature of the substance acted upon are eliminated.

This investigation on the action of lipases was therefore undertaken. It has now been in progress for a number of years; and a series of papers describing the experiments in detail have been published in the *Journal of the American Chemical Society* for the years 1912, 1913, and 1914. It is the purpose of this paper to summarize the more important results and conclusions. With reference to these it should be stated that the aim of this investigation has been, not so much to follow the changes which the lipase produces in other substances (for example, by measuring the rate at which it causes the hydrolysis of different esters), as to study the changes in the activity of the lipase itself under various conditions, in the hope of obtaining information with regard to the chemical and physical properties of the substance or substances upon which the lipolytic actions depend.

Preparation of Extracts Containing Two Kinds of Lipase from Castor Beans. Lipases were prepared from both vegetable and animal sources. The most satisfactory and interesting material was found to be husk-free and oil-free castor beans. From these beans two distinct kinds of enzyme were readily extracted and separated from each other. These two preparations differed from each other in their hydrolytic action upon esters. Under certain fixed conditions the one was found to exert a comparatively greater action on ethyl butyrate than on glyceryl triacetate; the other to exert a comparatively greater action on glyceryl triacetate than on ethyl butyrate. The two kinds will be called *esterase* and *lipase*, respectively. Ethyl butyrate was used in these hydrolysis experiments as an example of a simple ester not readily hydrolyzed by water; glyceryl triacetate, as an example of an ester analogous to the naturally occurring fats and oils, from which it differs, however, by its greater solubility in water which makes it more convenient in comparative experimental work.

The esterase of castor beans was found to be associated with substances soluble in water; for clear aqueous solutions of it are obtained by direct extraction with water, dialysis, and filtration. The lipase of castor beans was obtained by extracting the water-insoluble castor-bean preparation with 1.5 normal sodium chloride solution, in which it shows a maximum solubility, and removing the salt by dialysis. In this way a mixture containing the lipase in suspension is formed.

There was no indication of the presence of a co-enzyme with either the esterase or lipase. The identity of the esterase with glycerophosphatase described by Plimmer (*Biochem. J.*, 7, 43; 1913) was made probable.

Presence of These Lipases in Other Materials. Soy beans were found to contain no esterase, but to contain a lipase having an appreciable solubility in water, but showing again a maximum solubility in a 1.5 normal solution of sodium chloride. Both esterase and lipase were found to be present in human intestinal secretions obtained by means of duodenal tubes. The esterase predominated in the secretions when no food had been taken for some time previously; and it is therefore probably present in the intestinal juice (*succus entericus*). The lipase predominated after the ingestion of food; and it therefore doubtless occurs in the pancreatic juice and bile.

Effect of Neutral Salts on the Rate of Hydrolysis of Esters by Enzymes. The effect of a number of neutral salts on the rate of the hydrolytic actions produced by these enzymes was studied systematically over wide ranges of concentration. Similar results were obtained with the enzymes from different sources. In some cases the added salts showed very marked differences in their effect on the hydrolytic actions of the two enzymes on their respective esters. The esterase action, for instance, was retarded by the presence of sodium chloride or sodium bromide, the retardation being distinct even at a concentration of 0.005 normal and increasing with increasing concentration of the salts. The lipase action, on the other hand, was increased by these salts up to a concentration of 0.1–0.2 normal, and was then decreased, this decrease becoming considerable at high concentrations. Sodium fluoride produced a very strong retardation with both enzymes even in solutions as dilute as 0.1 normal or less. The retarding action of sodium iodide was intermediate between that of the chloride or bromide and the fluoride. Other uni-univalent salts and certain uni-bivalent and bi-bivalent salts were studied similarly. Some of the bivalent radicals or ions, for example sulphate, were found to increase the lipolytic actions.

The possibility that a deleterious action is exercised on digestive processes by bromide and iodide when administered therapeutically in large amounts or over long periods of time was indicated by these results. The inhibiting actions of these salts were shown to be due to the fact that they precipitate or coagulate the enzyme-material. The coagulations were to some extent reversible at first, but long contact with the salt rendered them irreversible.

Effect of Manganese Salts as Oxygen Carriers. Of all the salts studied manganous sulphate produced the greatest accelerating action with castor and soy beans. In all probability this increased action is due to a large extent to the effect of the manganese as an 'oxygen carrier' in converting inactive material present in the bean into active enzyme. For it was found that when an original castor-bean preparation has been made inactive by heating its solution, it can be partially reactivated by adding manganous salt and passing a stream of air through the solution. The active substance is also produced by placing a solution or suspension in contact with an anode and submitting it to a long-continued electrolysis. These facts evidently support the explanation of the formation of active enzyme from inactive material by oxidation; but hydrolysis is perhaps also a factor.

The cycle dead, living, dead, occurs here in perhaps one of its simplest manifestations, exemplified by the transpositions inactive material of the bean, active enzyme, inactive or 'killed' enzyme material. The possibility of such a regenerative action occurring in the growth and development of the castor bean led to testing the oil-free kernel for manganese. A definite test for it was obtained; and the amount present was estimated to be 0.0006% of the oil-free kernel, or 0.008% of its ash.

Effect of Alcohols and Esters on the Rate of Hydrolysis Caused by Lipases. Methyl and ethyl alcohols were found to exert retarding effects on the rate of hydrolysis—effects which continuously increased with increasing concentration of the alcohols. Methyl alcohol retarded the hydrolyses somewhat more than did ethyl alcohol. Glycerin, on the other hand, had no effect even at a concentration of 25%. The retardation was shown to be due to coagulation of the enzyme.

Since the simple esters are similar in physical properties to the simple alcohols, it was thought probable that they would exert similar coagulating or inactivating actions on the active enzyme. Methyl acetate should then exert greater retarding action on the enzyme than ethyl acetate, while with glyceryl triacetate, the retardation might well be negligible. Similarly, esters containing the lower acid radicals, such as ethyl acetate, might be expected to exert retarding effects; while esters containing the higher radicals such as ethyl butyrate, might have considerably less effect. These hypotheses were tested, and were found to be confirmed. From these results the glycerides of the higher fatty acids which occur in nature would be expected to exert no inhibiting action on the lipase materials.

These actions of the esters on the lipases serve to explain part of the selective actions of the lipases which have been described in the past. They make it evident that the action of the substrate (substance acted upon) on the enzyme must in all cases be taken into account when considering reactions of enzymes.

In the case of the lipase material it was shown that a definite quantity of the enzyme can react with only a definite quantity of glyceryl triacetate in a given time. When the enzyme and ester are present in this ratio an increase in the amount either of ester or of lipase material does not increase the extent of the action.

Preparation, Composition, and Activity of the Solid Lipase-Materials. Solid esterase preparations, active as a rule, were obtained by precipitating the filtered and dialyzed aqueous extracts with acetone. The dialyzed salt-extracts, which contained the lipase material in suspension, give an inactive preparation after filtration, washing of the precipitate, suspension of it in acetone, and standing in this solvent. On the other hand, on standing in water for about two weeks, the soluble esterase-preparation lost its activity, while the insoluble lipase-preparation retained its activity unchanged.

The nitrogen-content of different preparations of esterase, referred to the ash-free and moisture-free substance, ranged from 15.4 to 16.3%, and the phosphorous-content ranged from 0.36 to 0.90%. The ash from these preparations amounted to 5%. Tests made upon them showed the presence of no carbohydrate, of much tryptophane, of much aromatic-group compounds, and of a trace of tyrosine. The solid preparations from the lipase material showed a more constant composition, giving an average nitrogen-content of 16.8%, referred to the ash-free and moisture-free substance, and an average phosphorus-content of 0.68%. The ash was 4.3%. The preparations gave a negative test for carbohydrates, a faintly positive one for tyrosine, a distinctly positive one for tryptophane, and a strongly positive one for aromatic groups. The forms of combination of the nitrogen in the different preparations were found to be the same as those recorded in the literature for typical proteins from various sources, with minor differences in the relative amounts of the various amino acids present. About 25% of the nitrogen was present in the form of arginine, a characteristic of seed proteins. Similar results were found with the soy-bean preparations.

These analyses, taken in connection with the method of preparation which removed all fatty (ether-soluble) substances, show that both esterase and lipase preparations are essentially protein in character. The esterase preparation may be considered to be an albumin, the lipase preparation to be a globulin.

The inactivation of these preparations by water, by salt solutions, or by acetone may be compared with the inactivation of the original castor and soy-bean preparations by heat. The loss in weight of these preparations in a vacuum desiccator over phosphorus pentoxide was not accompanied by loss in activity; but the same loss in weight by heating at 100–110° was accompanied by a loss in activity of from 50–80%. By drying first and then heating, which caused only a 0.1–0.2% greater loss in weight, the same loss in activity was produced. The loss of activity of the esterase-preparation caused by salts is due to coagulation or precipitation; that of the lipase-preparation caused by treatment with acetone is apparently due to dehydration. Hydrolysis of the enzyme may also play a part under some conditions.

Relation between the Hydrolytic Effects of Lipases and Those of Proteins and Amino-Acids. The hydrolytic actions of lipases are intimately connected with protein material. If these actions are due to proteins, it is probable that only part of the complex protein molecule is directly responsible for them. This possibility was studied by measuring the effects of some aminoacids and peptides in causing the hydrolysis of a number of esters. Some of the results, obtained in part by Dr. M. L. Hamlin, are as follows: Glycine, glutamic acid, and aspartic acid exerted hydrolytic actions on methyl acetate, ethyl acetate, glyceryl triacetate, phenyl acetate, ethyl butyrate, ethyl benzoate, and phenyl benzoate. If these esters be arranged in the order of decreasing amounts of hydrolysis, the order is different in the three cases where the action is caused by water, by glycine alone, and by glutamic or aspartic acids. A comparison of the hydrolytic actions of glycine, alanine, and phenylalanine on the seven esters also indicated certain selective actions. The hydrolysis of methyl acetate and ethyl butyrate by solutions of glycine and acetic acid is less than that by corresponding solutions of acetic acid alone. The hydrolysis of these esters by glycine-hydrochloric acid mixtures was not even approximately proportional to the hydrogen-ion concentrations of the solutions. The dipeptides exerted a comparatively greater action on ethyl butyrate than on methyl acetate, while the dibasic aminoacids showed the reverse actions.

The fact that the different esters are hydrolyzed by aminoacids and peptides is not in itself surprising. In its bearing on the enzyme work its interest lies especially in the selective or specific character of some of the actions, which are apparently independent of the hydrogen-ion concentrations, but dependent upon the structure of the aminoacid or peptide. These effects were, to be sure, small, and the specific character of them was not very pronounced; but the possibility of reproducing such selective actions even in a small degree with simple groupings which themselves may occur in proteins, supports the view that more complex groupings may produce the greater hydrolytic and highly specific actions observed with the natural lipases. That the simple linking together of animoacids in peptide-union is not sufficient to account for the actions is shown by the fact that peptides exert, if anything, a smaller hydrolytic action than the simple aminoacids.

Conclusion as to the Specific Character of Lipolytic Action. From the investigations briefly summarized in this paper it appears that the specific character of the hydrolytic actions produced by lipases is mainly due to two effects; first, the effect of the substrate on the enzyme in causing its coagulation or precipitation, and second, the effect of the enzyme on the substrate arising from the presence in the former of special groupings which may be similar to those contained in simpler nitrogenous substances which also bring about the hydrolysis of esters.

THE HYDRATION OF THE IONS OF CESIUM CHLORIDE DERIVED FROM TRANSFERENCE EXPERIMENTS IN THE PRESENCE OF RAFFINOSE

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The hydration of ions has attracted the attention of chemists for a number of years, and a large amount of evidence has been accumulated to show that ions are hydrated to a greater or less extent. One of the strongest pieces of evidence in favor of this view has been obtained by transference experiments in the presence of a non-electrolyte. If at the end of such an experiment the ratio of water to non-electrolyte has changed in the solutions around the electrodes, either the ions have carried water from one electrode-portion into the other, or they have carried the non-electrolyte in the opposite direction.

In a previous investigation by E. W. Washburn[1] the relative ionic hydrations of the chlorides of lithium, sodium, and potassium in 1.2 molal aqueous solution at 25° were derived by means of transference experiments in the presence of a suitable non-electrolyte as a reference substance. The object of the present investigation was to extend these data so as to include cesium chloride, which seemed desirable since there was much reason to believe that the cesium ion is the least hydrated of all the alkali ions.

The method employed consisted essentially in passing at 25° a measured quantity of electricity through a solution of cesium chloride containing enough raffinose to produce the desired rotation placed between a silver anode and a cathode consisting of silver covered with solid silver chloride. The apparatus was constructed so as to allow the separation of the portions around the electrodes from one another and from three middle portions. After the electrolysis the separate portions were carefully analyzed; the salt being determined by precipitation with silver nitrate, the raffinose polarimetrically, and the water by difference. The changes in content of the portions were computed on the assumption that the raffinose remained stationary during the passage of the current. Inasmuch as solutions containing 10% of raffinose have been prepared which had a specific conductance of 2×10^{-6} mhos (without 'correcting' for the conductance of the water), it is clear that the raffinose cannot conduct electricity appreciably either as an electrolyte or as a colloid.

The apparatus and experimental procedure were essentially the same as those employed in the previous work; but the concentration of the reference substance (raffinose) was reduced from 0.1 to 0.07 formula-weight per 1000 grams of water. This reduction in the concentration of the reference substance, while theoretically desirable, would tend to decrease considerably the accuracy in the measurement of the water transference, especially where, as in the case of cesium chloride, this transference is very small. This loss of accuracy was, however, compensated by employing a more sensitive polarimeter with a layer of solution one meter in length and substituting the *E* line (from a quartz mercury-vapor lamp) in place of the *D* line (from a sodium flame) previously employed. Without this great refinement in the polarimetric measurements the water transference in the case of cesium chloride could hardly have been detected, much less measured. The actual rotation measured in the polarimeter was about 50° and was reproducible to about 0.004°.

A practice run using sodium chloride as the electrolyte confirmed the previous work with this salt; thus this run gave 0.82 mols of water and 0.623 equivalents of chloride-ion transferred per faraday, as against 0.81 and 0.617 obtained in the previous work.

Two runs were made with 1.1 molal cesium chloride solutions. One of the runs was completely successful; but only the results obtained from the cathode portion of the second run were reliable. The number of mols of water transferred from anode to cathode per faraday were in the first run 0.33 and 0.39, and in the second run 0.28, giving a mean result of 0.33 mols per faraday. The values obtained for the number of equivalents of the anion transferred per faraday were similarly 0.506, 0.511, and 0.510.

The experimental data are presented in Table 1. The data given for the middle portion are also those for the original solution.

TABLE 1

First Experiment

	ANODE PORTION	ANODE MIDDLE	MIDDLE PORTION	CATHODE MIDDLE	CATHODE PORTION
Angle of rotation	48.671	48.228	48.131	48.116	47.452
Density at 25°	1.1105	1.1357	1.1357	1.1357	1.1642
Per cent raffinose	3.0320		2.9319	2.9310	2.8197
Per cent CsCl	12.077	14.772	14.770	14.750	17.713
Silver in coulometers	5.5048				5.5044
Weight of electrode portion	132.58				119.77
Grams H_2O transferred	0.30				0.36
Grams CsCl transferred	4.243				4.198
Faradays of electricity	0.05103				0.05102
Cation transference-number	0.494				0.489
Mols H_2O transferred per mol CsCl	0.64				0.80
Mols H_2O transferred per faraday	0.33	(Mean	0.36)		0.39

Second Experiment

	ANODE PORTION	ANODE MIDDLE	MIDDLE PORTION	CATHODE MIDDLE	CATHODE PORTION
Angle of rotation	55.865	55.137	55.137	55.127	54.421
Density at 25°	1.1152	1.1409	1.1409	1.1409	1.1689
Per cent raffinose	3.4653	3.3433	3.3433	3.3428	3.2208
Per cent CsCl	12.383	15.152	15.160	15.161	18.063
Silver in coulometers	5.4787				5.4787
Weight of electrode-portion	*				121.12
Grams H_2O transferred					0.25
Grams CsCl transferred					4.190
Faradays of electricity	0.05078				0.05078
Cation-transference-number					0.490
Mols H_2O transferred per mol CsCl					0.56
Mols H_2O transferred per faraday					0.28

* The anode residues were scorched in drying in this experiment, thus rendering uncertain the weight of the electrode portion.

The fact established by these experiments, independent of any hypothesis, is that the electrolysis of a solution of cesium chloride containing also raffinose at a low concentration is attended by a decrease in the ratio of the quantity of water to that of raffinose at the anode and a corresponding increase at the cathode. If now the probable assumption be made that the raffinose is not transferred by the current, the further conclusion is reached that there is a net transfer of water to the cathode, indicating that the cesium-ion is hydrated, and hydrated to a greater extent than is the chloride-ion. Moreover, a quantitative relation between the number of mols of water N_w^{Cs} on the cesium ion and the number N_w^{Cl} on the chloride ion can be derived from these results, as follows:

$$N_w^{Cs} = 0.67 + 1.03 N_w^{Cl}.$$

For the purpose of comparison the corresponding relations previously obtained for the three other alkali chlorides[1] and for hydrochloric acid[2] at 25° at about 1.2 molal are here brought together with those for cesium chloride. The N_w's indicate the average number of water molecules carried by the ion indicated by the superscript as it moves through the solution. The values given are the means of the two results obtained at the two electrodes independently. The deviation of this mean from the two individual values is indicated in each instance by the term preceded by the ± sign.

$$N_w^{H} = 0.28 \pm 0.04 + 0.185 N_w^{Cl} \quad (1)$$
$$N_w^{Cs} = 0.67 \pm 0.1 + 1.03 N_w^{Cl} \quad (2)$$
$$N_w^{K} = 1.3 \pm 0.2 + 1.02 N_w^{Cl} \quad (3)$$
$$N_w^{Na} = 2.0 \pm 0.2 + 1.61 N_w^{Cl} \quad (4)$$
$$N_w^{Li} = 4.7 \pm 0.4 + 2.29 N_w^{Cl} \quad (5)$$

From equations (2) and (3) we get the relation $N_w^{K} - N_w^{Cs} = 0.65$, which shows that the cesium ion carries on the average 0.65 less molecules of water than the potassium ion and is therefore the least hydrated of the alkali ions.

From equations (1) to (5) it follows, if we assume that the chloride ion is unhydrated, that the numbers of mols of water carried by the other ions are:

H^+, 0.3; Cs^+, 0.7; K^+, 1.3; Na^+, 2.0; Li^+, 4.7.

If we assume that the chloride-ion contains 4 mols of water, then the numbers contained in the other ions are:

H^+, 1.0; Cs^+, 4.7; K^+, 5.4; Na^+, 8.4; Li^+, 14.

Some experiments were also made with potassium nitrate; and, although the results were not quantitative, it was definitely shown that the water moved from the anode to the cathode in the case of this salt also.

All of the results given above are based upon the assumption that the raffinose remains stationary during the passage of the current. There is thus far no evidence indicating that this assumption is invalid to an extent which would appreciably affect the results. With the polarimetric apparatus at present at our disposal it will be possible to work with considerably smaller salt concentrations and to reduce the concentration of the reference substance as low as 0.01 molal in the case of lithium chloride solutions. By substituting trehalose for raffinose it will probably even be possible to use a considerably lower concentration than this and thus to remove all doubt as to the validity of the assumption in question. Work along these lines will be continued in this laboratory.

A more complete description of the research described in this paper will soon appear in the *Journal of the American Chemical Society.* A detailed description of it has already been printed as part of a doctor's thesis submitted by Earl B. Millard to the Graduate School of the University of Illinois.

[1] Washburn, *J. Amer. Chem. Soc.*, 31, 322 (1909).

[2] Buckböck, *Zs. physik. Chem.*, 55, 563 (1906).

THE ORIGIN OF CORAL REEFS

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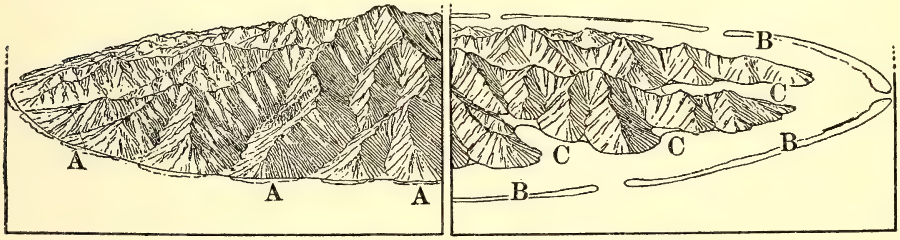
A Journey across the Pacific. A liberal grant from the Shaler Memorial Fund of Harvard University, supplemented by a subsidy from the British Association for the Advancement of Science, whose meeting in Australia during August I was invited to attend as a foreign guest, enabled me to spend the greater part of the year 1914 in visiting a number of islands in the Pacific Ocean with the object of testing the various theories that have been invented to account for coral reefs. Thirty-five islands, namely, Oahu in Hawaii, eighteen of the Fiji group, New Caledonia of which the entire coast line was traced, the three Loyalty islands, five of the New Hebrides, Rarotonga in the Cook group, and six of the Society islands, as well as a long stretch of the Queensland coast inside of the Great Barrier reef of northeastern Australia, were examined in greater or less detail. Darwin's theory of subsidence is, in my opinion, the only theory competent to account for the coral reefs there seen; thus my work leads to the same conclusion as that reached by several other recent students of this old problem. A full discussion of my observations will be published later, probably in the *Bulletin of the Museum of Comparative Zoology* at Harvard College. A brief statement of the chief results gained here follows.

Theories of Coral Reefs. Before setting out on the voyage I reviewed the various theories of coral reefs in an essay that was published during my absence under the title of "The Home Study of Coral Reefs" in the *Bulletin of the American Geographical Society* for 1914. Every one of the several theories is successful in explaining the visible features of the reefs themselves, provided the postulated conditions and the invisible processes of the past are accepted. Evidently then a study of the reefs alone will not suffice to discover which theory really provides a correct mental counterpart of their past and unobservable history. Hence appeal must be made from the non-committal reefs to competent witnesses of some other kind, which were present while the reefs were forming and which are willing to testify about the events which then took place.

Evidence derived from Barrier Reefs. In searching for such witnesses it should be borne in mind, first, that as far as fringing reefs A, A, A, figure 1, are concerned, their origin is hardly in debate; they are growing colonies of corals, initiated by the arrival from elsewhere of passively floating larvae, which establish themselves in shallow water close to a newly offered and suitable shoreline; second, that as far as atolls —or reef-rings enclosing shallow lagoons without central islands—are concerned, they are, unless penetrated by numerous and expensive borings, inscrutable, for they stand alone and bury their past; third, that as far as elevated reefs are concerned, their inner structure and their relation to the foundation on which they were formed would give important evidence regarding their origin, and should therefore be investigated; and fourth, that it is in connection with barrier reefs, B, B, B, figure 1, that the desired witnesses to the facts of the past can be most readily found; for the central volcanic island, rising from the lagoon within a barrier reef, was surely there while the reef was forming around it; and the features of the island shoreline will, as Darwin long ago pointed out for still-standing islands and as Dana a few years



later pointed out for subsiding islands, afford critical evidence regarding the changes which the island suffered contemporaneously with the formation of the encircling reef. These considerations may make it clear why my work has been almost wholly confined to the central islands of barrier reefs, though several examples of uplifted reefs were not neglected.

Coral Reefs around Still-standing Islands. The various theories of coral reefs may be divided into two groups. Those of the first group, some six or seven in number, postulate a fixed relation between land and sea level during the development of the reefs; these will be called the still-stand theories. The two theories of the second group postulate a change in the relative level of land and sea during the development of the reefs. As far as the barrier reefs that I have visited are concerned all the theories of the first group must be rejected, because within every one of these barrier reefs the embayments or drowned valleys, C, C, C, figure 1, by which the shoreline of the central volcanic island is indented, give through Dana's principle of shoreline development indisputable evidence of recent submergence. I believe that the same conclusion applies to all barrier reefs, first for the reason that those which I visited were not selected because they were thought to be in any way unlike other members of their kind, but because they were easily accessible; and second because all the charts of other barrier reefs that I have examined show that their central islands also have embayed shorelines. For none of these islands can any one of the still-stand theories hold good.

Coral Reefs and the Glacial Period. It remains to inquire which one of the two theories that postulate a change in the relative level of land and sea best accounts for the facts of barrier reefs and their associated central islands. One of these is Darwin's simple theory of a slowly subsiding ocean bottom, as a result of which the islands gradually sink, diminish in size, and eventually disappear, while their fringing reefs grow upwards and are converted into barrier reefs and atolls. The other is the more complicated "glacial control theory" lately elaborated by Daly with especial reference to atolls. It begins by assuming still-standing foundations, above and around which out-growing reefs of less or greater size, with shallow lagoons or none, were developed in preglacial times; a lowering of sea level is then inferred during the glacial period, when a significant amount of sea-water was withdrawn to form the continental ice sheets, and when in consequence of lowered ocean temperature the corals of most reefs were killed; next follows an abrasion of the unprotected preglacial reefs so as to reduce them to flat platforms a little below the lowered sea level; and finally, when a rising temperature melts the continental ice-sheets and the sea surface is raised and warmed, and the corals are permitted to grow again, reefs are built up to the present sea level around the margin of the abraded platforms, producing barrier reefs or atolls as the case may be, around the lagoon that covers the abraded platform. The embayments of the central island within a barrier reef are explained as drowned valleys that were eroded while the sea level was lowered.

After careful consideration I have had to discard this theory, except insofar as it may have produced small results that are altogether subordinate to the larger effects of some more efficient cause. My reasons are in brief that, if the lagoons of large atolls have been abraded across their whole diameter of 20 or 30 miles, the central volcanic islands within narrow-lagoon barrier reefs should have been strongly cliffed by the lowered sea all around their shores, and their lagoon waters should now rise on the cliffed spur ends; but this is not the case; the spurs generally dip gently into the lagoon with small cliffs or none. Further, if the embayments of the central islands within barrier reefs occupy new-cut valleys that were eroded during the lowered sea-stand of the glacial period, the up-stream parts of such new valleys should be visible beyond their embayed parts, and should there appear as incisions beneath the floors of preglacial valleys, producing a valley-in-valley landscape; but in the hundreds of embayments that I saw, no such composite valleys occurred. Finally, it is doubtful if the lowering of sea temperature generally sufficed to kill the corals and expose the reef flanks unprotected from sea attack; for on the atolls of the Paumotus Agassiz found many instances of slightly uplifted reef limestones, which he regarded as 'Tertiary,' and hence as preglacial, on the inner border of the present encircling reefs; and in such cases the lagoon floors could not be the result of marine abrasion in glacial time.

Submergence and Subsidence. It might now appear as if no other cause than subsidence, as postulated in Darwin's theory and apparently confirmed by Dana's explanation of embayed shorelines, remained available; yet all the observable facts of the case may be fully as well explained by a rise of the sea surface, caused by an upheaval of the sea-bottom elsewhere, as by a sinking of the islands and of the sea-bottom on which they stand; but a deliberate discussion of this alternative shows it to be highly improbable because it demands extravagant crustal deformation and because it involves all the coasts of all the continents as well as the coral-reef islands. If it be set aside, only

Darwin's theory remains, the simple sufficiency of which stands forth all the more clearly in contrast with the failing inefficiency of its competitors.

The Problem of Atolls. The conversion of barrier reefs into atolls by a continuation of the process that has converted fringing reefs into barrier reefs is a highly probable matter; for it would be unreasonable to suppose that this process, whatever it is, should always have stopped before the central islands of barrier reefs were wholly submerged, and should never have worked in neighboring areas where reefs of identical form, but without a central island, are given another name. And as the converting process has, with so high a degree of probability, been shown to be subsidence of the ocean bottom in the region concerned, and not change of ocean level during the glacial period or uplift of the ocean bottom in some other region, it appears reasonable to explain atolls as well as barrier reefs by subsidence. This conclusion becomes all the more reasonable when the intimate association of barrier reefs and atolls in the Fiji and Society groups and elsewhere is noted. Nevertheless the problem of atolls cannot be now regarded as absolutely solved, nor can it be absolutely solved until we make addition to our knowledge now undreamed of.

Reefs and Reef-Platforms. A modification of Darwin's theory has lately been proposed by Vaughan, who regards recent submergence, proved by the embayments of the central islands, as the determining cause for the up-growth of existing barrier reefs, but who interprets the deeper and larger part of the entire reef-mass as an independent "platform" of earlier origin. As this investigator has not yet published his views regarding the origin of the reef-platforms, his modification of Darwin's theory will not be here discussed farther than to note that it seems inapplicable to many barrier reefs in the Fiji and Society groups; that the discontinuity of certain barrier reefs seems to be explicable on the assumption of imperfect up-growth during and after a recent and rapid subsidence, as well as on the assumption of independent origins for the reefs and their platforms; and that, while the extension of reef-platforms outside of the coral zone, as in the case of the Great Barrier reef of Australia, truly suggests a dual origin of reef-masses, this does not exclude the contemporaneous growth of platform and reef within the coral zone during long-continued but irregular or intermittent subsidence.

Summary of Results. The general result of my voyage, already announced above, as well as several special results, may here be concisely stated:

Darwin's original theory of subsidence gives by far the most satisfactory explanation of all the barrier reefs that I have visited in the Pacific or studied on large-scale charts; and as atolls often occur in association with barrier reefs, the theory of subsidence appears to give the best explanation of such atolls also. Atolls that are not associated with barrier reefs may be of some other origin, but this does not seem probable.

The elevated reef along the south coast of Oahu, Hawaii, was formed during or after a sub-recent period of subsidence, for its limestones enter well-defined valleys of erosion.

The Fiji group has suffered various movements of subsidence and uplift by which its many islands were affected in unlike ways. Uplift has taken place at different times, for some of the elevated reefs are elaborately dissected, others are very little dissected, and still others remain at sea level. The uplifted reefs seem to rest unconformably on subaerially eroded volcanic centers, hence the centers must have been above sea level to suffer erosion; they must have been depressed to receive the unconformable reef deposits; and the compound mass must then have been elevated to lay bare the reef. In one island (Vanua Mbalavu) the uplifted reef has been maturely dissected and partly submerged, as indicated by its embayed border, and a new barrier reef has grown up outside of it. Thus all the Fiji reefs, those now elevated as well as those at sea level, appear to have been formed during periods of subsidence.

The extensive barrier reef of New Caledonia has grown up during a recent subsidence by which that long and maturely dissected island has been much reduced in size and elaborately embayed; but unlike most encircled islands this one was strongly cliffed around its southeastern end and along much of its northeastern side before the recent subsidence took place.

The two southeastern members, Maré and Lifu, of the Loyalty group, are former atolls, evenly uplifted 200 or 300 feet; Maré has a small knob of volcanic rock in its centre. Uvea, the northwestern of the three Loyalty islands, is a slightly tilted atoll.

The New Hebrides show signs of uplifts in their elevated reefs, and of depressions in their embayments. There is some evidence that certain uplifted fringing reefs on the island of Efate, near the center of the group, were formed during pauses in a subsidence that preceded their uplift, and not during pauses in their uplift as inferred by Mawson. Espiritu Santo, in the northwest, has several large embayments interrupting its fringing reefs; this is taken to mean that earlier fringing reefs were drowned by rapid submergence, so that they did not grow up in off-shore barrier reefs.

The Great Barrier reef of Australia, the largest reef in the world, with a length of some 1200 miles and a lagoon from 15 to 70 or more miles wide, has grown upward during the recent subsidence by which the Queensland coast has been elaborately embayed, as was pointed out by Andrews in 1902.

Five islands of the Society group exhibit unequivocal signs of recent submergence in their intricately embayed shore lines, as has lately been shown by Marshall; the cliff-rimmed island of Tahiti, the largest and youngest of the group, has suffered moderate submergence after its cliffs were cut, but its bays are now nearly all filled with delta plains; hence a pause or still-stand has followed its latest submergence. All the barrier reefs of this group appear to have been formed during the recent submergence, due to regional subsidence, that embayed their central islands.

A METHOD OF PROPHESYING THE LIFE DURATION OF SEEDS

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Seeds that will withstand drying in the air without injury retain their viability under herbarium conditions from one to one hundred and fifty years. Seeds that will not withstand drying in the air generally have a longevity of only a few months. Ewart[1] has suggested that seeds retaining their viability for three years or less be called microbiotic; for three to fifteen years, mesobiotic; and for more than fifteen years, macrobiotic. The cause of the loss of viability with storage has been a subject of considerable study. Two explanations have been offered: exhaustion of stored foods and degeneration of digestive and oxidizing enzymes. Both of these explanations have proved incorrect for both foods and enzymes are present in almost full force for some years after viability is lost. Certain facts have led us to surmise that the gradual loss of viability with storage is due to a slow coagulation of the proteins in the plasma of the embryo. The fall of longevity with rise of temperature and of moisture content of the seed indicate this.

Certain known facts concerning proteins make the experimental investigation of this hypothesis possible. Chick and Martin[2] have shown that the coagulation temperature of proteins is not a fixed point as is the melting point of metals and other substances. Coagulation is a function of duration of heating, percentage of water present, and the reaction of the protein as well as of the temperature. The lower the water content the more heating is required for coagulation. Acidity favors and alkalinity retards coagulation by heat. Buglia[3] established the following time-temperature formula for the coagulation of proteins: $T = a - b \log Z$, in which T = temperature in degrees Centigrade, Z = time in minutes, and a and b are constants.

If our surmise is correct that the loss of viability of seeds with storage is a matter of coagulation of cell proteins of the embryo, this time-temperature formula for the coagulation of proteins should be applicable as a temperature-life duration formula for seeds. In experiment, of course, the life duration determined must be at relatively high temperatures in order to make the time within the range of an experiment. This calls for temperatures ranging from 50–100°C. for air dry seeds. So far we have used the method of reflux of ethyl or methyl alcohol or mixtures of these with water as a means of obtaining constant temperatures. This gives a range of temperature from about 65 – 99°C. and a variation at any temperature of less than ± 0.1°C. Delayed germination and lack of resistance of heated seeds to fungal attack made sterilization imperative. Silver nitrate to an aqueous solution of which the coats of various seeds are impervious proved an effective sterilizing agent.

TABLE I

Record Sheet No. 21. Turkish Red Wheat

April 10, 1914. Temperature 87.5°C. Moisture 12%. Percent germinated in heavy figures. Percent partially germinated in light figures

Time, days for germination	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Control	0	2	2	4	3	3	0	0	0	0	0	0	0	0	0	0	0	0
	92	92	92	93	95	95	98	98	98	98	98	98	98	98	98	98	98	98
Heated for 7 minutes			0	15	12	7	8	7	8	7	5	3	3	4	4	4	3	2
			2	5	27	41	49	55	61	64	67	70	72	72	72	72	73	74
“ “ 8 “					4	8	8	7	8	7	5	3	1	5	1	1	1	0
					5	10	25	30	35	41	47	52	54	54	58	59	59	60
“ “ 9 “						2	4	4	5	4	5	4	5	2	3	6	5	4
						2	4	8	10	11	18	25	28	32	34	35	37	38
“ “ 10 “										1	3	4	6	4	3	3	4	5
										0	0	0	0	4	5	9	11	11
“ “ 11 “												1	1	0	0	1	2	1
												1	1	2	4	4	4	5
“ “ 12 “													2	2	2	2	2	2
													0	0	0	2	2	2
“ “ 13 “																		
																0	0	0

Table I shows the data gained from a single experiment. Three things stand out prominently as results of heating: delay in germination, fall in germination percentage, and appearance of abnormal germination. The abnormal germination (root without stem or stem without root) are shown in light type. All these characters appear in seeds stored for a long time at room temperatures, indicating the likeness of the change whether it takes place at a high temperature in a short time or at a low temperature acting for a long time. In Table II are shown the life durations at various temperatures as found by experiment and the calculated temperatures for the various life durations. We have supplied for another variety of wheat one record of longevity by White[4] in which the time was definitely known as eight years and the moisture estimated at 12 percent and the temperature at 20°C. In all these failure of 75 percent to germinate after nineteen days is chosen as the end point. The calculated temperatures of the table were found by solving for the constants *a* and *b* by the method of least squares from the found values of T and Z, and from these calculating T for the various values of Z.

TABLE II

Germination Record Turkish Red Wheat

Theoretical temperature calculated by formula $T = a - b \log Z$

T = Temp Celsius, Z = Time of Heating, $a = 98.88$ $b = 11.78$

Trial No.	A	B-C	D-E-D	G	H	I	J	K-L	N	N	O	P
Duration minutes	7	8	9	10	15	18	45	50	50	120	315	8 yrs.
Found temp.	89.2	87.7	87.5	87.5	84.4	84.4	78.9	79.1	78.5	75.8	71.3	20±
Calc. temp.	88.9	88.2	87.6	87.1	85.0	84.1	79.4	78.9	78.9	74.4	69.5	20.9

The rather close agreement between calculated and found values indicate that the time-temperature formula for the coagulation of proteins can be applied as a temperature-life duration formula for seeds, at least under the condition of these experiments. Much more work is needed to establish the general application of this principle. Several more life durations should be determined for wheat as a mesobiotic seed. The life duration with 12 percent moisture should be run at several temperatures ranging from 50–70°C. A large number of determinations should also be made with some moisture content between 16–20%. Such seeds will show much shorter life durations. Determinations should also be run for 2–3% moisture which will give greater life durations. Similar determinations should be made for a macrobiotic seed, such as sweet clover for which we have reliable records of longevity, as well as a microbiotic form. There are several matters that may limit the application of this formula:

1. Increase of the acidity of the seed will hasten coagulation of the cell proteins. Such a change is known to occur in seeds of certain Rosaceae, at least if stored in the imbibed condition.

2. Lepeschkin[5] found that in active plant cells a redispersal of cell proteins is going on coincidently with coagulation. As a consequence at high temperatures where the coagulation was rapid the found and calculated life durations agreed closely; while at lower temperatures where redispersal is prominent the calculated life durations were much shorter than the found values. In the low water content of air dry seeds it is possible that the redispersal of proteins is of little significance. This may limit the method to seeds of relatively low water content.

3. A slight error in a and b will give a relatively large error for a life duration at low temperatures such as 0° C. At higher temperatures the error becomes less. In the data above calculated temperature for a life duration of eight years varies little whether a and b are calculated by including White's data at 20° C. or merely from the determinations above 70° C.

4. The lower the water content of seeds the more heating they will withstand and the greater the longevity at moderate and low temperatures. This law has its limits, for excessive drying is itself injurious. In seeds that will endure dessication injury sets in with a reduction of the water much below two per cent, while in forms like *Drosera* it appears before air-dry condition is reached. Our method is, of course limited to degrees of dessication less marked than those producing injury.

5. Undoubtedly longevity under like conditions will vary with different varieties of the same species and even with different crops of the same variety; but the general conditions found for a given crop will probably apply to other crops of the same variety and to other varieties of that species. How far the five points mentioned above will limit the application of this method can only be determined by such experiments as those outlined above.

The work shows possibilities of throwing light on the nature of the process of loss of viability in seeds and of leading to a quantitative statement of the significance of various storage conditions (especially moisture content and temperature) upon the longevity of seeds.

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[1] Ewart, *Proc. Roy. Soc. Victoria*, **2**, 1–210 (1908).

[2] Chick and Martin, *Amer. J. Physiol.*, **40**, 404 (1910); **43**, 1 (1911).

[3] Buglia, *Ztschr. Chem. Indust. Kolloide*, **5**, 291 (1909).

[4] White, *Proc. Roy. Soc. London*, **81** B, 417 (1909).

[5] Lepeschkin, *Ber. Dtsch. Bot. Gesells.*, **30**, 703–714 (1913).

THE FERTILIZING POWER OF SPERM DILUTIONS OF ARBACIA

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My previous studies of fertilization led to the conclusion that initiation of development depends upon activation of a substance produced by the egg and located in its cortex. This substance, for which I propounded the name *fertilizin*, possesses two side-chains involved in fertilization, one of which unites with receptors borne by the spermatozoon, the other with the receptors borne by the egg. The latter is the fertilization reaction proper; it was postulated that any agent that may activate the fertilizin, as do the sperm receptors, so as to cause its ovophile side-chain to combine with the egg-receptors may act as a parthenogenetic agent. Fertilization and parthenogenesis are thus brought under one point of view with reference to initiation of development.

Certain data concerning the fertilizin of Arbacia, such as time of origin, its location in the egg, its disappearance after fertilization, etc., are known owing to its property of agglutinating spermatozoa of the same species, which serves as indicator. Some of its chemical and physical properties have also been studied to a certain extent. But up to the present the very existence of the so-called sperm receptors has remained hypothetical. On any theory of fertilization it is necessary to postulate the existence of a spermatic substance that induces development of the egg. But in spite of a considerable number of investigations, the existence of such a substance still remains an hypothesis. The present contribution presents demonstrative evidence, though of a negative sort, concerning this substance.

The phenomena, on which I rely for my conclusion, concern the fertilizing power of sperm suspensions of graded dilutions. On the basis of the usual supposition that a single active spermatozoon may fertilize an egg of its own species, all of the eggs should be fertilized in a series of sperm suspensions of increasing dilutions up to the place in the series in which each egg receives only a single spermatozoon. Beyond this the percentage of eggs fertilized should fall off at a certain rate to a vanishing point.

An approximate realization of this may be obtained if the interval between preparation of the more dilute sperm suspensions and their use in fertilization be made as short as possible. This may be done by the addition of eggs to measured quantities of sea-water, followed by the addition of a sufficient quantity of an accurately determined sperm suspension stirred in quickly to reach the desired dilution. The final dilution is then made in the presence of eggs, and the age of the final dilution is therefore practically zero with reference to the fertilization reaction.

Under these conditions 100% of the eggs may fertilize up to a dilution of about 1/3000 of a 1% sperm suspension. The curve of fertilizing power of the sperm suspensions measured in terms of the percentage of eggs fertilized then falls off slowly to 1/24,000 of 1% sperm, then rapidly to about 1/300,000%, then slowly again to about 1/90,000,000% where, however, about 1% of fertilization may still take place. Accompanying observations showed that beyond a dilution of about 1/5000 of a 1% sperm suspension only a single spermatozoon can possibly be concerned in the fertilization of each egg.

One obtains exceedingly contrasting results if a series of sperm dilutions in powers of 2 is made, beginning with 1%, by first transferring a certain amount of the 1% suspension to a second crystal of the series and adding an equal amount of sea-water, proceeding similarly from crystal 2 to crystal 3, and so on down the series. In such a case one finds that fertilization runs out absolutely from about a 1/64 dilution of 1% sperm suspension to 1/1024% in different cases. No comment is needed to emphasize the contrast.

As many as 20 to 40 active spermatozoa are found in association with each egg at 1/128%, which may be, however, absolutely ineffective. So that in an experiment running out in the seventh crystal of the series it would appear that a greater number of spermatozoa than this is required to fertilize an egg.

These results suggest at first glance that the order of adding eggs and sperm to the sea-water may be of significance. This is, however, not the case. The repeated handling of the sperm in successive half dilutions is also not the main cause for the result. Thus it would appear that the only real difference between the fertilizing power of the sperm in these cases is a time factor. In the first case the final dilution is made in the presence of the eggs; in the second case 20 to 30 minutes is consumed in the preparation of the sperm dilutions before the eggs are added.

The time factor is the real explanation as will be shown immediately. But at first sight this did not seem a very reasonable explanation for the following reasons: In the first place the time involved has never been considered sufficient to reduce fertilizing power of sperm; and in the second place the original 1% sperm suspension was shown in several experiments to be capable of fertilizing a high percentage of eggs at 1/30,000 dilution or less, after a longer interval of time. If the sperm suspensions lose their fertilizing power with time, it must be that the significance of time in this respect varies inversely to concentration.

This conclusion was abundantly verified by the following tests: A quantity of sperm suspension of a given concentration is prepared and divided in several equal amounts in a series of crystals; the same quantity of eggs is then added at time intervals to the crystals of the series, and the percentages of fertilization estimated by careful counts. Fourteen different grades of dilution between 1/300 and 1/240,000 of 1% sperm were thus measured. Loss of fertilizing power was shown in all of the suspensions thus tested in less than one hour; and in general the rate of loss increased with each successive dilution. The actual data are to be published elsewhere. Here we may summarize the results in the following table, showing the approximate time required A, for 66% loss of fertilizing power, B, for complete loss, at six different dilutions.

Dilution of sperm	1/1200%	1/3000%	1/6000%	1/30000%	1/60000%	1/120000%
A. 66% loss	32 min.	7 min.	5 min.	3 min.	2-3 min.	1 min.
B. 100% loss	64 min.	24 min.	?	20 min.	20? min.	7 min.

Suspensions of higher concentrations, than those included in the table exhibited a much slower rate of loss, which was measured by another method, showing that from 1/4% down, loss occurs in increasing amount within a period of 100 minutes.

Other possible factors than time influencing the fertilizing power of sperm suspensions are on the whole relatively slight in this series of experiments. There is a certain natural variation in different lots of ova and sperm, which is no doubt responsible for some irregularities in the data. Another factor is that of egg-concentration; but a series of determinations showed that the actual variations due to this cause, occurring in the experiments themselves, are not of significance.

There are two modes of explanation of these results theoretically possible, viz: First, that the loss of fertilizing power is due to loss of motility of the spermatozoa; in the first place this theory does not agree with the observation, that the spermatozoa lose their fertilizing power before they lose their motility; in the second place it renders the increase of rate of loss with dilution incomprehensible because on *a priori* grounds the exact opposite would be expected; in the third place as a result of several lines of work the theory that the fertilization reaction is primarily a function of motility of the spermatozoon has been given up; penetration of the ovum by the spermatozoon is due to the inception of the fertilization reaction, and not the reverse, as was previously assumed. We cannot therefore accept this theory.

The second theory is that the spermatozoa lose their activating substance, sperm receptors in my terminology, which agrees very well with the demonstrated ineffectiveness in spite of the observed persistent motility. That the rate of loss should increase with dilution is to be expected if we regard the loss of the sperm receptors as a diffusion phenomenon; and, if we regard it as an active process of secretion, we should expect such a result owing to general increase of functional activity under conditions which approach more nearly the normal.

The second mode of explanation, which I have actually adopted, fits in with the necessary postulate that the spermatozoon bears such a substance, and with the fact that the spermatozoon carries out the initial fertilization reaction while it is still intact and external to the ovum. The postulated activating substance of the spermatozoon must be liberated before penetration, and these experiments give us some idea as to the manner of its liberation.

Glaser (*Biol. Bull.* **26,** 84–91; 1914) has recently maintained that more than one spermatozoon is necessary for fertilization of the same form which I have studied. His observations may also find their explanation under the same point of view, inasmuch as he was not aware of the significance of the time factor in inseminations with highly dilute sperm.

We thus obtain the following additional point of view with reference to fertilization: the spermatozoon arriving at the egg while still intact liberates an activating substance which initiates the fertilization reaction; as one consequence among others of this reaction the spermatozoon is taken up by the egg, and completes the process of fertilization in its interior. This point of view is consistent, as far as it goes, with my own theory of the fertilization reaction; and it is also perfectly consistent with Loeb's quite different point of view.

My previous experiments had shown that eggs lose a certain substance in sea-water (fertilizin) which is necessary for their fertilization; fertilized eggs no longer produce this substance and are incapable of fertilization. Both eggs and spermatozoa, therefore, contain substances, more or less liable to loss, which are necessary for fertilization. The mechanism of fertilization cannot possibly, therefore, be regarded in the simple manner postulated by Loeb's theory. The existence of parthenogenesis demonstrates the efficacy under given conditions of the egg-substance alone; we must therefore regard the spermatic substance essentially as an activator of the fertilizin of the egg.

There remains of course the problem of identifying the free activator in the medium of the sperm suspension by its only known mode of operation, that of fertilizing the ovum. This problem, over which several investigators have broken their weapons, appears in a somewhat different light as a result of these experiments; and new experiments should therefore be undertaken.

VARIATION IN BACTERIA

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The bearing of slight physiological differences upon the classification of bacteria and upon the phenomena of infection has made the occurrence of variation among bacteria fully as conspicuous as in the higher forms of life. During the past ten years many observations have been recorded upon the extent and nature of bacterial variability. In these studies some confusion has arisen through the difficulty of distinguishing true variations from the development of latent characteristics, and from environmental modification.

By the term 'latent characteristics' is meant those qualities or properties that are dormant in the organism or cell and are manifested only in response to definite external influences. Thus, certain bacteria form spores in the presence, but not in the absence, of oxygen; some bacteria are known that develop conspicuous capsules when growing in the animal body, but lack these envelopes in part or altogether in artificial media; according to Wright, animal fluids seem to be essential for the production of the characteristic clubs of Actinomyces colonies. The sudden appearance in this way of a definite morphological character cannot be looked upon as an instance of variation. Such a manifestation is merely an immediate response to changed conditions of life and is of exactly the same kind as the marked difference in the aquatic and terrestrial forms of Polygonum amphibium referred to by DeVries, or as shown in the transformation of the shrimp Artemia salina into what some writers consider another species, Artemia milhausenii, when the former is transferred to water of a greater degree of saltiness. In no sense is the awakening of such a dormant character to be confounded with true variation. In other words the power to produce certain structures or certain physiological effects exists ready formed in the specific bacterial cell and is evoked by proper stimuli. These latent qualities often play an important part in bacterial identification, and it is frequently impossible to recognize a given organism until it is brought into a particular environment and has given the appropriate response. Bacteria are very sensitive to slight chemical differences in their surroundings and many alleged instances of variability in bacteria are simply differences in response evoked by variations in the composition of culture media. Herein lies the justification for the attempts at standardization of culture media and the maintenance of exact uniformity in the conditions selected for bacterial growth. The true latent character appears promptly and typically when the suitable conditions for its manifestation are afforded, and not under other circumstances. I have in my laboratory a culture of a bacillus discovered by one of my students (M. Didlake, *Centralbl. Bakt.* II, **15**, 193, 1905) that gives a brilliant red pigment in agar prepared from the soy bean, but fails to produce the pigment upon the ordinary meat broth peptone agar or upon any other medium tested. This is a clear case of a latent characteristic and is to be ranked with the fermentation of rare carbohydrates—as rhamnose by the paratyphoid bacillus—by microörganisms that in nature rarely if ever come in contact with the substances that they unhesitatingly attack.

What are called *environmental modifications* are sometimes hardly to be distinguished fundamentally from the phenomenon just under consideration. In a sense these modifications depend upon the possession of an innate capability for response to a definite environment, and as such must be regarded as expressions of latent characteristics.

Such modifications in the higher forms of life are typically the effects of use and disuse, of a more or less abundant food-supply or of climatic factors. One important difference between such modifications and the bursting into bloom of latent characteristics is that environmental influences of the nature indicated tend to produce similar results in different organisms. Among bacteria in general, latent characteristics are more susceptible of demonstration than environmental modifications. It is often difficult in practice to distinguish genuine environmental modifications from adaptations due to selection, although the phenomena are totally distinct. If one-half of an alpine plant be set in a low altitude garden, the other half being left in its original habitat, differences will arise which may be reasonably ascribed to environmental modification. With bacteria, experimentation of this sort is not so simple. The enormous number of generations through which a given culture of bacteria can pass in a short time may convey superficially the impression of an environmental modification similar to that observed in an individual organism transplanted into different surroundings, when in reality there has been simply elimination of unsuited variations and a selection of the variety adapted to the particular conditions. This is a serious obstacle in the way of obtaining a proper appreciation of direct environmental influence upon the individual bacterial cell.

The great majority of bacterial variations that have been described belong to the class of apparently adaptive modifications, whether such modification be considered as due to the direct action of the environment upon thousands or millions of bacterial cells, or to the superior advantage in the intra-cultural competition possessed by those cells that have some new peculiarity such, for instance, as that of fermenting a particular carbohydrate. A bacterial culture, brought in contact with a carbohydrate that it is unable to attack, may undergo a change so that it becomes able to split the carbohydrate, the change being then apparently adaptive. Such cases have been frequently cited as instances of bacterial mutation, but, as many of them are described, they might equally well be regarded as due to selective acting upon the so-called fluctuating variations. It is worth noting that most or all the cells of certain species seem to behave in the same way in the presence of certain fermentable substances, and that the newly acquired property is sometimes permanent for a long series of generations on other media, sometimes lost.

In the course of some experiments I have been carrying on during the past three years I have attempted to determine the width of swing in a pure line strain of bacteria, B.coli, cultivated under varying conditions. From a freshly cultivated feces culture a single cell was isolated by the Barber method, and from the descendants of this cell, numbering some hundreds of millions, two other cells taken at random were made the parents of two strains which have been used in a long series of experiments. Without space left to consider the technical details the following results may be stated. Both strains have been subjected to a series of influences, some of which might be expected to lead to particular adaptive modification, others of which were of a less specific character. Neither strain in the course of growth for over 500 generations at 37° C. (transfer every two days) on ordinary nutrient agar has shown any permanent change in ability to produce indol, to coagulate milk, or to ferment carbohydrates. On several occasions, however, variations in indol production were observed (four days at 37° C.—Ehrlich method) so that the results as recorded range from a mere trace to the maximum intensity observed in any culture. The tests were always carried out on the same lot of culture medium and were controlled by a number of cultures (usually about 20) from the same strains but of different cultural histories. Milk was always rendered acid but sometimes not coagulated in forty-eight hours. The milk used was certified milk always obtained from one dealer and treated in as uniform a manner as possible throughout. Nevertheless, differences in rapidity and completeness of coagulation were so great in different lots of milk as to make such changes of little value in attempts to study variation in the physiological properties of the organisms themselves.

The most fundamental change thus far observed is the acquisition of saccharose-fermenting powers by one of the pure line strains. This quality appeared in the seventh transfer on sodium chloride agar. It was not manifested by all the cells of the culture, but at the time of examination the saccharose-fermenting cells were greatly in the majority. On continuing the transfers they became the sole type found in the cultures, the non-saccharose type disappearing altogether. The power of fermenting raffinose was also possessed by the saccharose-fermenting strains. Gas is produced in both saccharose and raffinose solutions and over 4% of normal acid is formed. The newly acquired fermenting property has remained permanent throughout a series of over 500 test-tube generations (forty-eight-hour transfers), and is shown both by the cultures on sodium chloride media and by the strains transferred immediately on the acquisition of this property to ordinary nutrient agar and grown side by side with the two parent strains.

This instance of bacterial mutation therefore seems to fulfil the requirements (*a*) of appearing suddenly without intermediate stages, (*b*) of being irreversible, at least for three years and for some hundreds of test-tube generations, (*c*) of comprising change in two characters (saccharose and raffinose-fermenting power), and (*d*) of not involving all the cells of the parent strain.

It may be remarked that a differentiation of B.coli into species is commonly made on the basis of the power to ferment saccharose and most of the recent classifications of B.coli groups start with this as a fundamental distinction. In the great majority of strains of B.coli that have been tested by various observers saccharose fermentation is correlated with raffinose fermentation. Saccharose-fermenting streptococci on the other hand are often devoid of power to ferment raffinose.

It may be noted further that the acquisition of this new character by an originally non-saccharose-fermenting strain of B.coli has thus far developed only once and then on sodium chloride medium. Cultures of the parent organism grown in saccharose broth for a series of generations as yet show no gas production or acid production. This particular change therefore seems to be due to the intra-cellular or molecular changes brought about by non-specific influences and not to a direct adaptation to particular environmental conditions.

A DYNAMIC CONCEPTION OF THE ORGANIC INDIVIDUAL

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The organic world exists in the form of more or less clearly defined individuals, which may be completely isolated from others as self-maintaining organisms, or only partly isolated, like the members of a so-called colony among the lower animals and the different buds or growing tips and the parts associated with each in the multiaxial plants. A tree, for example, consists of a great number of plant individuals organically connected with each other.

In all except the very simplest organic individuals an orderly, definite sequence of events in space and time occurs which we call development. Development includes the series of changes from the reproductive cell or cell mass to the mature form of the organism. On the one hand, the organs arise in definite space relations to each other and to certain axes or planes which we can draw through the developing organism, and on the other hand, development consists in an orderly sequence of events in time. Certain regions always precede and others follow in regular order. The result of this orderly behavior is an organism of more or less definite form and structure, often exceedingly complex and with a high degree of constancy in successive generations. In the simplest individuals these space and time sequences are either less definite or else they are subject to frequent change and replacement by others, but in most organisms they are relatively permanent.

Most theories of the organism have failed to account satisfactorily for these orderly characteristics. Either they have simply assumed the existence of some sort of mechanism adequate to account for the facts, or some 'vitalistic,' i.e., non-mechanistic principle, as a controlling or ordering factor, or they have ignored the whole problem. Nevertheless, orderly behavior in development is one of the most fundamental and significant characteristics of the organism, for upon it depend the form and structure of the adult and the possibility of definite and coördinate function of parts.

During some fifteen years of experimental investigation of certain aspects of the problems of reproduction and development, I have been able to establish certain facts which throw some light on the problem of the mechanism which underlies the orderly character of development and at the same time point very definitely to certain conclusions concerning the nature of the so-called polarity and symmetry of the organism. Some of the more important of these facts and the conclusions drawn from them are briefly stated in the following paragraphs.

In all axiate forms (forms possessing one or more axes with reference to which an order is perceptible) which I have examined, including several species of ciliate infusoria, Hydra, and several hydroids, various flatworms, the eggs and developmental stages of several annelids, of starfish and sea urchin and of several fishes, and amphibia, there exists, at least during the earlier stages of development and in many cases throughout life, a gradient in rate of the metabolic processes or of certain fundamental metabolic reactions, which is coincident in direction with the longitudinal or chief axis of the body.[5, 7, 8, 9, 10] The region of highest rate in this gradient always becomes the apical region of the organism, i.e., the region which precedes in locomotion, or which is farthest removed from the region of attachment or insertion in the case of sessile forms, or is the chief region of growth or of reaction to external conditions according to the nature of the organism. Moreover, in those organisms in which a definite head appears, this head, or primarily the cephalic nerve ganglion which is its fundamental part and the first to appear in development, always arises from this region of highest metabolic rate.

This metabolic gradient can be distinguished in various ways; for example, the susceptibility of different regions of the body to various chemical agents such as the cyanides and at least many other narcotics in concentrations which kill in the course of a few hours, varies in general with metabolic rate.[6] Consequently when organisms and particularly the simpler forms and earlier stages of development, are killed by such substances a metabolic gradient appears as a death or susceptibility gradient, the regions of highest rate being most susceptible and dying first. Various other conditions such as lack of oxygen, high temperature, and in many cases low temperature also give the same death gradient and by means of the Tashiro biometer a gradient in CO_2-production in the same direction can also be demonstrated.

In much lower concentrations of the cyanides and narcotics which permit some degree of acclimation the susceptibility gradients of the organisms are reversed, because in these low concentrations the regions of high metabolic rate undergo more rapid and more complete acclimation to the reagent, while the regions of lower rate are less capable of acclimation and sooner or later die.

While the metabolic gradient along the longitudinal axis is usually the most clearly defined, similar gradients exist in relation to other axes or planes of symmetry. In at least most bilaterally symmetrical forms metabolic rate decreases from the median region laterally toward each side and in the invertebrates ventral regions have primarily a higher rate than dorsal, while in the earlier developmental stages of the vertebrates the metabolic rate decreases from the dorsal toward the ventral region. This difference between invertebrates and vertebrates is of interest in relation to the position of the postcephalic parts of the central nervous system, which in the former are usually ventral, in the latter dorsal.

When we examine the course of development in nature we find that it confirms these experimental results. In general, wherever we can distinguish an axis or a plane of symmetry we can distinguish a gradient or gradients in rate or sequence of development along the axis or in definite relation to the plane. This gradient corresponds in direction to the metabolic gradient and in all cases where a central nervous system appears, it is the first organ to become morphologically distinguishable, it arises from the region of highest metabolic rate in the primary gradients, and its cephalic part represents the region of highest rate in the whole organism.

There is also evidence that such metabolic gradients exist in plants, as well as in animals. Most plants are multiaxial forms, but as regards single axes, there are various indications of more or less definite metabolic gradients along these axes. We find, for instance, that the highest rate of growth occurs in the apical regions of each axis and that a more or less definite growth gradient extends from this region along the axis.

In general a relation of dominance and subordination exists between regions of higher and those of lower metabolic rate. If a region of high metabolic rate is once established in any way in an undifferentiated cell or cell mass a more or less definite gradient in rate extending to a greater or less distance from this region arises, because the changes in the primary region spread or are transmitted, but with a decrement in intensity or energy, so that at a greater or less distance from the point of origin they become inappreciable. It is evident that in general transmission of this sort must occur from a region of higher to one of lower rate of reaction, and S. Tashiro has found that this rule holds for the specialized form of transmission which is found in nerves. Thus the region of higher rate, once established, becomes an important factor in determining the rate of other regions, and since the rate thus determined is higher in regions nearer to it and lower in those farther away, a gradient results. Since the region of high rate of reaction determines the existence of the gradient, it becomes the chief factor in determining the rate of reaction in other regions within the range of its influence and therefore in determining the sequence of events in time and space in these parts. Consequently it appears to dominate or control other regions and they to be subordinate to it. It is evident that according to this conception the degree of dominance and subordination and the range of influence of the dominant part must depend primarily upon two factors. The first of these is the difference between the metabolic rate of the dominant part and the intrinsic rate of other parts before they were influenced by it. The higher the rate in the dominant region above the original intrinsic rate of the cell or cell mass the greater its effect in increasing the rate in other parts and the greater the distance to which its influence extends. The second factor is the efficiency of transmission. Where the transmitted changes influence other parts but slightly and soon die out, dominance is slight and limited to short distances from the dominant part, but where the decrement in the course of transmission is slight, as in the nerves of higher animals, the influence on the dominant part may make itself felt at very great distances.

I have been able to demonstrate experimentally in various ways this relation of dominance and subordination and to show that the axial gradient is an essential factor in determining the orderly formation of parts and their relations in space and time to each other. (For some of the experiments on *Planaria* see references 1, 3, 7. Various other data on coelenterates and flatworms are as yet unpublished.) For example, in the reconstitution of pieces of *Planaria* into new whole animals, the higher the metabolic rate in the developing head region, the farther away from the head do the pharynx and mouth appear, and vice versa, and other parts of the body show similar relations. Any piece of the planarian body is incapable of giving rise to any parts which are characteristic of levels of the body anterior to the level from which it was taken, unless a head first begins to develop. On the other hand, any piece is capable of producing parts characteristic of more posterior levels than its own, even in the complete absence of a head. These facts mean essentially that each level of the body is dominated by more anterior levels, but in their absence itself dominates more posterior levels and that the head region dominates all levels within a certain variable limit of distance.

It has also been shown by many authors that small isolated pieces of the body of various simple animals may undergo reconstitution into apical structures or heads without the presence or formation of any other parts of the body, but in no case has a posterior or basal structure arisen, except in connection with and as an outgrowth from more apical or anterior parts. In other words, the apical region or head of the organism is capable of developing independently and in the complete absence of other parts, while the formation of other regions of the body is dependent upon the presence of more apical or more anterior parts. This relation between apical or anterior regions and other parts is of fundamental importance for our conception of the organism, but, so far as I am aware, attention has not been called to its significance by those who have observed and recorded many of the facts.

In some of the lower animals it is even possible to eliminate experimentally the original gradient and then to produce a new gradient in the cell mass in a different direction. In such cases the original axis of the organism disappears and a new development takes place along an axis coincident with the newly established gradient, the region of highest rate in the gradient becoming the apical part of the new individual.

The dominance of the apical region, the growing tip, over other parts of the axis in the plants has long been known to botanists and it has been demonstrated repeatedly that this dominance can be decreased or eliminated and so the relations in space or time of other parts altered, by decreasing or inhibiting metabolic activity in the dominant region. Essentially the same relations undoubtedly exist along the axis in both plants and animals, but this very important fact has apparently not been recognized.

These and various other experimental data which cannot be mentioned here point us very definitely to the conclusion that the organic axis in its simplest form in both plants and animals is a gradient in rate of metabolism or of certain fundamental metabolic reactions, perhaps primarily the oxidations, and that such a gradient is at the same time the axiate organic individual in its simplest terms. In fact we may define the axiate individual as consisting primarily in a gradient or gradients in rate of metabolism or of certain metabolic reactions in a specific protoplasm. The organic individual is not then simply a hodgepodge of chemical substances: it is a physico-chemical complex with one or more reaction gradients. If this conception of the organic individual is correct, then we must admit further that physiological correlation is primarily a matter of the transmission of chemical changes rather than of the transportation of chemical substances. In other words, axiate organic individuation or integration is of the nervous type from the beginning, and the development of the nervous system is the morphological expression of physiological conditions which were present and began to act at the moment when the axis first arose and which in fact constitute the axis in its simplest terms. Undoubtedly specific chemical substances, hormones, products of metabolism, or whatever we prefer to call them, play very important parts in organic development, but the individual must already exist as an orderly whole before they can act in any definite and orderly manner.

We must now consider the question of the origin of the gradient, or more correctly of the region of high metabolic rate which determines the gradient. The only possible conclusion from many different lines of evidence is that it results from the differential action of factors external to the protoplasm, cell or cell mass concerned. We see gradients arising in nature in this way and it is possible to produce them experimentally. If this conclusion is correct, the so-called polarity and symmetry of organisms, which are in reality morphological and physiological expressions of these gradients, are not fundamental properties of protoplasm and do not result from polarities or symmetries of its constituent molecules or particles. They are therefore not comparable to the physical polarities and symmetries which are resultants of atomic or molecular constitution, for they represent molar differences in metabolic condition, which in the final analysis are of external origin. It is probable, however, that no cell or cell mass can continue to exist under ordinary natural conditions for any considerable length of time without acquiring at least a temporary gradient or gradients, for uniformity of action of external factors at all points of its surface is inconceivable, except perhaps for very short periods, and differences of action will result in a gradient or gradients. And it is certain that an orderly and definite sequence of events in time and space, such as occurs in organic development, is possible only where one or more of these metabolic gradients is present. Undoubtedly other factors besides the gradients play a part in determining the character of the results in many cases, but the orderly course of events is dependent upon the gradients.

The metabolic gradient cannot of course arise or persist simply as a gradient in rate of chemical reaction, for the metabolic processes and the colloid substratum, the protoplasm, in which they occur are always associated. The persistence of the gradient when once established, is due to the changes in the substratum connected with the differences in rate of reaction. For example, there is evidence that, at least in certain forms, a gradient in amount or activity of oxidizing enzymes exists along the axis.

According to this conception, the starting point of the process of differentiation is, in the final analysis, a quantitative gradient or gradients, i.e., a difference in rate rather than in kind of metabolic reaction. Is such a basis adequate? I believe it is. We know that in chemical reactions *in vitro* quantitative differences very often result in qualitatively different products. In the organism, where a great number of chemical reactions occur in a complex substratum which influences both their rate and character, the possibility of differences in quality arising from differences in quantity is much greater. The surface of the earth, with its physiographic features and its living forms, shows a remarkable differentiation along the equatorial-polar axis, and this differentiation results from the action of quantitative factors, viz., differences in light and heat. Moreover, it is possible to determine differences in the course and results of development which are manifestly qualitative by changes in external conditions which affect primarily the rate rather than the kind of reaction in the organism.

If the axiate individual is primarily a metabolic gradient or gradients, the lengths of these gradients represent the physiological limits of size of the individual. If the actual size exceeds this limit, either as the result of growth or of decrease in the length of the gradient, that portion which lies beyond the limit of size becomes isolated physiologically[2] This physiological limit of size is actually attained by the whole organism only in the plants and lower animals, where transmission is effective only over relatively short distances, because of the rapid decrement in intensity or energy. In the higher animals, where the chief conducting paths have become highly differentiated nerves, in which the decrement is very slight and transmission over very great distances is therefore possible, size is limited by other factors, such as the progressive differentiation of cells, which limit division and growth.

In the plants and lower animals the physiological isolation of a part usually results in reproduction of some sort, the character of the reproduction varying according to the conditions. Certainly the simpler and probably all agamic, or, as we commonly call them, asexual reproductive processes belong in this category, although additional factors are concerned in some. Reproduction under these conditions of physiological isolation means simply that the isolated part loses more or less completely its characteristics as a part, since the correlative conditions which determined them are no longer acting, and becomes a new individual, either with the persistence of the original metabolic gradients, or with the development of new ones through the action of external factors, both cases being represented in nature. We see these forms of reproduction very commonly in both plants and the lower animals. For example, in some of the worms, when the body attains a certain length, the posterior region becomes a new animal exactly as it does when we isolate it physically by cutting it off, and it can be shown that this reconstitution results from physiological isolation.[1, 4] In plants a new bud, which is a new plant individual, arises at a certain distance from the one previously formed, etc.

These reproductions have been variously controlled and modified in plants and I have found it possible to control them in animals. Growth is not necessary for their occurrence, for we can decrease the physiological limit of size of the individual, i.e., the length of the gradient, by decreasing the metabolic rate of the whole or of the apical region alone. One of the simplest ways of inducing such reproduction is to cut off the dominant region or decrease or inhibit its metabolic activity. This results in lowering the rate in all other parts and so in shortening the gradient, and this in turn results in physiological isolation of the more remote parts. Sometimes, when the gradient is nearly or quite eliminated, the individual breaks up into many new individuals, perhaps into single cells, each of which later repeats the process of development. Many cases of spore formation in the plants and lower animals are of this nature.

This conception of the organic individual throws light on many questions heretofore obscure. It not only accounts for the orderly behavior in space and time of the organism during development and for the agamic reproductions, but it enables us to understand how a multitude of axes or polarities, extending in all possible directions, can exist in different cells and organs at the same time with a general axis or axes of the whole organism.

This view also affords a basis for the interpretation and synthesis of the data of regeneration or form regulation which have stood heretofore as curious and remarkable facts, but without any adequate underlying general conceptions or working hypotheses. With this conception of the individual these data fall readily into line and many of them support and confirm it in a very definite way.

From this point of view the development of the nervous system as a conducting and correlating system and of its functionally dominant cephalic region are less completely mysteries than they have been, for we can see that they are the final results of conditions which existed at the first moment of individuation, and that the physiological integration of the organic individual is always fundamentally a relation of dominance and subordination. In the lower organisms and the earlier developmental stages of the higher, individuation approaches more or less closely to that type of social individuation which we find in the tribe or in certain forms of the primitive state, while in the later stages of development of the higher animals there are many indications of an approach toward democracy in the organism.

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A NOTE ON FUNCTIONS OF LINES

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A function of a line

$$\overset{b}{\underset{a}{F}}[y(x)] \tag{1}$$

may be regarded as a generalization of a function $F(y_1, y_2, \ldots, y_n)$ of a finite number of variables y_i $(i = 1, 2, \ldots, n)$. Instead of having a well defined value when a point $(y_1, y_2, \ldots, y_n)$ is given, the value of the function (1) is determined only when an infinitude of y-values belonging to an arc of the form

$$y = \varphi(x) \qquad (a \leqq x \leqq b) \tag{2}$$

is prescribed. The index i ranging over the integers $1, 2, \ldots, n$ in the function of a finite set of variables, is replaced in (1) by the index x ranging over the interval $a \leqq x \leqq b$. Examples of functions of this sort are the length of the arc (2), the time required by a heavy particle to fall from one end of the arc to the other, the area of the surface generated by revolving the arc about the x-axis, and many others.

For such functions Volterra has defined continuity and a derivative function.[1] The function (1) is said to be continuous at the arc (2) if for any given ϵ there always exists a δ such that

$$|\Delta F| = |F[\varphi(x) + \psi(x)] - F[\varphi(x)]| < \epsilon,$$

whenever $\psi(x)$ satisfies the conditions

$$|\psi(x)| < \delta \qquad (a \leqq x \leqq b).$$

Let $\psi(x)$ be further restricted not to change sign and to vanish identically except on an interval of length less than h containing a fixed value $x = \xi$. Then the derivative of F at the value ξ is defined by the equation

$$F'[\varphi(x), \xi] = \lim_{\substack{\delta = 0 \\ h = 0}} \frac{\Delta F}{\sigma},$$

where

$$\sigma = \int_a^b \psi(x)dx.$$

Further results of interest can be deduced[2] provided that the function F, the arc (2), and the value ξ, have associated with them a constant M such that

$$\left|\frac{\Delta F}{\delta h}\right| < M, \tag{3}$$

however $\delta > 0$, $h > 0$, and $\psi(x)$ are chosen, provided only that $\psi(x)$ is related to δ and h in the manner described above.

It is important that these considerations should apply to the integrals of the calculus of variations in terms of which the line functions cited above by way of illustration, with many others, are expressible. Such integrals in general are not continuous, do not possess derivatives, and do not satisfy the condition (3), in the forms specified by Volterra. It is the purpose of this note to prove this statement, and to call attention to the modifications of Volterra's definitions which apply also to the line functions of the calculus of variations.

Consider the simplest type of integrals of the calculus of variations.

$$F[y(x)] = \int_a^b f(x, y(x), y'(x))dx. \tag{4}$$

The length integral is a special case which is not continuous according to Volterra's definition. For in the figure $ac + cb$ is the length of each serrated line joining a with b and consisting of the slanting sides of the triangles with bases on ab and equal altitudes. There is one of these serrated paths in any neighborhood of the straight line ab. Hence the length integral is not continuous according to the definition given above.

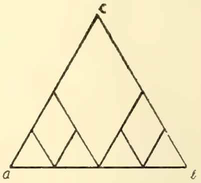

FIG. 1.

A function will be said to be of class $C^{(n)}$ if it is continuous and has continuous derivatives up to and including those of the n-th order. Consider then a curve (2) of class C'', and let the function f in the integral (4) also be of class C'' in a neighborhood R of the values (x,y,y') belonging to (2). Then the increment ΔF for the integral (4) is expressible in the form

$$\begin{aligned}\Delta F &= \int_a^b \{f(x, \varphi+\psi, \varphi'+\psi') - f(x, \varphi, \varphi')\}dx \\ &= \int_a^b \{A\psi + B\psi'\}dx,\end{aligned}$$

where[3]

$$A = \int_0^1 f_y(x, \varphi + \theta\psi, \varphi' + \theta\psi')d\theta,$$

$$B = \int_0^1 f_{y'}(x, \varphi + \theta\psi, \varphi' + \theta\psi')d\theta,$$

provided that ψ is continuous, and of class C'' except possibly at a finite number of values of x in the interval $a \leqq x \leqq b$, and provided also that the values $(x, \varphi + \psi, \varphi' + \psi')$ for $a \leqq x \leqq b$ are all in R. After the usual integration by parts of the calculus of variations, and an application of the mean value theorem for a definite integral, this becomes

$$\Delta F = \int_a^b \left(A - \frac{dB}{dx}\right) \psi \, dx = \left[A - \frac{dB}{d\,x}\right]_{x=x'} \int_a^b \psi(x)\, dx, \qquad (5)$$

where x' is a suitably selected value in the interval of length h or less including $x = \xi$, and on which ψ is not identically zero. Let ψ have the value

$$\psi = \sqrt{r^2 - (x - \xi)^2} - (r - \delta),$$

corresponding to the circular arc in Figure 2, on the interval

$$\xi - \frac{h}{2} \leqq x \leqq \xi + \frac{h}{2}$$

where it is not identically zero.

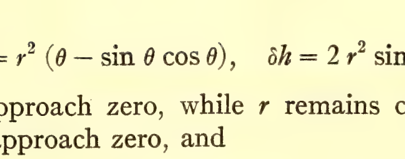

FIG. 2.

Then

$$\sigma = \int_a^b \psi \, dx = r^2 \,(\theta - \sin\theta\cos\theta), \quad \delta h = 2\, r^2 \sin\theta \,(1 - \cos\theta).$$

If δ is allowed to approach zero, while r remains constant, it follows that h and θ both approach zero, and

$$\lim_{\substack{\delta=0 \\ h=0}} \frac{\Delta F}{\sigma} = \left[f_y - \frac{d}{dx} f_{y'} + \tfrac{1}{2} f_{y'y'} \frac{1}{r}\right]_{x=\xi}, \qquad (6)$$

$$\lim_{\substack{\delta=0 \\ h=0}} \frac{\Delta F}{\delta h} = \frac{4}{3}\left[f_y - \frac{d}{dx} f_{y'} + \tfrac{1}{2} f_{y'y'} \frac{1}{r}\right]_{x=\xi}, \qquad (7)$$

where the arguments in f and its derivatives are x, φ, φ'. It is clear from these expressions that at a value ξ defining a point on the curve (2) where the derivative $f_{y'y'}$ is different from zero, the limits (6) and (7) may take any arbitrarily assigned values, one value only being excepted in each case, provided that r is properly chosen. The function (4) has therefore no derivative in the sense of Volterra at the value $x = \xi$, and does not satisfy the condition (3).

For any more general variation ψ satisfying the restrictions specified in the first part of the last paragraph, it is clear from (5) that

$$\frac{\Delta F}{\sigma} = \left[\int_0^1 \left(f_y - \frac{d}{dx} f_{y'}\right) d\theta - \psi' \int^1 f_{y'y}\, \theta\, d\theta - \psi'' \int_0^1 f_{y'y'}\, \theta\, d\theta\right]_{x=x'}, \tag{8}$$

where the arguments of f and its derivatives are x, $\varphi + \theta\psi$, $\varphi' + \theta\psi'$. Hence if the conditions

$$|\psi(x)| < \delta, \quad |\psi'(x)| < \delta, \quad |\psi''(x)| < \delta \qquad (a \leqq x \leqq b), \tag{9}$$

as well as those described above are satisfied, the derivative limit will exist and have the value

$$\lim_{\substack{\delta=0 \\ h=0}} \frac{\Delta F}{\sigma} = \left[f_y - \frac{d}{dx} f_{y'}\right]_{x=\xi},$$

the arguments of the derivatives of f being x, φ, φ'. Let N be the maximum of the absolute values of $f_y - df_{y'}/dx$, $f_{y'y}$, $f_{y'y'}$, in the neighborhood R. Then from (8)

$$\left|\frac{\Delta F}{\delta h}\right| = \left|\frac{\Delta F}{\sigma}\right| \left|\frac{\sigma}{\delta h}\right| \leqq \left|\frac{\Delta F}{\sigma}\right| \leqq N + N\frac{\delta}{2} + N\frac{\delta}{2},$$

and it is evident that the quotient $\Delta F/\delta h$ is bounded for all values of $\delta > 0$, $h > 0$, and ψ such that the inequalities (9) hold and the values $(x, \varphi + \psi, \varphi' + \psi')$ for $a \leqq x \leqq b$ are in R.

By similar arguments it will be clear what properties are possessed by an integral of the form

$$F[y(x)] = \int_a^b f(x, y, y', \ldots, y^{(n)})\, dx.$$

Let the curve (2) be defined by a function φ of class $C^{(2n)}$. In a neighborhood R of the values $(x, y, y', \ldots, y^{(n)})$ belonging to the curve, the function f is supposed to be of class $C^{(n+1)}$. Then[4] the function F has continuity of order n. In other words, for a given ϵ there always exists a δ such that

$$|\Delta F| = |F[\varphi(x) + \psi(x)] - F[\varphi(x)]| < \epsilon$$

whenever ψ is of class $C^{(n)}$, or continuous and of class $C^{(n)}$ except possibly at a finite number of x-values, and

$$|\psi(x)| < \delta, \ |\psi'(x)| < \delta, \ \ldots\ldots, |\psi^{(n)}(x)| < \delta \qquad (a \leqq x \leqq b). \qquad (10)$$

Further F has a derivative at any value $x = \xi$ which is approached with order $2n$; that is, if ψ does not change sign and vanishes except on an interval of length less than h including $x = \xi$, and if furthermore

$$|\psi(x)| < \delta, \ |\psi'(x)| < \delta, \ \ldots\ldots, |\psi^{(2n)}(x)| < \delta, \qquad (11)$$

then the limit

$$F'[\varphi(x), \xi] = \lim_{\substack{\delta=0 \\ h=0}} \frac{\Delta F}{\sigma}$$

exists. Further the absolute value of the quotient $\Delta F/\delta h$ will be bounded for all choices of $\delta > 0$, $h > 0$, $\psi(x)$ satisfying the relations (11) and such that the values $(x, y, y', \ldots, y^{(n)})$ on the arc $y = \varphi + \psi, a \leqq x \leqq b$, are all in the neighborhood R.

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[1] Volterra, *Leçons sur les equations intégrales*, ch. 1, art. 5: or his *Leçons sur les fonctions des lignes*, ch. 1, art. 2.

[2] Volterra, arts. VII and 2, 3, respectively, of the chapters referred to above.

[3] See Jordan, *Cours d'Analyse*, vol. 1, p. 247.

[4] See Fischer, A generalization of Volterra's derivative of a function of a curve, *Amer. J. Math.*, 35, 385 (1913).

A CLASSIFICATION OF QUADRATIC VECTOR FUNCTIONS

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There is probably no chapter of mathematics more worthy of attention, or more neglected at present, than the theory of vector functions. In the case of the linear vector function, it is true, a good deal has been found out in one way or another, and this by some of the very greatest of mathematicians. First investigated in detail by Hamilton[1] and again appearing as Cayley's matrix of the third order,[2] the linear vector function is essentially the same as the Grassmann open product[3] and the Gibbs dyadic.[4] In Germany the nonion or three-square matrix bears the name Tensor,[5] a word used by others in a different sense. On the other hand, we may make a clean sweep of all these operational concepts and, if it pleases us, define a vector function as a set of three algebraic forms X,Y,Z, homogeneous polynomials in three variables x,y,z, with nothing left of the original idea of a vector as a directed quantity except a definite order in writing the three forms X,Y,Z. The occurrence of the same mathematical entity under such a variety of names and algorisms is perhaps the natural consequence of its fundamental character.

From whatever point of view we prefer to start, it is well known in the linear case that a convenient classification of types may be made with reference to the axes of the function; an axis of a linear vector function ϕ of a vector ρ being a direction of ρ such that $\phi\rho$ and ρ are parallel, or $\phi\rho = g\rho$, where g is a mere number. In the language of algebraic forms this is the same as saying that an axis is a point, in homogeneous coördinates, satisfying the equations

$$yZ - zY = 0,\ zX - xZ = 0,\ xY - yX = 0. \tag{1}$$

In the longer work of which the present paper is an outline, a similar basis is taken for a classification of types of *quadratic* vector functions $F\rho$ of the vector ρ. Related mathematical problems which, by reason of their close kinship, suggest the study of vectors of higher degree are numerous. For example, if x,y,z, and X,Y,Z, denote points respectively in a first plane and in a transformed plane, the vector $F\rho$ obviously enough defines a geometric point-transformation. The worker who limits himself, however, to such an interpretation in homogeneous coördinates will lose sight of the conveniences of vector addition. We may with equal ease let $F\rho$ define a transformation in space of three dimensions with the origin invariant.

As another application, the properties of $F\rho$, by reason of their invariant character with reference to change of coördinate axes, are intimately connected with the whole theory of a set of three algebraic forms. That the study of the linear vector function led to the discovery of various invariants belonging to one function or to a system of several such functions, is well known.[6]

Again, the student of certain types of differential equation will find that the notion of a vector function comes readily into his work. The very appearance of equations like

$$\frac{dx}{X} = \frac{dy}{Y} = \frac{dz}{Z}, \tag{2}$$

where X, Y, and Z are algebraic forms as already explained; or like

$$(yZ - zY)dx + (zX - xZ)dy + (xY - yX)dz = 0; \tag{3}$$

suggests translation into some sort of vectorial language. With special reference to the quadratic case, there exist several long and interesting, although not very recent, French monographs, notably one by Darboux,[7] in which he takes advantage of the close relation of equations (2) and (3) with each other and with the equations, in non-homogeneous forms,

$$\frac{dy}{dx} = \frac{Y}{X}, \; z = 1, \; Z = 0. \tag{4}$$

It follows that a vector function offers a ready tool for inquiring into the nature of the functions defined by any equation of the type $dy/dx = R$, where R is a rational function of x and y. Darboux does not get much beyond an examination of a great variety of cases where (3) possesses one or more algebraic integrals, or else can be brought to depend on a Riccati equation. He does show very clearly the wide range of even this problem, indicating the very general character of the function which would satisfy (4) when X and Y are quadratics set down at random. For instance, the most general hypergeometric function satisfies a second order equation which is a resolvent for a very special case of (4) in the Riccati form. Darboux uses no vector algebra as such, but he brings out and uses a fact which, translated into vector language, is as follows: The addition to a vector $F\rho$ of another vector of the form ρt, where t is a scalar variable, does not alter the axes of $F\rho$. This is geometrically evident. Analytically expressed it means that if $X,Y,Z,$ satisfy (1) when a certain set of values of x,y,z is given the equations will still be satisfied by

$$X + tx, \; Y + ty, \; Z + tz,$$

written instead of X,Y,Z. This can be verified directly. In fact the variable t disappears automatically from (3). Roughly speaking, the connection of ideas consists in this, that if (3) has been completely solved (which requires a certain number of particular solutions), then both (2) and (4) can be solved by quadratures. In the quadratic case, the scalar t takes the form $S\delta\rho$, that is, it depends upon a single constant vector δ. Now if, for a value of δ, we can find a solution of (4), or (what is much the same here), of the partial differential equation

$$X\frac{\partial u}{\partial x} + Y\frac{\partial u}{\partial y} + Z\frac{\partial u}{\partial z} = 0, \tag{5}$$

this solution will be a particular solution of (3). Stated another way, all the different functions defined by (5) when all possible values are given to the vector δ can be found by quadratures when (3) has been solved completely; these functions constitute a family or set possessing some group properties,—just how far they form a group has not been investigated, so far as I am aware.

The axes of the vector function correspond to the singular points of (3), if we interpret in homogeneous coördinates. It is well known that these are $n^2 + n + 1$ in number,[8] when n is the degree of the forms X,Y,Z. If $n = 2$ we thus have seven axes, in general.

The necessity for careful examination and classification of types of quadratic vector functions appears from the fact that many differential equations like (4) do not yield vectors of the most general kind, having all seven axes distinct, but possess multiple or coincident axes of all orders up to seven. To take a simple example, if $X = xy$, $Y = yz$ and $Z = zx$, the vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are all double axes, and $\mathbf{i} + \mathbf{j} + \mathbf{k}$ a single axis, that is, in homogeneous coördinates, the points (1,0,0,), (0,1,0,), and (0,0,1), are higher singularities of (3), and (1,1,1,) is an ordinary singularity. By a proper choice of t, that is of δ, we can add a term $\rho S\delta\rho$ which shall make $Z = 0$, and have the equation (4) as

$$\frac{dy}{dx} = \frac{y(1-x)}{x(y-x)};$$

the value of t being $-x$, and that of δ being $\mathbf{i}$. The variables x,y,z themselves correspond to particular solutions of (3). Four particular solutions are needed, however, to complete the solution by quadratures; hence the rest of the functions of the family cannot be found by mere quadratures.

Again, a quadratic vector may have more than seven axes, but if so it has an infinite number, and equation (3) may be divided through by a scalar variable. Take for example one of the simplest types furnished by the technique of vector algebra, viz., $V\rho V\alpha\rho$, or in Gibbs' notation $\rho \times (\alpha \times \rho)$. This vector may be expanded as

$$\rho S\alpha\rho - \alpha\rho^2,$$

which differs from the vector $\alpha(x^2 + y^2 + z^2)$ only by the term in ρ, having no effect on the axes. Hence any element of the minimal cone $\rho^2 = 0$ is an axis, and α is the only other axis.

The most general quadratic vector function may be very elegantly defined by a sum of triads, that is, a triadic. A single triad $\alpha\beta\gamma$ multiplied (dot product) by and into ρ is the same as the Hamiltonian vector $\beta S\alpha\rho S\gamma\rho$. Evidently β is an axis. Also, any vector at right angles either to α or to γ is an axis. As a less special example, the vector $Vq\rho s\rho t$, where q,s,t are constant quaternions, has important geometrical applications; Hamilton showed that its properties include those of the most general cubic cone.[9] This vector cannot be so simply expressed in any other algorism. It has, in general, all its axes distinct; two of them are on the minimal cone $\rho^2 = 0$, and are easily found. The others are determined by an equation of the fifth degree.

In developing a classification of various types, I have made comparatively slight use of the technical processes of vector algebra, and have based my subclasses on configurations of the axes rather than on the possibility of simple algebraic expression. It appears that, *in the most general type, a normal form of vector is easily obtained in terms of the axes themselves.* If there are double axes, but no higher axis, there is still no particular difficulty, although the normal forms are less simple. It is shown that there is only one kind of triple axis; obviously, a quadratic vector can have at most two of these. A normal form of vector with two triple axes is developed, and has a number of properties in the way of symmetry. An axis of the fourth order, on the other hand, may be of two kinds. Quadratic vectors with an axis of higher order than the third fall naturally into two families, according as the axis is of the first, or of the second kind. An axis of the first kind is shown to correspond to a double point common to all three of the cubic curves defined by equations (1), if we interpret in homogeneous coördinates. The second kind is shown to depend on a partial differential equation satisfied by the vector $F\rho$. This differential condition depends in part on the results of my former papers, where the properties of a differential vector have been developed.[10]

Tests for the existence of axes of any order up to, and including the fourth have been given for vectors of any degree whatever. In the quadratic case, normal forms are given including all possible types.

The existence of over one hundred special types makes it very desirable to have, on the formal side, the means of covering in one comparatively simple algebraic expression as many of these types as possible,—and in such a way that their properties are easily correlated. The largest number of advantages for this purpose appears to be possessed by the form $V\phi\rho\theta\rho$, where ϕ and θ denote linear vector functions. In Gibbs' notation, this is the same as determining our vector function by the cross product of two dyadics. Besides compactness of expression, this vector product offers the following advantages:

1. It is easily interpreted as the most general birational quadratic point transformation in a plane.

2. It differs from a quadratic vector of the most general type only by a term in ρ, which, as already shown, does not alter the axes.

3. The properties of a quadratic vector are made to depend on those of linear vectors.

4. For the study of differential equations it is especially suited since it is a vector product. Equation (3) takes the factored form $S\phi\rho\theta\rho V\rho d\rho = 0$, or in Gibbs' notation $(\phi\rho \times \theta\rho).(\rho \times d\rho) = 0$.

5. Three of its axes are zeros, that is, for three values of ρ the vector vanishes in all its components.

It appears, therefore, as a problem of importance to determine how far the various possible configurations of axes, in special types of quadratic vectors, are included among possible configurations of the axes of $V\phi\rho\theta\rho$. This is the same as the problem of determining with what exceptions a quadratic vector of whatever type or sub-type can be written in the form

$$V\phi\rho\theta\rho + \rho S\delta\rho.$$

At this point of the investigation a certain difficulty presented itself. It is easy so to chose δ that the resulting quadratic vector shall have three zeros, distinct or multiple; if, then, the vector does not fall into a uniplanar, i.e., a binomial form, it is possible to factor vectorially into $V\phi\rho\theta\rho$. But a binomial quadratic vector cannot be so factored, hence the necessity of examining a very large number of choices of δ to find those which do not yield a binomial. For most types where such a vector δ can be found, I have contented myself with giving the value of the resulting δ, since its accuracy, when found, is easy to verify. In the cases where no value of δ can be found, I have, of course, demonstrated the impossibility.

The final result is, on the whole, highly satisfactory. It appears that *the form* $V\phi\rho\theta\rho + \rho S\delta\rho$ *includes all types of quadratic vectors except two simple sub-types both belonging to the family having a higher axis of the second kind.* Normal forms for these two very exceptional types have been given.

The existence of these exceptions is due to the fact that, as the form of the vector grows more and more restricted, the possible choices of δ, which avoid the binomial, decrease in number. Thus in general there are thirty-five possible values of δ; but if the determinant of the components of a set of three axes vanishes, the number falls to thirty-one; and the occurrence of multiple axes also reduces the number. The wonder appears to be, not that there are exceptions, but that there are so few, and these so simple.

Various properties of quadratic vector functions appear by virtue of the normal forms which characterize their types. Some of these bring out properties of the differential equations (3) and (4). For example, it is very easy to determine, when the type of the vector is known, whether these equations can be solved by quadratures. Again, it appears that the most general equation (4) never corresponds to the vector function of the most general type, but to a slightly restricted type. A consideration of the details of these normal forms would carry the discussion beyond the limits of the present paper.

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[1] W. R. Hamilton, *Lectures on Quaternions*, p. 480.

[2] Arthur Cayley, *London, Phil. Trans. R. Soc.*, **48**, 17 (1858).

[3] Hermann Grassmann, *Gesammelte Werke* (Die Lineale Ausdehnungslehre, 1844, § 172).

[4] Gibbs-Wilson, *Vector Analysis* (1902). A dyadic is a sum of dyad terms, where each dyad is a product of two vectors. When multiplied into the vector ρ by dot, i.e., scalar, multiplication, a single dyad is equivalent to a Hamiltonian term of the form $\beta S\alpha\rho$.

[5] W. Voigt, *Die fundamentalen Eigenschaften der Krystalle* (1898).

[6] Hamilton, *loc. cit.*, Lecture VII. A Hamiltonian invariant is a function of three vectors which is unaltered when these vectors are varied.

[7] Darboux, Mémoire sur les Equations Différentielles Algébriques, *Bul. Sc. Math., Paris,* **13**, 83 (1878).

[8] Clebsch, *Leçons sur la Geometrie* (Tr. Benoist), t. 2, p. 113; *Vorlesungen über Geometrie*, Bd. 1, 390, 1001.

[9] Hamilton, *Elements of Quaternions*, Art. 415.

[10] F. L. Hitchcock, *Phil. Mag.*, Ser. 6, **3**, 576 (1902); Ibid., **4**, 187 (1903).

ON THE RADIAL VELOCITIES OF FIVE NEBULAE IN THE MAGELLANIC CLOUDS

By Ralph E. Wilson
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Presented to the Academy, February 11, 1915

In the course of observations on the velocities of approach and recession (radial velocities) of southern nebulae whose spectra contain bright lines, made with spectrographs attached to the 37-inch reflecting telescope of the D. O. Mills Expedition from the Lick Observatory, at Cerro San Cristobal, Santiago, Chile, we have found that the following five nebulae have high radial velocities. The results depend upon the observed positions of the Hβ hydrogen line and the nebulium lines at 5007A and 4959A, in each case. The velocities are not corrected for the solar motion.

Nebula	*Right ascension*	*Declination*	*Radial velocity km/sec.*	*Number of plates*
N. G. C. 1644	1h 6.2m	−73° 44′	+158	3
N. G. C. 1714	4 52.0	−67 06	+301	2
N. G. C. 2111	5 52.6	−69 33	+268	2
N. G. C. 1743	4 54.6	−69 21	+254	1
N. G. C. 2070	5 39.4	−69 09	+276	1

It is of special interest that all of these nebulae lie in the Magellanic Clouds, and that they are the only nebulae thus far observed by us in the Clouds. The first object, N. G. C. 1644, is in the Smaller Cloud, and the other four are in the Greater Cloud.

It is difficult to doubt that these nebulae are actually within the structure of the two Clouds, respectively: N. G. C. 1644, seen upon the Smaller Magellanic Cloud as a background, is the only known bright-line nebula in that region of the sky;[1] and the others are four of the nineteen known bright-line nebulae closely clustered upon the background of the Greater Magellanic Cloud, which are almost equally isolated from other nebulae of their kind.

Again, the radial velocities of the four nebulae observed in the Greater Cloud lie between 250 and 300 km. per second, recession. We should not expect the substantial equality of such high velocities unless the four objects bear a close relationship to each other or to the structure of the Greater Magellanic Cloud as a whole. It seems desirable that an effort should be made to measure the radial velocities of as many of the faint stars in the Magellanic Clouds as time and means will permit, in order to determine whether the average velocities of the stars in the Clouds approximate the velocities of the nebulae existing in the Clouds; that is, to determine whether the Magellanic Clouds possess high velocities of recession with reference to our general stellar system.

One is also inclined to inquire whether a more or less intimate resemblance may exist between the characters of the Magellanic Clouds and of the spiral nebulae, inasmuch as the spirals have been observed by Slipher to possess abnormally high radial velocities.

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[1] For list of nebulae known to have bright-line spectra, see *Annals of Harvard College Obs.*, 76, 21 (1914).

NOTICES OF SCIENTIFIC MEMOIRS

Monograph of the Bombycine Moths of North America, including their Transformations and Origin of the Larval Markings and Armature. Part III. *Families ceratocampidae (exclusive of ceratocampinae), saturniidae, hemleucidae, and brahmaeidae.* By ALPHEUS SPRING PACKARD, edited by THEODORE D. A. COCKERELL. First Memoir of Volume 12 of the Memoirs of the National Academy of Sciences. Washington, 1914. 1–502 p. 34 fig. 113 pl.

The third part of the late Dr. A. S. Packard's *Monograph of the Bombycine Moths of North America*, published on December 31, 1914, by the National Academy of Sciences, and containing 16 pages including 34 figures and 113 plates, many of which are colored, covers the Saturnioid moths, including many species of large size, important as producers of silks, and others injurious to cultivated plants. This does not complete the work planned by Dr. Packard, but includes all the material which was sufficiently elaborated at the time of his death to be available for publication. As the work developed under Dr. Packard's hands, it became increasingly apparent to him that for a broad and philosophical view of the subject it was necessary to take into account the exotic genera and species, and thus the later work is of larger scope than the original title (*Bombycine Moths of North America*) would suggest. The work on the Saturniidae was, in fact, planned on the lines of a monograph of the species of the entire work, with special reference to the transformations, including elaborate new descriptions of all the larvae which could be obtained from any source. The editor found it impracticable to bring the work up to date, to include all known species, and sought only to include some account of lately discovered genera, and to elaborate somewhat more fully the part dealing with North American species. To do more than this would be to unduly increase the size of the volume without corresponding advantages, especially since the added matter would represent merely the republication of descriptions which have already been printed elsewhere. To do less would inconvenience the user of the book, who would look in it for up-to-date information, so far as the plan of the work might lead him to expect. The final result is necessarily imperfect and suffers greatly from the lack of the development and revision the work would have received at the hands of the illustrious author, had he lived; but on the other hand it represents a large positive contribution to entomology, especially on account of its numerous new and detailed descriptions, and the beautiful new illustrations of larvae.

Dr. Packard left comparatively few illustrations of the adult insects, and the absence of pictures of most of the genera discussed was felt to be a serious drawback. Fortunately this difficulty was overcome through the generosity of Mr. J. H. Watson of England, and of the U. S. National Museum, through Dr. H. G. Dyar, and so the published work contains figures, made from photographs, of a very large number of species, many of which are represented by the actual types, while many others are here figured for the first time.

The work here concluded is a continuation of previous investigations published as Volume 7, First Memoir, and Volume 9, Second Memoir, of the Memoirs of the National Academy of Sciences.

The Turquois: A Study of its History, Mineralogy, Geology, Ethnology, Archaeology, Mythology, Folk-lore, and Technology. By Joseph E. Pogue. Third Memoir of Volume 12 of the Memoirs of the National Academy of Sciences. Washington, 1915.

This treatise contains 206 pages and is illustrated by 20 half-tone plates, 2 colored plates, and 6 text-figures. It is concerned with the treatment of a single precious stone in all its aspects and, though scientific in scope and technical in detail, aims to have a popular appeal and a general interest as well. The first chapter deals with turquois from the historical standpoint, tracing its position in literature from remote times to the present. Special emphasis is laid upon the mediaeval ideas that obtained regarding this precious stone, and the review affords an epitome of the development of early mineralogic science. Chapter two describes in detail the physical properties of turquois and discusses its chemical composition. Chapter three is largely geological and is descriptive of the turquois localities of the world. Special attention is given to the early history of the interesting deposits in the Sinai Peninsula, which were extensively exploited by the Egyptians during the Dynastic Period; and the famous mines near Nishapur in Persia are brought before the reader in the detail their significance deserves. The importance of scarcely known deposits in Central Asia, which have for centuries contributed a notable supply of stones to the Orient, is also pointed out. Chapter four goes into a careful discussion of the origin of turquois. In chapter five the use of turquois from a world point of view is taken up and traced from antiquity to the present. The human element is here uppermost and interesting analogies appear in parallel applications by independent peoples. Its employment by the American aborigines leads to especially full treatment of a phase of the subject of considerable ethnologic significance. The position held by turquois in Tibet and China is discussed in the light of recent investigations by Berthold Laufer. In chapter six the identity of *chalchihuitl*, a species of precious stone used by the Aztecs, is investigated and its bearing on turquois discussed. The seventh chapter depicts the position of turquois in the mythology and folklore of the various peoples that have chanced to use this mineral. The superstitions in which the turquois is concerned are many and varied and afford an interesting parallel to its ornamental application. Chapter eight is given up to the more technical aspects of the subject, wherein the mining, cutting, means of imitation, and production are considered. The final section is an annotated bibliography, comprising upwards of 400 titles, which forms a reasonably complete record of the works that touch on turquois.

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PHOTOGRAPHIC DETERMINATION OF STELLAR PARALLAXES WITH THE 60-INCH REFLECTOR

By Adriaan van Maanen

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Presented to the Academy, March 4, 1915

The determination of stellar parallaxes had been attempted from the days of Tycho Brahe down, but without success until Bessel in 1838 succeeded in demonstrating and measuring the parallax of 61 Cygni. Since that time the work has been advancing steadily, first with the use of the meridian-circle, heliometer, and micrometer, until in 1886 Pritchard applied photography with great success. Although several astronomers have given much time to the problem, results are still comparatively few. In 1910 Kapteyn and Weersma published a list of well-determined parallaxes, which, although nearly complete, contains only 365 stars. The accuracy of these parallaxes varies considerably; their probable errors are anywhere between 0″.004 and 0″.151, while the mean probable error is 0″.032. Since then a few lists of parallax determinations have been published, which show a good improvement; the list of Slocum and Mitchell (14 stars) has a mean probable error of 0″.011; that of Miller (8 stars) of 0″.011.

The material published so far cannot, however, help us very much in forming an idea of the distribution of the stars in space, as it is very one-sided. Most of the stars were chosen on account of their brightness or their large proper motions; but for the distribution of stars in space, it is clear, that we need as well the parallaxes of stars which are not supposed to be our nearest neighbors. But here arises a new difficulty; the quantities sought become so small, that only the greatest accuracy in their determination can give useful results. According to Kapteyn the mean parallax of a star of, for instance, the sixth magnitude and a proper motion of 0″.100 annually is only 0″.018. It is clear that in such cases the probable error must not exceed a few thousandths of a second of arc if the results are to be valuable.

In *Contributions from the Mount Wilson Solar Observatory*, No. 79, Adams and Kohlschütter have given the absolute motions and absolute magnitudes of 100 stars with known parallaxes, for which they have determined the radial velocities. Nearly all these stars have large proper motion and are between the spectral types F and M. In the hope of completing this list by the addition of stars of the same spectral type, but of smaller proper motion, an investigation has been made to determine the accuracy with which parallaxes can be obtained with the 60-inch reflector. Although in general the method employed was that used by Schlesinger and Slocum with the 40-inch Yerkes refractor, the following deviations from their procedure may be mentioned:

The equivalent focal-length of the Cassegrain combination of mirrors employed is 80 feet, thus increasing the scale of the plates by one-quarter.

The exposure time of 15 minutes gives stars as faint as magnitude 13; by avoiding the very faintest objects visible on the plates, we can still use stars between magnitude 9.5 and 12 for comparison purposes. The fainter stars must as a rule have smaller parallaxes, so that we have here the double advantage of using comparison stars with smaller parallaxes and with a smaller mean distance from the central star.

The plates used are Seed 23, which have a fine grain; the star images therefore are sharper and the grain is less troublesome in the measuring.

The plates are measured with the blink-arrangement of the stereocomparator; two plates are compared directly without the use of any scale such as in ordinary measuring instruments. That the stereocomparator can, with some precautions, be used successfully for this kind of work was proved in a previous article (*Astronomical Journal*, **27**, 140; 1912).

Special care has been taken to make the hour angles of the two plates to be compared nearly the same. The difference in hour angle has in no case been larger than 5°, while in most cases it was less than 2°.

In comparing two plates directly we avoid in the final solution for the parallax the use of a third unknown quantity; this increases the weight of the resulting parallax.

Finally, care has been taken to make the weight of the parallaxes resulting from a certain number of plates as large as possible by distributing the exposures as symmetrically as practicable within the year.

There is, however, a difficulty sometimes affecting the use of the 60-inch reflector for this work. After a rapid change in temperature during the time when the dome is open, the mirror occasionally shows astigmatism. Under such conditions the images are more or less asymmetrical and the measures may not be as accurate as usual. This happened with three of the exposures on the last star given in the table below.

The result of the investigation has been rather promising; 5 stars have been finished, the results of which are given in the table.

Name of Star		π	*P.E.*	*Number of Exposures*
Boss P. G. C.	96	+0″.026	0″.007	14
	672	−0 .009	0 .004^5	14
	1549	+0 .001	0 .001^5	16
	2921	+0 .078	0 .006	10
	3233	+0 .003	0 .010	12

For the mean of 13 exposures the mean probable error of a parallax is not quite 0″.006. If we compare this result with the best known photographic determinations of parallaxes listed below, it will be seen that we have gained considerably.

Instrument	*Observer*	*Mean P.E.*	*Number of Exposures*
Helsingfors (11-inch)[1]	Donner Kapteyn de Sitter	0″.023	12
Bonn (11-inch)[2]	Küstner Kapteyn de Sitter	0″.020	12
Cambridge, England (12.5-inch)[3]	Russell	0″.028	27
Yerkes (40-inch)[4]	Schlesinger	0″.013	37
Yerkes (40-inch)[5]	Slocum Mitchell	0″.011	21
Yerkes (40-inch)[6]	Slocum Mitchell	0″.009	28
Swarthmore (24-inch)[7]	Miller	0″.011	Not published

[1] Groningen, *Pub. Astr Lab.*, 20, 28; 1908.

[2] *Ibid.*, 23, 56; 1909.

[3] *Pub. Carnegie Inst Wash.*, 147, 65; 1911.

[4] *Astrophys. J.*, 34, 27; 1911.

[5] *Astrophys. J.* 38, 25; 1913.

[6] *Pub. Astr. Astrophys Soc. Amer.*, 16th Meeting, 19; 1914.

[7] *Ibid.*, 17th Meeting, 1914.

The material is of course insufficient to yield any information as to possible systematic errors, but we have one evidence that these can not be very large; the mean parallax is +0″.020, while according to Kapteyn's table for stars of this type, magnitude, and proper motion it is +0″.014.

ON CHONDROSAMINE

By P. A. Levene and F. B. La Forge

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Presented to the Academy, February 8, 1915

Theoretically there are possible as many α-amino hexoses as there exist hexoses, namely, eight in the *d*-series, and as many in the *l* and *dl*-series. Of the eight possible amino-sugars only one has been discovered in nature, namely, glucosamine; this also was prepared synthetically by Fischer and Leuchs,[1] and to it the configuration of glucose was ascribed. More recent investigations have shown that two explanations for the configuration of the sugar are possible: one of glucosamine and the other of mannosamine, and that at present it is not possible to give preference to either one of the two.[2]

In the course of our investigations on the structure of chondroitin sulphuric acid we isolated an α-amino hexose isomeric with the one previously described. It was named chondrosamine.[3] On the basis of further investigation the conclusion was reached that chondrosamine has the configuration of either α-altrosamine or of α-allosamine. The grounds for the conclusion are the following:

1. On oxidation with bromine the amino-hexose was transformed into a tetroxyaminocaproic acid having $[\alpha]_D^{28.5} = -29.23$ (glucosaminic acid has $[\alpha]_D = -14.65°$). On reduction with hydroiodic acid a hydroxyaminocaproic acid was obtained. The properties of this were identical with the acid obtained on reduction of glucosaminic acid. By this was demonstrated that chondrosamine contained a normal carbon chain.

2. On oxidation with nitric acid of the deaminized chondrosaminic acid an anhydrodicarboxylic acid (epichondrosic acid) was obtained. M.P. = 201–202°C., optically inactive. C = 37.58%; H = 4.42% (theory, C = 37.50; H = 4.20).

3. On oxidation with nitric acid directly of the deaminized amino-sugar an acid (chondrosic) was obtained, identical in composition: C = 37.11%; H = 4.15%; but different in properties: M. P. = 179–181°; $[\alpha]_D^{28} = -16.56°$.

4. On heating in a sealed tube at 150° with equal parts of hydrochloric and hydrobromic acid, chondrosic acid was converted into dehydromucic acid: C = 45.84%; H = 3.08% (theory, C = 46.16, H = 2.57).

5. On heating at 200°C., chondrosic acid was converted into pyromucic acid. C = 53.56%; H = 3.57 (theory, C = 53.37, H = 4.14). M.P. = 135°C. Isosaccharic acid is converted into dehydromucic

and into pyromucic acids under the same conditions as chondrosic acid, hence it was concluded that the two acids were similar in their structure, namely, that in both an oxygen bridge existed between the α- and α_1-carbon atoms.[4]

6. The anhydrodicarboxylic acids (derived from hexoseamines) have as many asymmetric carbon atoms in their molecule as the corresponding acids derived from hexoses. Therefore, there are possible only two optically inactive anhydrodicarboxylic acids: namely, one corresponding to mucic and the other to allomucic acid. This consideration limits the possibilities of configuration of epichondrosic acid to anhydromucic, or anhydroallomucic. The fact that chondrosamine forms a phenyl osazone identical in its properties with that of allose decides the choice between the two configurations in favor of anhydroallomucic.

7. Regarding the configuration of chondrosine the choice remains between that of *l*-allosamine or of *l*-altrosamine. Both anhydroallomucic and anhydrotalomucic acids are obtainable from chondrosamine depending on the procedure in preparation.

It was attempted to prepare chondrosaminic acid synthetically from ribosimine by the action of prussic acid. The acid obtained in this manner had the composition of hexosaminic acid, C = 37.02%, H = 6.58%, and N = 7.44% (theory, C = 36.92, H = 6.66, N = 7.18). M. P. = 198°C., $[\alpha]_D^{28} = -9.4°$. The acid was evidently epimeric with chondrosaminic, and on treatment with nitric acid it should have yielded chondrosic acid. Unfortunately for lack of material this experiment had to be deferred.

[1] Fischer, E., and Leuchs, H., *Berlin, Ber. D. chem. Ges.*, **36**, 24 (1903).

[2] Irvine, J. C., and Hynd, A., *London, J. Chem. Soc.*, **101**, 1128 (1912); **105**, 698 (1914).

[3] Levene, P. A., and La Forge, F. B., *J. Biol. Chem.*, **18**, 123 (1914).

[4] Fischer, E., and Tiemann, F., *Berlin, Ber. D. chem. Ges.*, **24**, 2139 (1891).

THE FREEZING-POINT-SOLUBILITY LAW FOR IDEAL SOLUTIONS

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When a solution is formed by mixing together two completely miscible pure liquids, it is found that when the two constituents resemble each other very closely the formation of the solution takes place without appreciable heat-effect or volume-change. Such solutions have been termed *ideal solutions;* and experiments have shown that the vapor pressure p_A of any constituent A in such a solution is proportional to the mole-fraction x_A of that constituent in the solution. This law is expressed in differential form by the equation: $dp_A = k dx_A$.

From this relation, with the aid of thermodynamics, we may derive the other laws of ideal solutions. Most of the experimental data at present available for illustrating the application of these laws to specific cases have been derived from direct measurements of the vapor-pressures themselves. It seemed therefore desirable to secure some additional experimental data which would serve as examples illustrating the character of the results given by some of the other laws.

The freezing-point-solubility law was chosen as the first one to be studied in this connection because it happens that this law offers a convenient as well as exact method for testing the accuracy with which a given solution obeys the laws of ideal solutions.

For an ideal solution of two substances, A and B, it can be shown thermodynamically that the following equations express the change of freezing-point caused by an increase in the mole-fraction of each component:

$$\frac{dT}{dx_A} = \frac{RT^2}{L_A x_A}; \quad \frac{dT}{dx_B} = \frac{RT^2}{L_B x_B}$$

where R is the gas constant, T the absolute temperature, L the heat of fusion per mole, and x the mole-fraction, in solution, of the substance indicated by the subscript. At the eutectic point it is evident that these two equations must hold simultaneously; and by eliminating x_A and x_B from them we can calculate the eutectic temperature for the system under consideration. The purpose of this investigation was to test in this way the validity of these equations for the three two-component solutions which can be formed from the three substances benzene, diphenyl, and naphthalene. For this purpose it is necessary to know the heat of fusion and melting-point of each substance.

Satisfactory values of the molar heats of fusion of benzene and naphthalene were available; that of benzene had been determined by Demerliac and by J. Meyer to be 2370 calories, and that of naphthalene by Alluard and by Pickering as 4560 calories. It was, however, necessary to determine that of diphenyl. For this purpose, weighed amounts (about 30 grams) of diphenyl in stoppered test-tubes were transferred from a mercury bath at 71° to a calorimeter, consisting of a silvered vacuum-tube filled with water; and the rise in temperature (about 3°) was measured with a Beckmann thermometer. The calorimeter was then cooled to its original temperature by dropping in a weighed piece of dry ice taken from an air bath at $-2°$ where it was suspended on a silk thread. As an additional check on the water-equivalent of the calorimeter the first operation was then repeated, using however test-tubes containing naphthalene instead of diphenyl. The heats of fusion of ice and of naphthalene being known, that of diphenyl could be readily calculated from these experiments. It was thus found to be 4020 calories per mole.

The freezing-points of the pure substances and the eutectic points of the three solutions were determined by the equilibrium method. The apparatus consisted of a Baudin thermometer standardized by the National Bureau of Standards and a small silvered cylindrical vacuum-tube which was heated (or cooled) to the required temperature previous to beginning the experiment. For temperatures higher than that of the room, the crystals (pure substance or eutectic mixture) were melted in a small casserole, and the liquid (about 25 cc.) was poured into the vacuum-tube and the thermometer inserted. When the temperature of crystallization had been reached more of the crystals were added, after which the thermometer reading soon became constant and remained so (within less than 0.1°) for ten or fifteen minutes, with constant stirring and tapping of the thermometer. For temperatures lower than room-temperature the solid crystals (previously cooled, in the case of the eutectic mixture) were placed in the vacuum-tube, and equilibrium between the crystals and liquid in contact with the thermometer bulb was obtained as before. In this way the three melting-points and the three eutectic temperatures could be easily measured with an accuracy of at least 0.1°. The constancy of the three melting-points within this limit is sufficient criterion of the purity of the materials employed.

In order to calculate the eutectic temperatures from the freezing-point equations given above, these equations were first integrated on the assumption that L is independent of T for the ranges involved. This assumption is justified in the case of benzene and naphthalene by the small differences between the specific heats in the solid and liquid states. In the case of diphenyl specific-heat data are not available, but the assumption seemed justifiable by analogy. The two integrals are:

$$2.303 \log_{10} x_A = -\frac{L_A(T_{0A} - T)}{RT_{0A}T}\ ;\ 2.303 \log_{10} x_B = -\frac{L_B(T_{0B} - T)}{RT_{0B}T};$$

where T_{0A} and T_{0B} are the absolute melting-points of the pure substances. By solving these two equations simultaneously (with the aid of the axiom, $x_A + x_B = 1$) values of the eutectic temperature T were obtained.

The melting-points of the pure substances were found to be: Benzene,

C_6H_6: 5.48°. Diphenyl, $C_6H_5C_6H_5$: 68.95°. Naphthalene, $C_{10}H_8$: 80.09°. The directly measured values and the computed values of the three eutectic temperatures were found to be as follows:

Benzene-naphthalene; measured: $-3.48°$; computed: $-3.56°$.
Benzene-diphenyl; measured: $-5.8°$; computed: $-6.1°$.
Naphthalene-diphenyl; measured: 39.4°; computed: 39.4°.

The close agreement between the calculated and observed values of the eutectic temperatures shows that these three solutions are very close to ideal solutions in their behavior; for this method of testing them is a very rigorous one.

The above given differential equations represent the most generally applicable freezing-point and solubility law which we have at present. By means of this law we can calculate the freezing-point of a given solution or the solubility of a given solid in a liquid for all solutions which fulfil the necessary conditions. These conditions are (1) the heat of dilution of the solution for the range of concentration involved must be practically zero; and (2) the dilution of the solution must not be accompanied by the occurrence of a chemical reaction of any kind. The quantity L in the equations represents in general the molal heat of solution of the crystalline substance in its own saturated solution. For ideal solutions this heat of solution is of course identical with the heat of fusion. The above law should be generally employed in place of the more common 'dilute solution' laws, which are only special cases of it.

In closing this paper it may be worth while to point out two examples of the use of the equation given above which do not seem to be familiar to most chemists, and especially to those chemists in whose special fields of work they are of chief interest.

1. The results of the search for some general principle concerning the solubility of substances in each other is usually summed up by restating the old principle that *similia similibus solvuntur.* In the light of our present knowledge we can, however, go somewhat further than this simple qualitative statement, and can formulate the following quantitative principle:

When a crystalline substance A is dissolved in any solvent B with which it forms an ideal solution, its solubility (expressed in terms of its mole-fraction in the saturated solution) is *entirely independent of the nature of the solvent B*, and is therefore the same in all such solvents. The solubility depends merely upon the temperature and upon the melting-point and heat of fusion of the crystals A, from which data it can in fact be readily calculated. This important principle, which follows directly from the above equations does not seem to have been generally appreciated, although it was discovered in 1893 by Schröder (*Zs. physik. Chem.*, **11**, 449).

2. Another important use to which the general freezing-point–solubility law could be put would be in settling the frequently discussed question as to whether racemates exist as such in the liquid state, and if so, to what extent. This question could be most certainly and definitely decided for any given pair of optical isomers simply by determining the freezing-point diagram for the system and applying the general law. The exact amount of racemate present in the liquid state and the manner in which it varies with the temperature could also be computed; for a solution composed of two optical isomers forms one of the most perfect examples known of an ideal solution, and the laws of such solutions would apply with great exactness over the whole range of concentrations. In addition to the freezing-point diagram, the latent heats of fusion and the heat of dilution would also be needed in order to work out the problem completely. The main point which it is desired to emphasize here is simply that the ideal solution laws are capable of giving a complete and quantitative answer to such a question as this, an answer concerning the correctness of which no reasonable doubt could be felt.

MERCURY DERIVATIVES OF AROMATIC AMINES. I. STRUCTURE OF PRIMARY AND SECONDARY *p*-AMINOPHENYLMERCURIC COMPOUNDS

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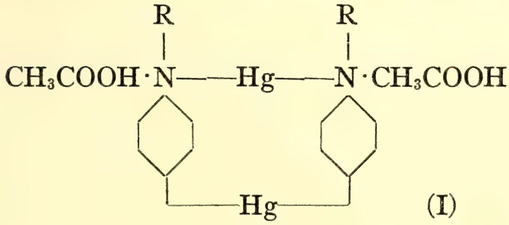
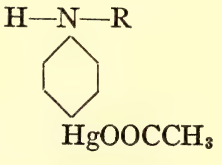
During the year 1902, the question of the structure of the mercury derivatives of the aromatic amines was the occasion of a polemic between L. Pesci[1] and O. Dimroth.[2] The former ascribed the complex structure (I) to the mercuriated primary and secondary amines (R = H or alkyl), while Dimroth advocated the simple monomolecular formula (II), basing his views upon arguments which Pesci was unable to controvert. Subsequent workers have used the monomolecular formula



(I)



(II)

indicated by Dimroth's reasoning, although no evidence of a direct nature was adduced and only the reactions of the mercury were considered in assigning the formula. It seemed to the writers that a study of the amino group in these compounds would furnish more direct evidence as to the structure than any at hand, for, obviously, a compound of the formula (I), where R = H, could not undergo the reactions characteristic of the primary amino group. This work was therefore undertaken as a preliminary phase of a more extended study of the mercuriated aromatic amines. It was found possible to prove formula (II) for *p*-aminophenylmercuric acetate by demonstrating the presence of the primary amino group in two ways: (1) the formation of a salicylidene derivative; (2) the ready formation of a diazo solution, from which were prepared the azo dyes with dimethylaniline, diethylaniline, phenol, resorcin, and α-naphthylamine-5-sulfonic acid. It was also found possible to prove the monomolecular structure of *p*-methylaminophenylmercuric acetate by demonstrating the presence of a secondary amino group through the preparation of the nitroso derivative. A more detailed discussion and a description of the compounds prepared will appear in the *Journal of Biological Chemistry.*

[1] *Zs. anorg. Chem.*, 32, 227 (1902).

[2] *Berlin, Ber. D. Chem. Ges.*, 35, 2038 (1902); *Zs. anorg. Chem.*, 33, 311 (1902).

MOLECULAR REARRANGEMENTS OF TRIPHENYLMETHANE DERIVATIVES. I. GENERAL DISCUSSION

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Introduction.—In most chemical actions taking place at moderate temperatures, a change affecting an atom or a group of atoms in a molecule leaves the remaining atoms in undisturbed relations to one another. This stability of groupings of atoms led, especially in the domain of organic chemistry, to the theory of the structure of molecules and underlies our methods for determining these structures. Occasionally, however, the study of the product of a reaction shows that a profound rearrangement of the grouping of atoms in a reacting molecule has taken place. Molecular rearrangements of this kind are of special interest for the reason that their study should shed some light, from a purely chemical side, on the forces holding atoms in place in molecular structures. The evidence thus obtained should aid in the study of these forces by purely physical means.

The recent development of electron theories of valence has given the investigation of the causes underlying molecular rearrangements an opportunity for much greater precision in formulating the problems and in expressing the results of their study. Thus, the original electron theory of valence proposed by J. J. Thomson[1] has been used with advantage in interpreting the molecular rearrangements of hydroxylamine and halogen amine derivatives of certain organic compounds independently and practically simultaneously by L. W. Jones[2] and myself.[3] These interpretations differed in some important particulars, but agreed as to the fundamental relations.

The electron theory will here be used in expressing the results of recent investigations of molecular rearrangements of triphenylmethane derivatives. Inversely, certain definite conclusions may be drawn from the study of the rearranging compounds in regard to assumptions made in the various theories of valence; and it is proposed to present this evidence first as a contribution to the greater problem of valence from the field of pure organic chemistry—the very field in which newer views are most urgently advanced by physicists and physical chemists.

The Electron Theories of Valence and the Molecular Rearrangements of Halogen Amines and Similar Derivatives. When an acyl halogen amine R.CO.NH(Hal) is treated with bases, it undergoes the Hofmann rearrangement, the radicle R (which represents an alkyl or acyl radicle or hydrogen) being torn from its union with carbon and carried to the nitrogen.[4] We have, for instance:

$$\mathrm{R.CO.NH(Cl) + NaOH \rightarrow NaCl + HOH + CO{:}NR.} \qquad (1)$$

Triphenylmethyl halogen amines, we have found,[5] undergo the same kind of rearrangement:

$$\mathrm{(C_6H_5)_3C.NH(Cl) + NaOH \rightarrow NaCl + HOH + (C_6H_5)_2C{:}NC_6H_5} \qquad (2).$$

Now, in both these classes of chlorine derivatives, the chlorine unquestionably is present as the specific radicle Cl of hypochlorous acid:[6] for the compounds are prepared from hypochlorous acid; they form the acid by hydrolysis; they form hypochlorous esters with alcohols, hypochlorous amides with amines; in all of these reactions the specific oxidizing power of the hypochlorous acid radicle remains intact; all attempts to replace the chlorine atom by negative groups (OH, OR, NH_2) have proved futile; all attempts to prepare chloroamines by means of the hydrogen chloride radicle Cl and its derivatives (HCl, PCl_3, etc.) have likewise proved ineffectual. On the other hand, in compounds such as the alkyl chlorides $R_3C.Cl$ and acyl chlorides R.OC.Cl, the chlorine is unquestionably present as the specific hydrogen chloride radicle Cl: the compounds are prepared from hydrogen chloride or its derivatives (PCl_3, etc.); they form hydrochloric acid by hydrolysis, by treatment with ammonia and a variety of other reagents; the chlorine can be readily replaced by a great number of negative radicles; and in these chlorides, the chlorine has no oxidizing power.

There is, evidently, a fundamental difference in the condition of the halogen in these organic compounds, the rearranging chloroamines and the (non-rearranging) alkyl and acyl chlorides. This difference is sharply and precisely expressed in terms of Thomson's original theory of electron valences by the electron structure (Cl^+) for the hypochlorous acid radicle,[7] the positive charge indicating that the atom has lost an electron, and by the structure Cl^- for the hydrochloric acid radicle, the negative charge being due to an electron captured by the atom. The chloroamines then have the structure $R_2N^{-+}Cl$, the alkyl and acyl chlorides, the structures $R_3C^{+-}Cl$ and $R.OC^{+-}Cl$. (Here and throughout these articles only the valences in question are indicated.) These electronic structures give a complete and exact history of all the striking characteristics of the two groups of compounds, as just enumerated.

According to the more recent suggestions of J. J. Thomson,[8] the valences holding atoms in molecules, especially in the case of non-electrolytes, may be formed, not only in the fashion assumed in his original theory (called now 'intramolecular ionization'), but also by the electrical attraction between the normal electrons of one atom and the 'positive core' of the other, each atom remaining, as a whole, electrically neutral. Practically the same suggestion (following one made by A. A. Noyes[9] in 1907 that we might have two kinds of molecules of one and the same compound, as, for instance, HCl and H^+Cl^- for hydrogen chloride) had previously been made by Bray and Branch[10] and by G. N. Lewis,[11] who speak both of 'non-polar' valences as uniting atoms, especially in non-electrolytes, and of 'polar' valences, of the type assumed in Thomson's original theory. Thomson defined and elaborated more precisely than did the American chemists the nature of 'non-polar' valences. Both Lewis and Thomson consider that the valences may be polar in one part, non-polar in another part of the same molecule. (The terms non-polar and polar will here be used as signifying the two kinds of valence postulated also by Thomson's later theory, inasmuch, as they are much more convenient than the terms 'intra-molecularly ionized' and 'non-ionized' valences.)

It is evident, from the development given above, that even according to these newer views of valence, the electronic structure of the chloroamines, as far as the chloroamino radicle is concerned, must be a *polarized* one, $R_2N^{-+}Cl$ and not the non-polarized union $R_2N \rightleftarrows Cl$ permitted by the new theories, and that, similarly, the electronic structures of the alkyl and acyl chlorides should be considered polar, $R_3C^{+-}Cl$ and $ROC^{+-}Cl$. The non-polar structures $R_2N \rightleftarrows Cl$ and $R_3C \rightleftarrows Cl$ would not account at all for the persistent, fundamental differences in the properties of the chlorine in the two series of derivatives. Now, while both the chloroamines and the alkyl and acyl chlorides may be minutely ionizable, they certainly are not electrolytes in the ordinary acceptance of the term. We thus find that strongly polarized combinations of atoms need not produce electrolytes. This being the case in an instance open to demonstration, one is led inevitably to the inquiry, why it should not be true for a great many polarized combinations and whether the assumption of non-polar unions is at all necessary.

Very strong, if not so obvious, evidence, leads to the conclusion that the relations developed for the chloramines and the alkyl chlorides are duplicated by other series of compounds, for instance by the hydroxylamines[12] and the alcohols, whose relations may be expressed precisely and completely only in the polar structures $R_2N^{-+}OH$ and $R_3C^{+-}OH$. Again we have no exhibition of markedly basic properties by the alcohols (which form oxonium salts with acids and are 'base-forming' like ammonia, but not bases, ionizing directly simply as weak acids) in spite of the strong evidence of intramolecular ionization.

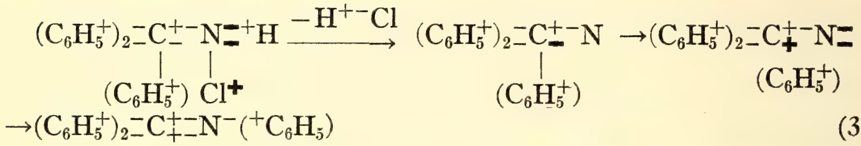
Finally, it should be said in this connection, that exactly the same persistent fundamental differences as exist between positive and negative chlorine in organic compounds are clearly manifested between nitrogen as the nitrous acid radicle N^{+++}, say in the diazo compounds[13] and nitrogen as the ammonia radicle $N \equiv$ in the amines, amides, and nitriles. Here again, the relations are precisely expressed by polar structures for the nitrogen radicles without attendant indications of any tendencies towards ionization.

Nothing stated in this article is, however, to be interpreted to the effect that one radicle may not be converted into the electromeric one of opposite sign; but all such transformations represent explicit, easily recognized oxidation-reduction reactions.

Turning now to a further phase of this problem, we find that in the molecular rearrangements expressed in equations 1 and 2, the hypochlorous acid radicle of the chloroamine is converted into the common negative chloride ion of sodium chloride. According to the original Thomson theory of valence, in this change the positive chlorine atom of the chloroamine must capture two electrons from some other atom or atoms;

and, since the reagent used remains in the same condition of oxidation throughout the reaction, these electrons must be taken from some neighboring atom or atoms in the original molecule. This, obviously, must disturb the valences holding the atoms in the molecule, and a molecular rearrangement is altogether inevitable.

The evidence as to the charges on the individual atoms and radicles is clearest for triphenylmethylchloroamines and its analogs,[3] and we must formulate the rearrangement in the following way, the charges that are changed being indicated by heavy type:



$$(C_6H_5^{+})_2{}^{-}_{-}\underset{(C_6H_5^{+})}{\underset{|}{C}}{}^{+}_{-}{}^{-}\underset{Cl^{\boldsymbol{+}}}{\underset{|}{N}}{}^{-}_{\boldsymbol{-}}{}^{+}H \xrightarrow{-H^{+-}Cl} (C_6H_5^{+})_2{}^{-}_{-}\underset{(C_6H_5^{+})}{\underset{|}{C}}{}^{+}_{\boldsymbol{-}}{}^{-}N \rightarrow (C_6H_5^{+})_2{}^{-}_{-}C^{+}_{\boldsymbol{+}}{}^{-}N^{-}_{\boldsymbol{-}} \;\; (C_6H_5^{+})$$

$$\rightarrow (C_6H_5^{+})_2{}^{-}_{-}C^{+}_{+}{}^{-}_{-}N^{-}(^{+}C_6H_5) \qquad (3).$$

It is assumed that Cl^{+} takes the two electrons first from the nitrogen originally holding it. (The same result would be obtained if the electrons were taken directly from the carbon by the positive chlorine. Evidence on this part of the problem is presented in the following article.) The nitrogen is thus rendered incapable of holding the H^{+} and Cl^{-} and these are lost, forming sodium chloride and water with the reagent sodium hydroxide, as indicated in equation 2. The unsaturated (univalent[14]) nitrogen, in turn, takes two electrons from the carbon atom holding it, and the new positive charge produced thereby on the carbon atom must release one of the positive phenyl groups and this must move, under the force of the electrical field, to the negative nitrogen.

We recognize that the disturbing, labile element in the original structure is just the unstable, positive chlorine, so admirably represented by Thomson's first theory, and that its undeniably great tendency to go over into its common stable negative form is *the original source of the molecular disturbance and rearrangement.*

The product of the rearrangement, phenylimido benzophenone, in harmony with the electronic structure resulting for it according to equation 3, is uniformly hydrolyzed by acids to benzophenone and aniline:

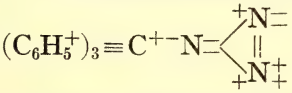
$$(C_6H_5)_2C^{+}_{+}{}^{-}_{-}N^{-}(^{+}C_6H_5) + HOH + HCl \rightarrow (C_6H_5)_2C^{+}_{+}{}^{-}_{-}O + (C_6H_5^{+}{}^{-}N\overset{-}{\equiv}H_3^{+})^{+-}Cl \qquad (4).$$

It is evident that the application of the original theory of Thomson to the valence relations in triphenylmethylchloroamine gives us a complete and precise picture of the cause, mode, and product of its molecular rearrangement.

If we attempt to express the rearrangement in terms of the theory that we may have non-polar, as well as polar valences existing simultaneously in molecules, we find the following: (1) It is necessary to assume polar valences in the chloroamine radicle, as was proved above. (2) We recognize the positively polarized chlorine as the unstable element in the molecule. (3) In the product, phenylimido benzophenone, we must assume at least partial polarization of the methylene carbon and nitrogen, in order to express precisely its mode of hydrolysis (equation 4). (4) It is necessary to assume therefore that at least one electron has migrated from the methyl carbon to the nitrogen (or chlorine). (5) To these definite conclusions may be added the fact that a presumptively negative group ^{-}OR attached to carbon never migrates[15] to nitrogen under these conditions, it is only presumably positive alkyl (CH_3^+) or alphyl ($C_6H_5^+$) radicles and hydrogen radicles H^+ that migrate thus. In other words, so far as experiment penetrates into these molecular rearrangements, we find in the original molecule, in the reaction itself and in the product of the rearrangement evidences only of polar valences. The later modifications of the Thomson theory are of no assistance in formulating the theory of these reactions.

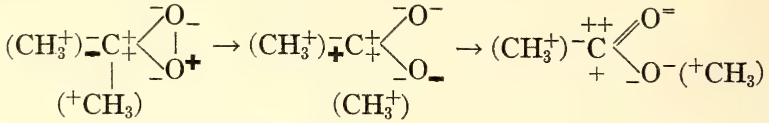
Finally, there is a whole group of rearrangements analogous to the rearrangement of chloroamines. In each instance one can plainly recognize, as the common characteristic of all the rearranging compounds, that the labile component in the original molecule is an unstable positive radicle or atom, which tends to go over into its stable negative form. In every instance, this tendency is found satisfied in the action, and as this involves the absorption of electrons from neighboring atoms in the original molecule, one must conclude that in all of these cases the tendency mentioned and its satisfaction bring about the rearrangement. These relations hold for the following groups of compounds:

1. *Hydroxylamine* derivatives, such as $(C_6H_5)_3C.HN^{-+}OH$ (considered in the following article), in which the positive hydroxide group[12] is unstable, tending to form the well known negative group. The molecular rearrangement may be expressed[12] by the same equation 3 as was used for the chloroamine, if ^{+}OH is substituted for Cl^+ and HO^- for Cl^-.

2. *Azides*, such as  $(C_6H_5^+)_3{\equiv}C^{+-}N{=}\!\!<\!\!\begin{array}{l}{}^{+}N{=}\\ \;\,\|\\ {}_{+}N^{+}_{+}\end{array}$ (see the following article), which tend to lose the neutral molecule N_2 with a gain of two electrons for the radicle N_2^{++}. Again equation 3 may be used to express the whole action, the loss of N_2 being substituted for the loss of HCl. (The azide is formed by the action of nitrous acid $N^{++}_{+}\overset{-}{\underset{-}{}}O$ (^{-}OH) on the hydrazine $(C_6H_5)_3C.HN^{-+}NH_2$,[16] which leads to the structure used.

The same net result would be shown if the alternative structure[17] $(C_6H_5)_3C^{+-}N^{-}_{-}(^{++}N^{+++})\equiv N$ were accepted for the azide.)

3. *Peroxides*, in whose rearrangement a positive charge on an oxygen atom is converted into a stable negative charge:



(5).

In conclusion, the effects of an electric current in decomposing the salts of organic acids are entirely analogous, fundamentally, to these molecular rearrangements.[12] With the current the effect is produced by the positive electrode charge, with these compounds, by the positive polar charges on the specific atoms, as just shown.

The original Thomson theory of electron valences is evidently extremely useful in the precise formulation of these molecular rearrangements, involving a large group of organic reactions. Of the newer theories of Thomson, Bray and Branch, and Lewis, we find occasion to use only so much as is already found in the original theory. This, of course, does not exclude the possibility that the newer assumptions may reveal their usefulness in other fields of investigation. (Thomson's structure for benzene, developed on the basis of the newer theory,[18] interpreted rigorously and without further limiting assumptions, seems to make possible at least *four* di-substitution products C_6H_4XY, while only the *three* well known series have been recognized.)

(For references see end of following article.)

MOLECULAR REARRANGEMENTS OF TRIPHENYLMETHANE DERIVATIVES. II. EXPERIMENTAL PART

By Julius Stieglitz and Collaborators
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Read before the Academy, December 9, 1914. Received, February 20, 1915

MOLECULAR REARRANGEMENTS OF TRIPHENYLMETHYL HALOGEN AMINES[5]

WITH ISABELLE VOSBURGH † AND AGNES FAY MORGAN

The study of the mechanism of the molecular rearrangement of acyl halogen amines RCO.NH(Hal) is complicated by the fact that the salts formed by the bases used as reagents to start the rearrangement may have the tautomeric structures, RC(OMe) (:NHal) or RCO.N(Me)Hal. This complication does not exist in the case of triphenylmethyl halogen amines $(C_6H_5)_3C.NHHal$, whose metal derivatives could only have the definite structure $(C_6H_5)_3C.N(Me)Hal$. A successful molecular rearrangement of these halogen amines by bases would give convincing evidence as to the nature of the salt formed in initiating the rearrangement and would give strong presumptive evidence in favor of the view that in the rearrangement of acyl halogen amines, the salts RCO.NMeHal are the ones directly concerned in the rearrangement.

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† My young collaborator, Miss Vosburgh, who carried out the first part of the experimental work of this investigation most skilfully and conscientiously, died December 4, 1914, as the result of an automobile accident.—J. S.

Further: chloroimidoketones $(C_6H_5)_2C{:}N^{-+}Cl$ have been found to resist rearrangement[19] in spite of the presence of the unstable positive chlorine atom (see part I). This result was not unexpected from the point of view of the theory of the one of us concerning the mechanism of these molecular rearrangements:[4] the theory postulates that the easiest path to a rearrangement is through the intermediate formation of univalent nitrogen derivatives $RCO.N<$, $R_3C.N<$ (see equation 3), the free valences of the univalent nitrogen being considered sufficiently powerful to tear the radicle R from the carbon and thus effect the rearrangement to a more stable molecule (see equation 3). The significance of the intermediate formation of a univalent nitrogen derivative in relation to the electron theory is discussed later on in the text. Triphenylmethyl halogen amines, $R_3C.NH(Hal)$ and $R_3C.N(Hal)_2$, in contrast with the chloroimidoketones $R_2C{:}NCl$, present the opportunity for an unhampered formation of the univalent nitrogen derivative $R_3C.N<$ by the loss of H(Hal) or $(Hal)_2$ under suitable experimental conditions. They were therefore expected to be easily subject to rearrangement.

This investigation was undertaken with the object of deciding the questions indicated and thus throwing light on the whole problem of these molecular rearrangements. The experimental results confirmed in every respect the theoretical anticipations. Of the facts established and their bearing, the following are the most important:

1. A number of triphenylmethyl halogen amines—$(C_6H_5)C.NHBr$, $(ClC_6H_4)_2(C_6H_5)C.NHCl$, $(BrC_6H_4)(ClC_6H_4)(C_6H_5)C.NHCl$, and $(C_6H_5)_3C.NCl_2$ and $(ClC_6H_4)(C_6H_5)_2C.NCl_2$—were found to undergo under suitable conditions a molecular rearrangement into phenylimido benzophenones $R_2C{:}NR$ (see equation 3). The rearrangement was established in the case of the first compound listed by the identification of the phenylimido benzophenone $(C_6H_5)_2C{:}NC_6H_5$ (yellow crystals, m.p. 110°; the melting-point of a mixture of the product with synthetic substance (m.p. 111°–112°) was found at 111°) and by the identification of the benzophenone (m.p. 47°) and aniline (bleaching-powder test; the

chloroplatinate gave 32.41% platinum, theory 32.74%) produced by the hydrolysis of the imido ketone (see equation 4). In the rearrangement of the chloroamines containing substituted phenyl groups, mixtures result: in a fraction of the rearranging molecules the substituted phenyl radicle migrates to the nitrogen, in the remainder the phenyl group itself leaves the carbon for the nitrogen. Thus, in the rearrangement of di-parachlorophenyl-phenyl methylchloroamine (the second compound listed above), the two substances $(C_6H_4Cl)_2C{:}NC_6H_5$ and $(ClC_6H_4)(C_6H_5)C{:}NC_6H_4Cl$ are formed. This was established by the separation and identification of their respective products of hydrolysis (equation 4), dichlorobenzophenone $(ClC_6H_4)_2CO$ (m.p. 141°) and aniline (bleaching-powder test), and chlorobenzophenone $(ClC_6H_4)(C_6H_5)CO$ (m.p. 77°) and chloroaniline (m.p. 70°). In each instance the melting-point of a mixture of the substance obtained with the corresponding synthetic product was taken to complete the identification. The ratios in which the various radicles migrate to the nitrogen in the rearrangements was determined by quantitative examinations of the anilines formed: for the instance just discussed, the ratio of chloroaniline to aniline was found to be 68/32, in molar terms—in other words, the two radicles share in the migration very closely in the same ratio as they are found in the original molecule.

2. The reagents used to effect the rearrangements are those which the univalent nitrogen theory would postulate: bases (sodium and calcium hydroxides, sodium methylate) were used for the rearrangement of the monohalogenamines to facilitate the loss of halogen acid; and, characteristically, heat alone was sufficient to cause the loss of chlorine with explosive violence by the dichloroamines, the rearrangement ensuing:

$$(C_6H_5)_3C.NCl_2 - Cl_2 \rightarrow (C_6H_5)_3C.N< \rightarrow (C_6H_5)_2C{:}NC_6H_5 \qquad (6).$$

3. A compound, triphenylmethylchloromethylamine $(C_6H_5)_3C.N(CH_3)Cl^+$, analogous to these rearranging bodies, with the exception that the well known stability of the union between nitrogen and the methyl radicle would interfere with the formation of a univalent nitrogen derivative, was found *not to be rearranged*, either by treatment with bases or by heat. A rearrangement would have produced a derivative of methylaniline (see below), which by hydrolysis would have given methylaniline. All the tests for methylaniline, even the exceedingly sensitive chloride of lime color test, showed its absence in the products of hydrolysis of the compounds formed under the conditions mentioned.

This important result shows that the tendency of a positive halogen atom to go over into its stable negative form is not alone sufficient, as a rule, to cause the characteristic rearrangement of these compounds, but that the path to the formation of a univalent nitrogen derivative must usually, if not always, be open, this derivative being the substance that suffers the actual molecular rearrangement (see equation 3). This must signify that a positive halogen atom can secure two electrons from the nitrogen atom to which it is directly linked much more readily than from the carbon atom holding the nitrogen (see equation 3). Otherwise the rearrangement would proceed by a direct exchange of radicals following a migration of the electrons from the carbon to the halogen atom. Furthermore, the characteristic contrast shown in the behavior of the rearranging compounds ($R_3C.NH(Hal^+)$—losing halogen acid—and $R_3C.NCl_2^+$ — losing chlorine —), on the one hand, and the non-rearranging compounds ($R_3C.N(CH_3)Cl^+$ and $R_2C{:}NCl^+$), on the other hand, leads to the important conclusion that the electrons most easily captured from the nitrogen atom by the positive halogen atom are *the two valence electrons* beyond the normal electrons of the nitrogen atom rather than any of the electrons of the normal atom. Otherwise, again, we should have a rearrangement by a direct exchange of radicals following the exchanges of electrons. This conclusion is included in equation 3 of the preceding paper.

The conclusions reached—(1) that a positive halogen atom can secure electrons more readily from the atom to which it is directly attached (nitrogen, in this case) than from one further removed from it, and (2) that the more readily available electrons are the two extra valence electrons of the nitrogen atom rather than its normal ones, seem not unreasonable. The second conclusion is in agreement with the observations of physical chemistry on the oxidation of negative ions like $S^=$, I^-, etc. This result seems to us the most important one established by the investigation, giving us a logical explanation, from the point of view of the electron theory of valence, of the real significance of the univalent nitrogen in the author's theory of the mechanism of this and similar rearrangements.

THE MOLECULAR REARRANGEMENT OF β-TRIPHENYLMETHYL β-METHYL HYDROXYLAMINE

WITH BERT ALLEN STAGNER

While a great mass of evidence, gathered by the author as well as independent investigators,[4] has shown that the molecular rearrangements of halogen amines, hydroxylamine derivatives and azides seem to proceed preferably by the path of an intermediate formation of univalent nitrogen derivatives, the theoretical possibility that another path might be taken toward the formation of more stable molecules has always been kept in mind. As a consequence, in the investigation of every new type of rearrangement, experiments have been included to attempt the rearrangement of compounds which could not readily form univalent nitrogen derivatives (see an instance above under 3). From this point of view, an attempt was made by the one of us and P. N. Leech to rearrange β-triphenylmethyl-β-methyl hydroxylamine $(C_6H_5)_3C.N(CH_3)OH$ by the same methods that had proved successful with triphenylmethylhydroxylamine $(C_6H_5)_3C.NHOH$. Indications of a rearrangement and the formation of a methylaniline derivative were indeed obtained,[12] but the evidence was quite uncertain, being limited to a general color reaction for monoalkylanilines and to some melting-point determinations of an acetyl derivative of the aniline.

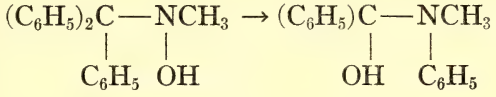
We have now succeeded in establishing the fact that β-triphenylmethyl-β-methyl hydroxylamine is rearranged in a perfectly normal fashion by the action of phosphorus pentachloride on the ether solution of the hydroxylamine. The compound formed in the reaction was shown to be $(C_6H_5)_2C(OH)N(CH_3)(C_6H_5)$ [or its chloride $(C_6H_5)_2CCl.N(CH_3)(C_6H_5)$] by the identification of its products of hydrolysis, methylaniline and benzophenone:

$$(C_6H_5)_2CCl.N(CH_3)(C_6H_5)+HOH\rightarrow(C_6H_5)_2CO+C_6H_5N(CH_3)H_2Cl. \quad (7)$$

The formation of the more important component methylaniline was proved (1) by the analysis of its chloroplatinate (found, 31.40% platinum; theory 31.27%); (2) by the comparison of its hydrochloride with the hydrochloride of a known preparation of methylaniline (its melting-point was 119°–122°, that of the pure hydrochloride the same, and the melting-point of a mixture of the two salts the same); (3) by the navy blue color produced by the action of bleaching powder solution on the methylaniline. To these tests may be added the melting-point observations made with Leech on the acetyl derivative of the base. The benzophenone formed was also isolated and identified by its melting-point (47°).

It was further shown that chlorophenyl-diphenylmethyl-β-methyl hydroxylamine $(ClC_6H_4)(C_6H_5)_2CN(CH_3)OH$, like the corresponding chlorophenyl-diphenylmethylhydroxylamine $(ClC_6H_4)(C_6H_5)_2C.NHOH$, shows a migration of the phenyl group in a fraction of the rearranging molecules, a migration of the chlorophenyl radicle in the remainder. The molar ratio of methylaniline to chloromethylaniline was 75/25.

It is, therefore, now for the first time an established fact that substances which could not form simple univalent nitrogen derivatives may be rearranged in the same fashion as those which would easily lead to such derivatives and the theory of the rearrangements must now count with this fact. Whether we actually have a direct exchange of radicles



$$(C_6H_5)_2\underset{\substack{|\\ C_6H_5}}{C}-\underset{\substack{|\\ OH}}{N}CH_3 \rightarrow (C_6H_5)\underset{\substack{|\\ OH}}{C}-\underset{\substack{|\\ C_6H_5}}{N}CH_3 \qquad (8).$$

as postulated by Beckmann[20] for the analogous rearrangement of oximes $(C_6H_5)_2C:NOH$, or whether in both of these cases the rearrangements go by way of *salts* of univalent nitrogen derivatives, will be considered in detail in another place. It must suffice to point out here that whichever path the rearrangement follows, the final result is that the positive hydroxyl group of the hydroxylamines and of the oximes undoubtedly becomes a negative group in the course of the reaction: it still would be the unstable radicle in the original molecule and the satisfaction of its tendency to become negative would still be the prevailing influence in the rearrangement (see part I).

THE REARRANGEMENT OF TRIPHENYLMETHYL AZIDES AND OF SYM. DI-TRIPHENYLMETHYLHYDRAZINE

WITH JAMES KUHN SENIOR

While there may be some question as to the mechanism of the rearrangement of hydroxylamine derivatives (see above), the theory of a direct exchange of radicles (equation 8) would be quite inapplicable to the rearrangements of azides. Triphenylmethyl halogen amines and hydroxylamines having been found, in this laboratory, to undergo normal rearrangements, the problem of the rearrangement of triphenylmethylazides $(C_6H_5)_3C.N(N_2)$ was undertaken by us. It was intended to settle three main questions. The first two were: (1) Whether a rearrangement could be effected at all; (2) Whether heat alone would not be sufficient to effect the rearrangement, this being the agent indicated by the univalent nitrogen theory of the rearrangement:

$$(C_6H_5)_3C.N(N_2) - N_2 \rightarrow (C_6H_5)_3C.N \rightarrow (C_6H_5)_2C:NC_6H_5. \qquad (9)$$

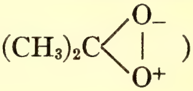
Both of these questions were easily answered in the affirmative. When the azide is heated to a high temperature, it is decomposed, with a loss of nitrogen. The course of the rearrangement was followed by the hydrolysis of the product and identification of the substances thus formed, namely aniline and benzophenone. These are the compounds which phenylimido benzophenone $(C_6H_5)_2C:NC_6H_5$ (equation 9) should form (equation 4). The aniline obtained gave the characteristic bleaching-powder test and formed a chloroplatinate which was analyzed (33.35, 32.32% platinum found, theory 32.78%); the benzophenone was identified by its melting-point 47.5°, which was unchanged when the product was mixed with some known, pure benzophenone.

(3) The third question raised in this investigation concerned itself with the following relations: in the rearrangement of a substitution product of triphenylmethyl azide, such as $(ClC_6H_4)(C_6H_5)_2C.N(N_2)$, a certain part of the substance would form an aniline derivative $(ClC_6H_4)(C_6H_5)C:NC_6H_5$, another part would yield a derivative of chloroaniline $(C_6H_5)_2C:NC_6H_4Cl$ (see above). The ratio may be easily ascertained by hydrolysis of the product and titration of the anilines with bromine. Now, the same situation exists in regard to the rearrangement of the corresponding hydroxylamine (ClC_6H_4) $(C_6H_5)_2C.NHOH$ and the chloroamine (ClC_6H_4) $(C_6H_5)_2C.NHCl$. If in each instance the path to the rearrangement lies through an intermediate univalent nitrogen derivative, we should have the identical product (ClC_6H_4) $(C_6H_5)_2C.N$ in each of these cases, the azide losing nitrogen, the hydroxylamine water and the chloroamine halogen acid. The environment of the univalent-nitrogen derivative would be different in the three cases on account of differences in the physical state, the temperature, etc. But since the rearrangement is an intramolecular one, it might be little affected by environment, once the compound (ClC_6H_4) $(C_6H_5)_2C.N$ is formed. A comparison of the ratios of anline to chloroaniline formed in the different series could then shed some light on the question of the probability of the formation of the hypothetical intermediate univalent nitrogen derivative. On account of the difficulty of securing experimental material only a beginning has been made on this phase of the problem of these rearrangements. We have already found, however, that in the rearrangement of (ClC_6H_4) $(C_6H_5)_2C.N(N_2)$ the ratio of aniline to chloroaniline formed is 70/30 in molar terms. This compares with the ratio[12] 71/29 found for the rearrangement of (ClC_6H_4) $(C_6H_5)_2C.NHOH$. The result speaks for itself.

That hydrazine and hydroxylamine are fundamentally similar compounds[21] is a well-established fact, shown both by their reactions and also especially by their forming parallel derivatives, of similar properties, with a great many organic substances. One strange difference in the chemistry of these derivatives, as thus far developed, seems to have been overlooked, and that is that molecular rearrangements of hydrazines, corresponding to the rearrangements of hydroxamic acids, ketoximes, triphenylmethylhydroxylamines, etc., have not been observed, or, apparently, even attempted; for the 'benzidine' and related rearrangements of hydrazines are of a different type. This seemed the more strange since derivatives of the third member[21] of the group, hydrogen peroxide, have been found to undergo the same typical rearrangement[22] (see part I, equation 5) as the hydroxylamine derivatives.

When we undertook to bridge this gap in the chemistry of the hydrazine derivatives, no difficulty was anticipated in securing such rearrangements. Not only the fundamental analogy between the groups of compounds indicated this, but, from the point of view of the univalent nitrogen theory of the rearrangements, it seemed as if there should be no difficulty in removing ammonia from triphenylmethylhydrazine and thus starting a rearrangement:

$$(C_6H_5)_3C.NH.NH_2 - NH_3 \rightarrow (C_6H_5)_3C.N \rightarrow (C_6H_5)_2C{:}NC_6H_5. \qquad (10)$$

We were very much surprised to find that all our attempts to cause a rearrangement of this hydrazine and of a great many others by means of acids or zinc chloride (reagents to combine with ammonia) were complete failures. Although the experiments are being continued in spite of the original failures, these were sufficiently numerous and impressive to lead us to reconsider in greater detail the theoretical relations involved. Now, for rearranging chloroamines (*e.g.*, $(C_6H_5)_3C.HN^{-}\ {}^{+}Cl$), hydroxylamines (*e.g.*, $(C_6H_5)_3C.HN^{-}\ {}^{+}OH$) or peroxides (*e.g.*,  $(CH_3)_2C\langle\begin{smallmatrix}O_{-}\\ |\\ O^{+}\end{smallmatrix}$), the distribution of the charges on the atoms in the unstable parts of the molecules can be determined with a great degree of reliability.[12] For a hydrazide, this is not the case, no criteria having as yet been established by means of which we can distinguish, for instance, between the electromeric structures:

$$(C_6H_5)_3C.HN^{-}.{}^{+}NH_2 \quad \text{and} \quad (C_6H_5)_3C.HN^{+}.{}^{-}NH_2$$

Obviously, however, the question of the molecular rearrangement of triphenylmethyl hydrazine could very well, and probably would, depend altogether on its electronic structure. For instance, a decomposition $(C_6H_5)_3C.HN.NH_2 \rightarrow (C_6H_5)_3C.NH_2 + (HN)$ would not lead to a rearrangement and the electronic structure of the hydrazide might insure this decomposition in place of the one expressed in equation 10.

These considerations made it appear desirable to attempt the rearrangement of a symmetrical molecule, di-triphenylmethylhydrazine, about whose electronic structure there could be no doubt: $(C_6H_5)_3C.HN^{+}.{}^{-}NH.C(C_6H_5)_3$. A loss of triphenylmethylamine from either side would leave a univalent nitrogen derivative $(C_6H_5)_3C.N$ and rearrangement should inevitably follow. Whether this argument should ultimately be found to be altogether right or not, it is a noetworthy fact that it led us to the first realization of the rearrangement of a hydrazine derivative along these lines. From the product of the action of zinc chloride on the symmetrical hydrazine, aniline was obtained and identified (by a positive chloride of lime test; by analysis of its chloroplatinate, which gave 32.80% platinum, theory 32.78%). The question as to the mechanism of the rearrangement awaits the identification of the further products of the reaction, but aniline could be formed only by a migration of a phenyl group from carbon to nitrogen, the type of rearrangement we desired to produce. This is, we believe, the first rearrangement of a hydrazine along the lines so characteristic for the hydroxylamine derivatives, which have been known for forty years. It is significant that the way to its accomplishment was pointed by the application of Thomson's original theory of electron valences to the subject of these rearrangements.

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[12] J. Stieglitz and Leech, *loc. cit.*

[13] J. Stieglitz, *J. Amer. Chem. Soc.*, **30**, 1797 (1908). See also W. A. Noyes, *loc. cit.*

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ECOLOGY OF THE MURRAY ISLAND CORAL REEF

By Alfred Goldsborough Mayer

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Presented to the Academy, February 13, 1915

A quantitative ecological study was made of the Madreporian corals of the fringing reef of Maër Island, the largest of the Murray Island Group, which lies in Torres Straits, Australia, in 9°55′ S. Lat.; 144°2′ Long. E. from Greenwich.

Maër Island is volcanic and has broken through an old limestone-bearing platform which is now submerged to a depth of 15–30 fathoms, and upon which the modern coral reefs of Torres Straits have grown.

The volcanic center of the Island is surrounded by a modern fringing reef which has grown seaward over its outer edges from the shores, and is now 1800–2200 feet wide on the windward (S. E.) side of the Island, and only 175–780 feet wide on the leeward (N. W.) side. It is wide in regions exposed to the full force of the breakers, and where it is not interfered with by silt from the three principal streams of the Island; while on the other hand in regions protected from breakers or inundated by sand or silt the fringing reef is narrow. Hurricanes are unknown and thus the corals grow on uninterrupted by periods of wholesale destruction such as affect the reefs of the West Indies, and most parts of the Pacific region.

A line running S. 39° E., and 1869 feet long, was surveyed across the S. E. reef flat of Maër Island. The shore end of this line was 1496 feet in a N. E. direction from the mouth of Haddon brook. Squares 50 feet on the side (2500 feet in area) were then surveyed and staked out at intervals of about 200 feet along this line; and all living coral heads growing within these staked areas were counted.

This reef flat is peculiar in that the water is dammed by the lithothamnion ridge which extends in a narrow barrier along the seaward, breaker-washed edge of the flat. Thus at low tide the water over the reef flat becomes a marine basin about two miles long, 1680 feet wide and only about 18 inches deep; but the water being impounded by the lithothamnion ridge, the reef flat is never laid bare even by the lowest spring tides. About 3,600,000 living coral heads are found upon this submerged area. For the first 370 feet out from shore there are no corals. The following table gives the number of living coral heads found on each 50-foot square (2500 square feet) at intervals of about 200 feet apart across the S. E. reef flat or Maër Island.

Distance[1]	*Living*[2]	*Species*[3]
400	3	2
525	110	11
625	126	9
825	413	13
1025	529	15
1225	962	9
1425	1838	18
1650	1512	27
1750	201	15

[1] Distance in feet of the center of the square from shore.

[2] Number of living coral heads growing within each square (2500 square feet).

[3] Number of different species of corals on each square.

About 40 different species of corals are found growing within these squares. Of the 22 genera of corals found on these squares, four constitute 91% of the living coral heads. Thus: Porites 38%, Seriatopora 25%, Acropora 18%, and Pocillopora 10%.

Seriatopora is the most successful coral of the calm waters of the middle zone of the reef flat 1100 feet from shore where it covers 40% of the area of the bottom, and constitutes 70% of the entire number of living coral heads. In this region where Seriatopora is dominant all other species of corals are reduced in number. Yet Seriatopora cannot live within 500 feet of the shore, owing to the high temperature of the water in this region, nor can it survive in places more than 1650 feet from shore due to the destructive action of the breakers upon its fragile stems.

The coral heads are most densely clustered in a region about 200 feet inward from the usual inner 'wash' of the breakers, but an even greater variety of *species* of corals are found to the seaward of this place where the water is strongly agitated by the surges. Thus in the place where coral growth is most dense there are only 18 species, but 200 feet to seaward of this zone there are 27 species, although owing to the rough water the coral stocks are much broken and are either large massive heads capable of resisting the waves or small ones protected within crevices.

Of the 22 genera from the squares over this reef flat 13, or more than half, are confined to the seaward parts of the flat and do not commonly appear within 1200 feet of the shore. In fact only two species, Bernard's 'Porites No. 12' and a form allied to 'Siderastrea' sphaeroidalis Ortmann are practically confined to the shore flats within 1100 feet of the beach.

Temperature is the dominant factor, and is even more important than silt in determining the habitat of corals. The most sensitive genera such as Acropora, Seriatopora, and Pocillopora are killed at from 36.2°–37°C., while Siderastrea and some species of Porites can withstand heat up to 38° or even 38.5°C. Generally speaking those forms which are sensitive to high temperature are correspondingly affected by being smothered under mud, or subjected to the influence of CO_2. This suggests that high temperature produces death by asphyxiation as postulated by Winterstein. There are however some exceptions, for certain corals such as Favia fragum and Mæandra æreolata die at a relatively low temperature but are quite resistant to asphyxiation. It appears from experiments conducted at Tortugas, Florida, that some corals can adjust themselves to a wide range of metabolic activity and can thus survive in a reduced oxygen supply by corresponding lowering their metabolic processes. Others, however, cannot effect this adjustment and must live at a fairly uniform rate of metabolism.

The more sensitive corals such as Acropora, Seriatopora, Euphyllia, or Pocillopora, which are all off-shore, pure water forms, are killed by being buried eleven hours under the mud, whereas the shore flat species of Porites, Siderastrea, and Mæandra can survive being buried from 24 to 73 hours.

The corals of this Australian reef which are never subjected to cold can nevertheless withstand low temperature quite as readily as can the the corals of the 'cold devastated' reefs of Florida; and conversely the Florida corals can withstand high temperatures quite as well as do those of Australia. In other words corals are, physiologically speaking, of similar constitution whether in the Atlantic or the Pacific; and natural selection has apparently not operated to improve their cold-withstanding or heat-resisting powers. The reef building forms must live in water which is warmer than 15°, and cooler than 38°C.

It was found that the shallow waters of the Maër Island reef flat receive most of their heat by direct radiation from the sun and lose it by radiation into outer space at night. Thus at 3 p.m. the water is from 1.2° to 6.7° higher than the air, while at 6 a.m. it is from 0.3° to 3°C. lower than the air temperature.

The range in water temperature was thus greater than that of the air; for during September and October the extreme range in air temperature was only 3.4°C. whereas the water of the reef flat 200 feet from shore ranged through 12.5°, and at 1860 feet from shore, in the breakers, the range was only 3.5°C.

Thus the waters of the shore flats within 400 feet of the beach must become too hot for coral life during the calms of the 'north west season' between November and April.

There is clear evidence of the submarine solution of a thickness of about two feet of limestone in the middle region of the Maër Island reef flat between 400–1600 feet from shore; but this may be chiefly due to the carbon dioxide washed outward from the densely forested shores of the Island during the rainy season, and not to the sea water as such.

Indeed the experiments and observations of Dole, and of Vaughan would lead to the conclusion that the sea water of coral reef lagoons lacks free CO_2 and is therefore probably incapable of dissolving limestone.

All species of reef corals survive without apparent injury an immersion for 4 to 5 hours in sea water diluted with an equal volume of rain water, and many species can withstand 11 hours of this treatment, and thus it appears that even torrential rains cannot be an important factor in the destruction of the reef flat corals of Maër Island through dilution of the water, for the tidal range is about seven feet and the incoming sea water would soon offset any dilution due to rains.

The injurious effects of rains upon coral reefs is due solely to the silt which they cause to be washed outward over the flats.

The research of which the above is an abstract, will be published by the Carnegie Institution of Washington.

CHANGES IN SHADE, COLOR AND PATTERN IN FISHES AND THEIR BEARING ON CERTAIN PROBLEMS OF BEHAVIOR AND ADAPTATION

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It is well known that the surface of many organisms is in appearance much like their environment. That is, the organisms simulate their surroundings and consequently are more or less inconspicuous. In some of these organisms the characteristics which produce such simulation are fairly permanent and the creatures correspond in appearance with, what may be termed, the general average of their environment. In others these characteristics may change rather rapidly, in such a way that the animals appear almost continuously like their surroundings, no matter how much they may change. These phenomena have always excited a lively interest in naturalists and a great amount of work has been done on them. They have been particularly prominent in discussions bearing on evolution and consequently most of the investigations concerned questions of function and origin and development, both individual and racial.

This paper however deals primarily with the extent and the accuracy of the process of adjustment to the environment, the factors involved in it, with its bearing on certain associated problems in behavior. Nearly every investigator assumes that the adjustment to the environment in an organism serves to conceal it from other organisms. This assumption rests upon the further assumption that vision in animals is similar to that in man. Thus we have the problem of vision closely associated with the question concerning the function of simulation of the environment. There are also some other associated problems in behavior which are of interest, particularly the one concerning the influence of experience on the rate of adjustment.

1. Extent, accuracy and nature of simulation. Simulation of the environment in fishes involves changes in shade, changes in color and changes in pattern. Nearly all fishes assume a light shade over bright bottoms and a dark shade over dark bottoms, and a considerable number assume the predominating colors of the environment, but in only a few does the pattern change so as to harmonize with that of their surroundings.

In some of the flounders simulation of the background is probably more extensive, accurate, and rapid than in any other animals. Two of these forms, Paralichthys and Ancylopsetta, were thoroughly studied regarding this response. It was found, in brief, that in glass dishes, either on artificial or on natural backgrounds, changes occurred in the skin such as to make the animals very closely resemble the background in shade, color and pattern regardless of great variations in these respects. On a white background they became remarkably nearly white; on gray they became gray of practically the same shade; and on black they became very nearly black. On colored backgrounds varying from dark blue to dark red they assumed colors very similar to those of the background in all but the red. On backgrounds having small figures the figures in the skin became correspondingly small and on those having large figures they became, within certain limits, correspondingly large. There is, however, no indication of an actual reproduction of patterns. While the size of the light and the dark areas in the background and the relative amount of surface covered by them have a profound effect on the pattern produced in the skin, their form and special interrelationship have, within wide limits, no effect.

2. Rate of adaptation to the background. The time required to produce adaptive changes in the skin in Paralichthys and probably also in other genera varies greatly. Under some conditions changes resulting in maximum adjustment have been observed to occur in two minutes or less, under others it requires several days. In general it is considerably longer for old specimens than for young. It is much longer in individuals kept continuously on a given background than it is in those frequently changed from one kind to another. This is clearly shown by the following: An individual, after having been in a white granite pan continuously for two weeks and long since maximum white, was transferred to a black pan, August 18, 2.05 p.m. At 4.30 p.m. it was about one-half maximum black; August 19, 12 m. about three-fourths maximum black; August 22, 10 a.m. nearly maximum black; August 23, 10 a.m. maximum black. This same individual, after having been frequently transferred from white to black and vice versa, from August 23 to August 30, was taken from the white background where it was maximum white and put into the black pan at 7.27 a.m. One minute later, 7.28, it was already five-sixths maximum black, and after one minute more, 7.29, it was maximum black. The change from black to white however was never observed to be so rapid as this; in all of the experiments it required an hour or more. Thus while it required five days to produce a complete change in the skin from white to black after continuous sojourn of two weeks on white, it required only two minutes after repeated transfers from one to the other.

Changes in color require, in general, much more time than changes in shade or changes in pattern. There is however much variation regarding this among the different colors. Yellow for example, is a color that the fish assume much more readily and rapidly than green or blue. This may be due to the fact that yellow ordinarily predominates in their environment.

3. Factors involved in adjustment to the background. a. Chromatophores: In the skin of the flounders there are colored cells known as chromatophores. Some of these are black and others are yellow of various shades. Associated with these there are other cells which contain numerous highly refractive crystals said to be guanin. These cells are called iridocytes. They appear pure white in reflected light. The pigment in all or in any group of the chromatophores can be concentrated in small globular masses, or it can be spread out so as to cover relatively large surfaces. Moreover, the iridocytes may assume such a position as to hide all or any portion of the pigment in the chromatophores. Thus, changes in shade, color and pattern are produced by reactions in these bodies. And these reactions are regulated by stimuli received through the eyes, as the following experimental observations demonstrate.

b. Eyes: Specimens with the anterior end on white and the posterior end on black become uniformly maximum white over the entire pigmented surface, with the anterior end on black and the posterior end on white or on a glass plate with intense light reflected through it from below they become uniformly maximum black; with one eye on white and the other on black they assume an intermediate shade (uniform gray) over the entire surface. Specimens, with one eye on a background having a given pattern and the other eye on a background having a different one, assume uniformly over the entire pigmented surface of the body, a pattern intermediate between those assumed when entirely on either background. Careful observation shows that this pattern really consists of a sort of superimposition of a coarser and a finer pattern; for large areas similar to those ordinarily produced when the fish is entirely on the coarser grained of the two backgrounds can still be faintly seen, as well as small areas similar to those ordinarily produced when the fish is entirely on the finer grained background. Thus the configuration in the skin when one eye is on a background having a given pattern and the other on one having a different pattern consists of a superimposition of the configuration produced by each of the two different backgrounds acting alone. The influence of the light received by each of the two eyes is evidently distributed over the entire body and the resulting pattern in the skin is due to a combination of the specific effect of stimuli received from the two eyes.

On a background consisting of any combination of black and white, only black and white is seen in the skin, no color whatever. But if a yellow card is placed within 3 cm. of the anterior end of the fish it becomes strikingly yellow. All of these results indicate that if light has any direct effect on the chromatophores it is insignificant in the process of adjustment to the background. This conclusion is further supported by results obtained with blind specimens.

The removal of either eye interferes but little with the activities of flounders. If the operation is carefully performed the wound heals in a few days and the animals respond normally. They move about and feed apparently with the same degree of freedom as they do with both eyes functional and there is no difference in the extent or the rate of adaptation to the background, neither in color nor in shade nor in pattern.

Specimens with both eyes removed also learn to get around in the aquarium without serious difficulty; but they do not simulate the background, either in shade, color, or pattern.

c. Direction of the light: Specimens on a white background, if they respond at all, become maximum white even if the intensity of the light is so low that they can scarcely be seen and the background appears dark gray to the human eye. On a gray background, however, even if it appears much lighter than the white in low light intensity, they become gray, on a black, black, etc. If however conditions are so arranged that the eyes receive no light direct from above, that is, receive only light reflected from the background, they become maximum white no matter what the shade or pattern of the background may be. This shows that the changes in shade and pattern and probably also color are in some way dependent upon the relation between the light received directly and that received by reflection from the background.

In accord with this, if the light from above is abnormally strong the fish ought to become dark on a white bottom. Apparatus was devised by means of which such illumination could be produced but it was found that when the light from above was abnormally strong the fish did not respond at all. The significance of this negative result is still somewhat problematic.

d. Comparison of the skin with the bottom: In specimens with the ventral eye removed it was possible to keep the anterior part, with the exception of the eye, so thoroughly covered with sand that the fish could not see any of the skin. Under these conditions adaptation occurred just as accurately and rapidly as it did when the skin could be seen. This shows that the skin is not necessarily compared with the bottom by the fish in the process of adaptation.

4. Selection of background. After flounders have been, for some time, on a given background they tend to return to this background when put onto another near it. That is, they tend to select the background on which they are least conspicuous. This was conclusively proved with reference to shade and fairly definitely with reference to color. The response is, however, of such a nature that, under normal conditions, it is probably of little value in concealing the animals; but it does throw some light on the question of vision referred to later.

5. Biological significance of simulation of the background. Some hold that the phenomenon of simulation is purely accidental and that it has no biological value: others maintain that it functions as a protection from enemies or in capturing prey; a few contend that it serves chiefly to regulate the temperature of the body; and some even hold that it functions in all of these ways and in still others. Unfortunately, however, none of these ideas are supported by experimental evidence. In no case has it ever actually been demonstrated that the response in question has any value whatsoever. This statement includes the results obtained in work on the flounders, although the experiments outlined have as yet been only partially completed.

6. Vision. Since the changes in the skin which result in simulation of the background are controlled by stimuli received through the eyes, the nature and accuracy of the process constitutes an excellent criterion of vision in so far as this term may be used in a purely objective sense.

On the basis of this criterion it was found that, in regard to shade and color, vision in fishes is essentially the same as it is in man; but that in regard to size it is less acute. It was found that flounders distinguish between dots 2 mm. and 3 mm. in diameter respectively, that they recognize dots 1 mm. in diameter but that they do not recognize those 0.5 mm. in diameter.

The most interesting and convincing evidence that we have, however, regarding vision, refers to motion. By means of a background consisting of a rotating disk composed of alternate black and white sectors, it was found that the fusion-rate of images in flounders corresponds very closely with that in human beings. This seems to show conclusively that, in regard to motion, vision in fishes is as acute as it is in man.

There has been so much contention concerning color-vision in animals that it may be well to add a few words in explanation of our evidence bearing on it.

On a background containing only gray or black and white no color is produced in flounders regardless of the shade or pattern or the intensity of the light. Simulation in color is consequently dependent upon the length of the waves of light, not upon difference in its intensity. This is one of the essential characteristics of color-vision in man. We have, however, still further evidence that bears on this matter. Flounders adapted to a given color tend to select a background of the same color: and this selection is of such a nature that it cannot be accounted for on the basis of difference in the intensity of the light reflected by the different colors. It must, therefore, be associated with the length of the waves. Thus the contention that fishes have color-vision is supported both by the reactions of the animals and by the reactions of the chromatophores in the skin. This evidence has, of course, no direct bearing whatever on the problem of color-sensation.

This work is a contribution from the U. S. F. Biol. Sta., Beaufort, N. C., U. S. A. Presented in this form by permission of the Commissioner of Fisheries it is to be published in full, with numerous photographs and autochromes, in the *Bulletin of the U. S. Bureau of Fisheries.*

SOME EXPERIMENTS ON SPERMATOGENESIS IN VITRO

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Unforeseen circumstances that forced me to interrupt my normal work offered a chance for trying some experiments, that I have had in mind for some time, to apply the method of tissue culture to the study of sex-cells with the purpose of rendering the history of these cells accessible to an experimental analysis. The results obtained in this direction during the past winter constitute of course nothing more than the first steps into a very promising field of research. For the experiments the sperm-cells of the moth, Samia cecropia L., were used. The method of cultivation is simply Harrison's method. By tearing the testis of the pupa the thousands of sperm follicles (spermatocysts) are isolated and can easily be mounted in a hanging drop of haemolymph. Naturally the whole procedure must be carried out strictly aseptically. In November most of the follicles contain spermatogonia or young spermatocytes during the growth-period. They could be kept alive in the cultures about three weeks, and during this time a great many follicles finished the process of spermatogenesis up to the full grown spermatozoa under the eye of the observer. The time required depends, as was to be expected, upon the temperature. It seems that the stages from the spermatogonia up to the maturation divisions need weeks for their development but maturation and spermatogenesis occur in a few days even in one day when kept warm. The most striking, although in no way surprising result of the observations is the complete harmony between the facts as observed in the living cell and those described from preserved specimens. The formation of the axial filaments in the young spermatocytes and their subsequent behavior, as well as the mitochondria and their distribution during the maturation divisions, could readily be followed. The interesting and hitherto unobserved behavior of the axial filaments in the formation of the regular sperm-bundles could also be made out. In addition interesting observations were made upon the behavior of the follicle membrane after the death of the sperm cells. At this time an extensive outgrowth of these cells begins, forming a kind of tissue which grows luxuriantly for some weeks more.

The observations made upon normal spermatogenesis suggested the hypothesis that the specific changes undergone by a sperm cell in its development into that strange thread-like spermatozoon may be due not so much to internal changes inside the cell as to changes in the physical constitution of the surrounding medium, controlled by the follicle membrane, forcing the cell to react in that specific way. In order to test this hypothesis experiments have been made in influencing the osmotic conditions inside the follicular membrane. One of the clearest results was that hypertonic condition in the haemolymph forces all sex-cells from spermatogonia to spermatids to grow out in one direction, namely, against the follicle wall. The resistance of the latter forces the cell to grow along its inner surface so that finally the follicle appears as a whorl of thread-like cells. The amount of growing out depends upon two factors: the age of the cell and the hypertonicity of the medium. Spermatocytes during or before the maturation divisions form long spermatozoa-like threads but with inverse polarity, the axial filament on the inner surface of the cell (present as it is well known in butterflies from the spermatocyte stage on) not being involved at all. The whole process proved to be reversible without change of the medium, as were also the slight changes produced by a hypotonic medium, since after some days all cells returned to their normal form. This means that the follicle membrane has the faculty of active control of the osmotic conditions inside the follicle. In addition to these experiments an effort was made to induce the cells of different age to grow in the normal direction along the axial filaments. Up to the present only the first steps have been obtained by treatment with diluted methyl alcohol. Another normal process that could be induced at a different point in the cell history was the formation of the axial filament. In these experiments interesting observations have been made upon the formation of flagella by the plasm of all sperm cells from the spermatogonia to ripe spermatozoa. They arise in Ringer's Fluid in a similar way as described for other cells by Merk, Kite, Oliver, Chambers, and they can be produced and made to disappear at will by change of temperature.

All these experiments together with the observations of the normal process point to the probability that the general processes of spermatogenesis are necessary reactions of the cells to a systematic regulation of the osmotic conditions on the part of the follicular membrane. The individual specific processes are caused by the specific properties of the reacting cells. This affords us the possibility of an experimental attack upon other closely related problems, viz., the function of Sertoli's cells, which may prove to be the same as the function of the follicle cells in the present case, and the problem of apyrene spermatozoa, which, according to some of my observations, may prove nothing else than a *lusus naturae*, an abnormality produced by slight changes in the physical properties of the follicle membrane that might readily occur under ordinary conditions. These questions will be discussed in the full account of the work to be published in the *Archiv für Zellforschung.*

GROWTH AND VARIATION IN MAIZE

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The Problem. The investigation reported in this paper is an attempt to analyze the normal variation of an organism in a particular case, from the standpoint of *Entwicklungsmechanik.* It was suggested in the first instance by the study of variation and differentiation in *Ceratophyllum* made by Pearl some years ago. We have here attempted to approach the problem of *inter*-individual variation from the same point of view and with similar methods to those applied to the problem of *intra*-individual variation in the case of *Ceratophyllum.*

The problem and the point of view may be most clearly defined by considering briefly certain well-known, indeed obvious, facts about variation. If one brings together a homogeneous group of individual plants or animals and measures the same character in each individual, there may be formed from the resulting data a characteristic variation curve for that group and character. The precise form of this curve, as well as the location in it of any particular individual, are functions of two basic variables. Of these one is the hereditary or germinal constitution of the individual. The other is the complex of environmental stresses and strains, which, each acting on the individual during its ontogeny have influenced the end result of the activity of the hereditary determiners or genes.

Now it is altogether usual in discussions of variation and heredity to take the two end terms of the ontogenetic series, the gene on the one hand and the adult soma on the other hand, as things given. What comes between the two is neglected. But clearly what goes between is a part of the very essence of heredity itself.

In any group of adult individuals each one will show some particular variation, in the sense of a deviation from the typical condition of the group. We take it to be one of the final objects of investigations in genetics to find out why (in the sense of locating the essential causal factors involved) a particular individual *A* exhibits a particular variation *a*, and not some other variation out of the indefinitely large number of possibilities. By means of Mendelian modes of research considerable progress has been made towards answering this question for many kinds of qualitative variations. For fluctuating or quantitative variations also, progress has been made through the researches especially of East and his associates, following those of Nilsson-Ehle on colors determined by multiple factors.

What we have tried to do in the present investigation is, by studying the growth of the individual, to analyze the adult variation curve into its component elements. We have attempted, in other words, to make a beginning at an understanding of the developmental physiology of the genes concerned in the production of the characters studied.

A given individual, at an early stage of growth, may exhibit an exceptional condition of a character, as compared with other individuals of the same age or stage of growth. It may be, for example, very short, the shortest plant in the whole population at three weeks of age. Will this same individual be the shortest plant in the adult population, after all growth is completed? If not, where is it in the adult variation curve, and how did it get there? This example will give a concrete idea of the general mode of approach to the problem of variation followed by this paper.

Results. Measurements were made at twice-a-week intervals of the height of each individual of three series of plants of a variety of sweet corn used in other experimental work in this laboratory and described in earlier papers. The heights were measured to the tip of the tallest leaf. In addition to these, separate sets of measurements were also made of the tassel height as soon as the tassels appeared. The growth curves obtained by plotting the mean height at each measurement are relatively smooth. After July 3, the time of tasseling, the plants grow in height much faster than before. Growth in height ceases entirely as soon as the tassel blooms.

The relative variability considered for the whole season shows a marked progressive diminution. It thus follows the general growth law of diminishing variability. Considered in detail, however, the relative variability first shows an increase. After June 19 there is a rapid decrease until the time of tasseling. During the period of tasseling there is a very rapid increase in the relative variability. This is followed by an equally rapid decrease. A stable condition is finally reached which is some 10 per cent lower than the variability at the beginning of the season.

The quantitative relations of tasseling to the growth and variability of the height of corn plants are very similar to the quantitative relations of puberty to the growth of children. This does not necessarily mean that the two processes are analogous physiologically, but it is a fact of interest and worth further consideration from a physiological point of view.

It appears probable that the corn plant grows in a series of cycles. Each cycle is characterized by the special development of one set of organs. They are, in order, the root cycle, the leaf cycle, the tassel cycle, and the ear cycle.

With these general facts regarding the growth of the maize plant in hand the analysis of the variation in terms of individuals was undertaken. Individual plants and groups of plants having the same relative size at one stage of their growth, were followed as individuals through the remaining growth stages. We have endeavored to ascertain how such plants are distributed as to relative size in the successive growth stages and to discover the factors determining these distributions.

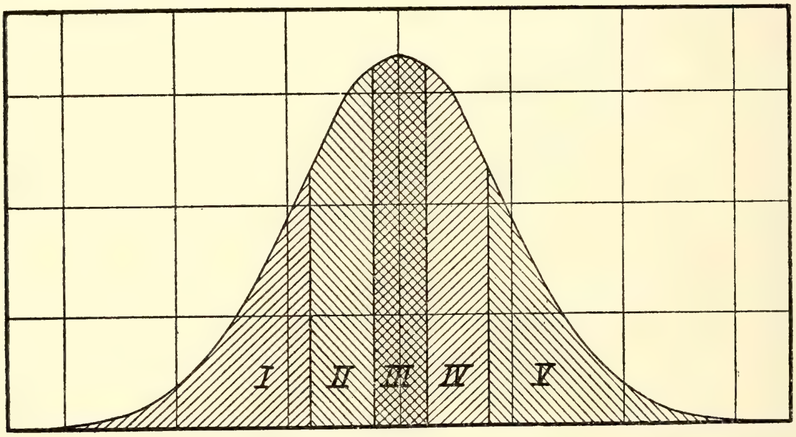

FIG. 1. DIAGRAM TO ILLUSTRATE THE DIVISION OF A FREQUENCY CURVE INTO QUINTILES. IN THIS DIAGRAM A NORMAL CURVE HAS BEEN DIVIDED INTO 5 EQUAL AREAS BY THE FOUR PERPENDICULAR SOLID LINES. THE FREQUENCY IN ANY ONE OF THESE AREAS IS EQUAL TO THAT IN ANY OTHER.

To study these questions it is necessary to have a measure of the *relative* size of the plants at each growth stage. For this, each distribution was divided into five equal areas or quintiles. In any distribution the relatively small plants are in quintile I and the relatively large ones in quintile V. Figure 1 illustrates the exact meaning of a quintile.

The problem was first approached by studying the quintile distribution of all the measurements, throughout the season, of a group of plants starting in a given quintile. It is shown that there is a strong tendency for the plants to remain in or near the quintile in which they started. As a measure of this tendency use is made of the root-mean-square deviation of these observed distributions from the most probable distribution of such measurements on the theory of chance. The tables of these constants are too extended to be given here, but the figures show that in every case (except one) the deviation from the theoretical mean is very much greater than would occur on the basis of chance. The deviation of the very small and the very large plants (quintiles I and V) are much greater than any of the others. Thus there is a much more marked tendency for the extreme plants to remain in the extreme classes than for the medium sized plants to remain in a particular medium sized class. There is a similar, though less marked, tendency for the plants *ending* in a given quintile to have remained in or near that quintile throughout their growth.

The second step in the analysis was to study the *mean quintile position* of each group of plants in the successive growth stages. Here the main conclusion is entirely clear. Extreme variants at the beginning of the season tend strongly, on the whole, to remain extreme variants during the whole season. At the same time such extreme variants regress slightly towards the general population mean as growth continues. This second tendency is, however, by no means so strong as the first.

The third step in the analysis of the variation curves was a study of the average relative size (mean quintile position) of the individual plants and of the variability of individual plants with respect to relative size. Definite quantitative evidence is presented showing that:

1. The observed differences in the manner of growth of individual plants and of groups of plants cannot be explained as the effect of external, environmental factors.
2. These differences are rather to be looked upon as the effect of internal factors.
3. The distribution of the average relative size (mean quintile position) of individual plants is such as to suggest the random distribution of these factors among the plants. The same thing is brought out by the distribution of the relative measurements of plants starting or ending with a given relative size (quintile).
4. The simplest method of explaining these facts is to regard the differences in the manner of growth as due to independent Mendelian factors which are distributed at random in any population of open fertilized maize plants. These factors would occur in the proportions found in a stable Mendelian population mating at random.
5. By assuming the presence of two independent growth factors and weighting each with the proper value, it is possible to obtain a theoretical distribution agreeing very closely with the observed distribution.

6. The interpretation of the growth of these plants by Mendelian factors is further strongly supported by the distribution of the standard deviations of the plants with different relative sizes. Thus it has been shown that the extreme plants which would be more nearly homozygous and for this reason less variable are, as a matter of fact, some 50 per cent less variable than the plants in the middle class after all allowance has been made for the difference in the size of the means.

This is a preliminary abstract of a paper with the same title now in press in the *Zeitschrift für induktive Abstammungs- und Vererbungslehre.*

ON A NEW GROUP OF BACTERICIDAL SUBSTANCES OBTAINED FROM HEXAMETHYLENETETRAMINE

By Walter A. Jacobs and Michael Heidelberger

ROCKEFELLER INSTITUTE FOR MEDICAL RESEARCH, NEW YORK

Presented to the Academy, February 17, 1915

Ever since the introduction of hexamethylenetetramine into medicine, this drug has attracted wide attention owing to its far-reaching possibilities as an internal antiseptic. In addition to its rapid appearance in the urine it is found after ingestion in practically all the body secretions, including even the cerebrospinal fluid. It is fairly well established that the antiseptic action of the drug is due to the liberation from it of formaldehyde by virtue of the acidity of the medium in which it acts, but that its diffusion throughout the body is a physical property of the undecomposed molecule. In other words, hexamethylenetetramine is but a vehicle for formaldehyde, and if the two conditions are satisfied, firstly, its appearance in large enough amount in the locality desired, and secondly, its cleavage to formaldehyde, a local internal antiseptic action might be expected. Owing, among other things, to the alkalinity of most of the body fluids the limitations set by these prerequisites are obvious.

Numerous attempts have already been made to achieve by proper chemical variation an improvement in the action of the drug. Such attempts, however, which have been confined almost entirely to combinations of hexamethylenetetramine with acids and phenolic compounds, have offered few advantages over the original drug. It seemed to us that more promising results might be obtained by a more direct variation of the hexamethylenetetramine molecule itself and a brief outline of the facts observed in the course of this work is the subject of the following:

The molecule of hexamethylenetetramine functions as a tertiary base, and very readily adds organic substances containing aliphatically bound halogen, forming quaternary salts, a few of which are already described in the literature. In the present work over two hundred such preparations were made, and, after chemical characterization, subjected to biological study.

When tested against various strains of bacteria *in vitro* it was found that with but few exceptions these compounds were bactericidal, the bactericidal power varying with the character of the alkyl or aryl group introduced into the hexamethylenetetramine molecule. These substances are, on the whole, easily soluble in water, but on standing or warming the solutions rapidly decompose, and in the case of the aromatic derivatives, develop a very finely divided emulsion of oil droplets or a colloidal suspension which precipitates more or less readily according to the concentration. On investigation, these decomposition products proved to be the methylene derivatives of the aryl or alkyl amines corresponding to the halides originally used. In the case of the hexamethylenetetramine addition products of the halogenacetylamines these precipitates were more closely studied. In several instances the methyleneglycyl derivatives were isolated in pure crystalline form, and upon testing for bactericidal power, the results were found to agree closely with those obtained with the parent quaternary salts. We are, therefore, led to believe that at least in this type of hexamethylenetetramine derivatives, the bactericidal properties of the parent substances are due to the methyleneglycyl derivatives produced on decomposition, or more particularly, to the methyleneamino group contained in these decomposition products. Since all types of hexamethylenetetramine quaternary salts (with the exception of a few of aliphatic nature) seem to undergo this form of decomposition, we are inclined to believe that in general the methyleneamino compounds are responsible for the bactericidal effect. With the confirmation of this we are at present engaged.

The form in which these methyleneamino compounds separate from the solutions of their parent compounds is a contributing factor in the action of these substances. For when once precipitated from suspension, they cannot readily be redissolved unless they contain a solubilizing group. The use of hexamethylenetetramine in this manner would seem, therefore, but a convenient artifice for obtaining these substances in the form of emulsions or colloidal solutions, conditions necessary for their bactericidal action. In the presence of serum or protein this precipitation is in the majority of cases either diminished or prevented entirely. In this connection it may be stated that with a large number of the compounds the bactericidal effect was found to be but slightly inhibited by serum or protein, a property essential for their use as internal antiseptics. The degree of bactericidal effect obtained depends not only upon the chemical nature of the compound but also upon the organism employed for the test. Comparative experiments with different bacterial species and with different varieties of the same species have demonstrated in many instances remarkable specificity. One class of substances, for example, killed *B. typhosus* in a dilution of 1 to 200 in two hours, whereas, in the same time it killed streptococcus in dilutions of 1 to 50,000 or more. The toxicity of the substances varies naturally with the character of the group added to the hexamethylenetetramine molecule.

A further study of the bactericidal properties of methyleneamino compounds in general is now in progress. A detailed description of the chemical and bacteriological work will follow in the appropriate journals.

CHEMICAL AND PHYSIOLOGICAL STUDIES OF A MAN FASTING THIRTY-ONE DAYS

By Francis G. Benedict

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Presented to the Academy, March 9, 1915

The important relationship between numerous diseases and the various stages of nutrition and the fact that many pathological cases border on complete or nearly complete inanition make a study of the physiology and chemistry of normal fasting of special importance to the physiologist and the clinician alike. It is rarely practicable to make exhaustive and simultaneous observations in several physiological and chemical fields of study with a subject undergoing a prolonged fast but the arrival in Boston in 1912 of a Maltese, A. Levanzin, who wished particularly to fast thirty-one days under strict scientific observation, presented such an opportunity. It was thus possible to supplement earlier data secured with men during relatively short periods of fasting (2 to 7 days) in the chemical laboratory of Wesleyan University. (Benedict, Carnegie Institution of Washington Publication No. 77, 1907.)

In accordance with a previously arranged program and in coöperation not only with the entire scientific and computing staff of the Nutrition Laboratory but with several members of the faculty of the Harvard Medical School, an extensive study of this man was made. The experiment began April 14, 1912, with a preliminary period of three days with food, followed by thirty-one days of complete abstinence from food and drink other than distilled water, and finally a three-day realimentation period. In all save the last period the experiment was most successful.

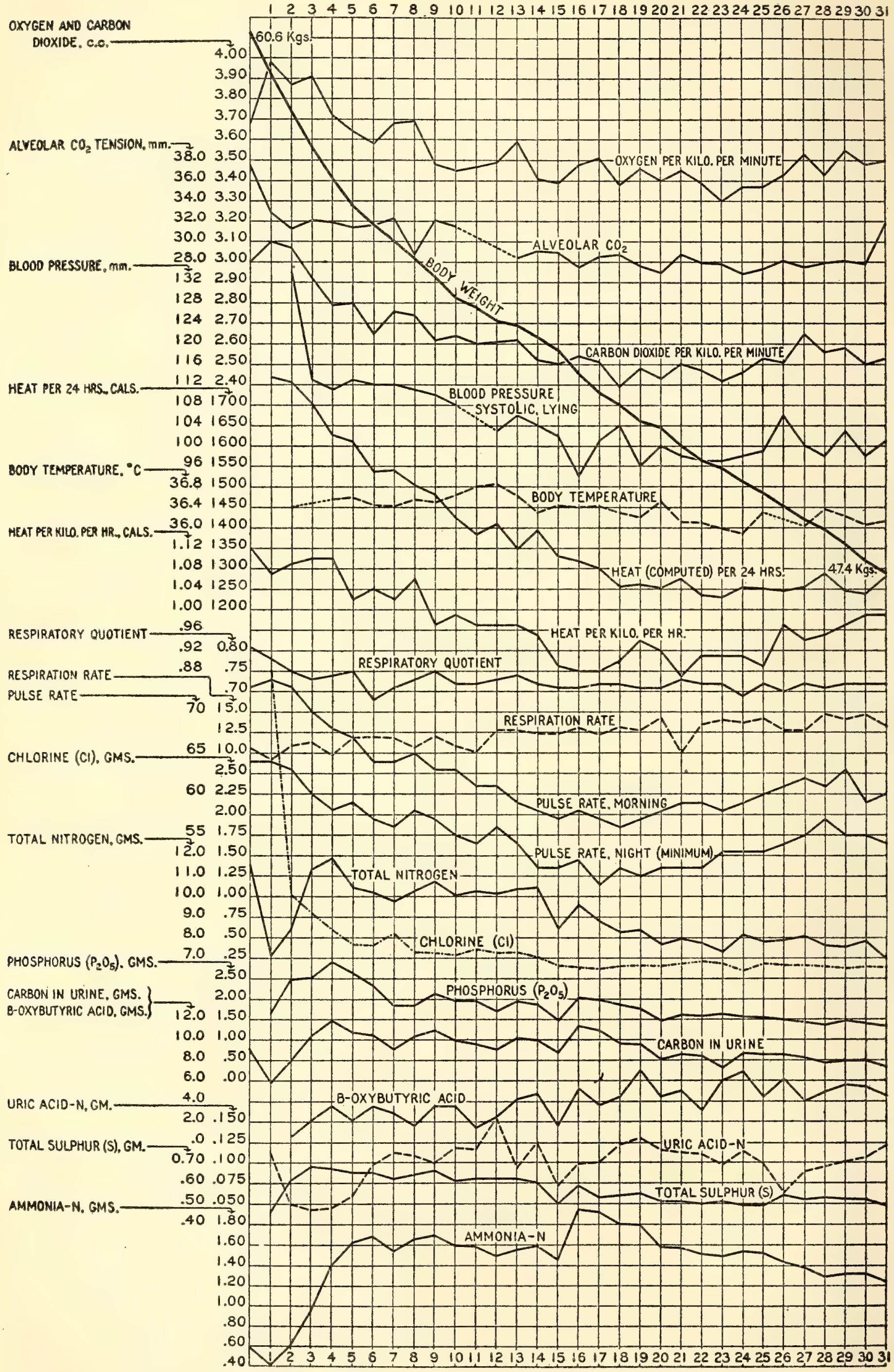
Measurements were made of the body-weight, insensible perspiration, rectal temperature fluctuations, pulse-rate, blood pressure, and the morphology of the blood. The mechanics of respiration and the alveolar air tension were also observed. A complete clinical examination was made every second day with a record of subjective impressions and psycho-physiological observations. The intestinal flora were studied and the skin excretion determined. A complete chemical examination was also made of the urine, including the partition of the nitrogen and the determinations of the chlorine, phosphorus, sulphur, total acidity, and B-oxybutyric acid. The mineral constituents, particularly the calcium, magnesium, sodium, and potassium excretion, were studied and the reducing power, the carbon, and the energy of the urine were determined. Throughout the fast the urine was examined daily with a microscope and tested for albumin. Of special significance was the study of the respiratory exchange, both with the universal respiration apparatus and with the large bed respiration calorimeter.

In the respiration calorimeter it was possible to measure simultaneously the carbon-dioxide excretion, the oxygen consumption, and the water vaporized from the lungs and skin. Direct heat measurements were likewise made which, when corrected for body temperature fluctuations, gave the exact heat production. From the results obtained with the respiration apparatus the effect of various factors upon the metabolism was studied, such as changes in the body position, the work of writing, and breathing oxygen-rich atmospheres. The metabolism of the subject during sleep was also compared with his metabolism in the waking condition. The energy transformation per kilogram of body weight and per square meter of body surface was computed. From the numerous respiration experiments with both apparatus and a careful record of daily activity, the total balance of income and outgo for this man for the thirty-one days could be computed with great accuracy.

Of special clinical importance are the records of the losses in weight; the studies of the rectal temperature, blood pressure and blood; the acidosis induced by the fast; and the effect of the fasting upon the composition of the urine and the respiratory exchange. The psychological studies and the general observations of the man showed that for thirty-one days the subject was able to exist in a fairly normal mental condition, entirely out of proportion to the physical decline in the body functions. The ill-advised insistence of the subject led to his breaking the fast by

METABOLISM CHART OF A MAN FASTING 31 DAYS

APRIL 14 - MAY 15, 1912



the selection of excessive amounts of acid fruit juices, thus inducing a most distressing colic and intestinal disorder; a short sojourn at a hospital was followed by convalescence.

It is impossible in the space available in these Proceedings to attempt an adequate summary of the results but the accompanying chart indicates the trend of some of the most important factors measured. The detailed results of the research are incorporated in a 416-page monograph issued as Publication No. 203 of the Carnegie Institution of Washington.

THE STUDY OF INDIAN MUSIC

By Alice C. Fletcher

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Presented to the Academy, March 2, 1915

Thirty-five years of acquaintance with Indian music gathered from tribes of different linguistic families widely scattered over North America and a study, still in progress, of the music of a particular group has revealed facts relative to their music and its uses that possess an anthropologic value.

The term music as applied to the Indian refers solely to vocal music; for the natives of America possess few varieties of musical instruments beside the drum and rattle, both of which are used mainly to accent time and rhythm.

The number of Indian songs which have come to my personal attention number many hundreds. Those to which intensive study is being given have been secured from people classed as Plains Indians. All of these songs have been examined under their native conditions and in connection with the ceremonies, secular or religious, of which they were a part. When I began to observe and gather Indian songs, the graphophone was not available for field work. Securing songs by dictation was a difficult and unsatisfactory task for the reason that the Indians so frequently were averse to repeating the songs under observation, particularly when they were religious in character. This obstacle has been entirely overcome by the use of the graphophone, as one singing will give a record that can be repeated any number of times for the purposes of transcription, verification, and other study. For over twenty-five years I have used the graphophone when engaged in the field study of native ceremonies containing rituals and songs.

The word 'song' to our ears, suggests words arranged in metrical form and adapted to be 'set to music,' as we say. The native word which is translated 'song' does not suggest any use of words. To the Indian, the music is of primal importance, words may or may not accompany the music. When words are used in a song, they are rarely employed as in a narrative, the sentences are not apt to be complete. In songs belonging to a religious ceremony the words are few and partake of a mnemonic character. They may refer to some symbol, may suggest the conception or the teaching the symbol stands for, rarely more than that. Vocables are frequently added to the word or words to eke out the musical measure. It sometimes happens that a song has no words at all, only vocables are used to float the voice. Whether vocables alone are used or used in connection with words, they are never a random collection of syllables. An examination of hundreds of songs shows, that the vocables used fall into classes; one class is used for songs denoting action, another class for songs of a contemplative character, and it is also noted that when once vocables are adapted to a song they are never changed but are treated as if they were actual words.

As Indian music is exclusively vocal, its range is confined to the compass of the human voice, which rarely exceeds three full octaves. This limited range, taken with its strictly vocal character, places Indian music in a class apart from the 'culture music' of our race. The remarkable development of 'culture music' has been due, in a large measure, to the invention and use of musical instruments. By means of these instruments the range of musical sounds has been increased far beyond that possible to the human voice and different qualities of tone have been secured. As a result, 'culture music' has been able to use diversity in musical sounds, to employ various melodic and harmonic forms and to acquire an objective and intellectual character quite impossible to the simple song from which it sprang. By the study of Indian music it is possible to retrace some of the steps that have led from song to 'culture music.' In pursuing this task, an important and helpful factor is found in certain conditions that have here obtained, namely: the natives of this continent, previous to the coming of our race, had not been subjected to inroads from alien peoples who might have disturbed the continuity of aboriginal culture; consequently the music and the life of the Indians can be observed in a simple, rather than a 'compound' environment.

Indian music presents several aspects to the student, all of which fall into two classes; the one technic, the other psychic in nature. The technic class has already received considerable attention from scholars, as to the tones used, the order in which they occur, the relation of this native order to the formulated scales of 'culture music,' native musical form, melody, rhythm and the interrelation between different rhythms maintained by the voice and the drum or rattle.

From a present study of some of the psychic aspect of Indian music I have already reached certain conclusions, some of which are here presented.

Among the group of Indians whose music is under observation, the following customs, observed in some of the tribes, illustrate something of the fundamental character of the personal and emotional elements found in Indian song.

When at puberty the youth passes through the fasting rites introductory to the duties pertaining to manhood, 'the vision' that then appears to him, becomes his most sacred, personal experience, one that he never fully shares with another, and the cadence, or song, that generally accompanies the 'vision' remains in his memory and is to the man his secret personal appeal to the unseen powers he believes to have control over his life. This strain of music, constitutes his personal prayer, his cry for help in his hour of need. These 'songs' are strictly individual and emotional. They are rarely, if ever, heard by anyone save the singer.

Again: Certain societies require that each member have a special song, this song is generally of the man's own composition, although sometimes these songs are inherited from a father or a near relative who when living had been a member of the society. These individual songs are distinct from songs used in the ceremonies and regarded as the property of the society, although the members are entitled to sing them on certain occasions. When this society holds its formal meetings a part of the closing exercises consist of the simultaneous singing by all the members present of their individual songs. The result is most distressing to a listener, but there are no listeners, unless by chance an outsider is present, for each singer is absorbed in voicing his own special song which is strictly his personal affair, so that he pays no attention to his neighbour, consequently the pandemonium to which he contributes does not exist for him. Another phase of Indian song is here exhibited, but it is more directly connected with the Indian's manner of singing than with the music itself. This manner has tended to influence the estimation of outsiders of Indian song.

In a general way, an Indian singer makes no special effort, nor is he much concerned, to present his song in such a way as to give to the listener a musical picture. Practically no attempt is made to give what we call 'expression.' The song is apt to move along in strict time; any change or break in the time or rhythm disrupts the flow of the music and, to the Indian, destroys the symmetry of the song. The Indian's manner of singing, his method of rendering his song is the outcome of the close emotional relation between the singer and the music. To a degree, it may be said that the Indian does not listen to his song as something objective to himself, yet he is found to be keenly aware of any slight change in the rhythm or tones of a melody—any mistake in the rendition of a song. In some societies a fine is imposed upon a member who makes mistakes in singing the songs. Years of experience has shown an unfailing demand for accuracy in the transcription and reproduction of a song. This demand betrays a consciousness and a recognition on the part of the Indian of musical form, both as to rhythm and the succession of tones that form a melody. In this consciousness is discerned one of the very early steps in the long path that leads to an artistic development of music.

Accuracy in the observance of religious rites arises from certain beliefs concerning the ability of these rites to open a way to the unseen powers, mistakes 'make the path crooked.' Indian ceremonies are ritualistic and symbolic objects are generally present. These objects have to be handled in peculiar and definite ways, with certain movements of the body and hands and accompanied by long recitations either chanted or intoned. Mistakes must be avoided as any carelessness is believed to be punished by supernatural means, therefore various mnemonic devices are employed by those who officiate, to insure against mistakes. In some tribes, if a mistake chances to be made in certain ceremonies, everything stops at the instant it is discovered. When the ceremony is resumed, it must start afresh, as though it had never been begun. Ceremonies that have elaborate rituals are generally attended only by the initiated and the rituals recited are known only to those who have paid the costly fees required from those who seek to be instructed in their use. In some ceremonies where lengthy intoned recitations occur these rituals are broken in upon by songs, sung by those officiating. These interpolated songs partake of the general character of Indian music and are emotional in their nature. The few words usually refer to the symbolic objects belonging to the ceremony,—the mind is thus directed toward the sacred object and the emotion evoked by the thought of it and its helpful relation to the people finds expression in the music. An old priest explained this introduction of songs into the long rituals, by saying: "The words talk to us, but harmonious sounds unite the people!" This explanation given by the old priest is confirmed by observations made as to the effect of songs pertaining to ceremonies having for their purpose the stimulation, among the people taking part, of a common recognition, as of the gift of food, the value of tribal unity, of peace within or without the tribe. Many of the songs belonging to this class of ceremonies, although led by those who officiate in the rites, are sung by all present, men, women, and even children joining in these songs. The singing is always in unison, the personal emotion of each singer adds to the psychic force bred of a social accord, as all the people in one voice sing their thanks to the Unseen Giver of Life.

On a similar but lower plane are the songs belonging to vocations, by these not only are the hands strengthened for their task, but the unison singing of the song helps to development of social consciousness.

From what has been stated above it will be apparent that a wide field is opened by the study of Indian music not only concerning the beginnings of 'culture music' but in revealing the psychic influence of song upon the individual and upon his social development.

SOME RECENT ANTHROPOLOGICAL EXPLORATIONS

By Aleš Hrdlička

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In 1912 an arrangement was entered into by the Smithsonian Institution and the Panama-California Exposition at San Diego, by which it became possible to send out jointly a number of anthropological expeditions for the purpose of furnishing the Exposition with original material for its exhibits, and of serving the Smithsonian Institution and the science of anthropology by advancing knowledge in directions in which progress was especially desirable.

The objects particularly aimed at in the latter respect were the promotion of research into man's antiquity; the survey of certain parts of northern and eastern Asia in quest of possible traces of racial connection with America; and the accumulation of reliable data on child development in certain primitive races under widely differing environments. Owing to illness in the personnel of the expeditions, to the European war, and other untoward circumstances, only a part of these plans could be fully carried out; but the results are of sufficient importance to encourage further activity in the directions named.

The several expeditions and their results, briefly outlined, were as follows:

Ancient Man in Europe and Asia. Field work was done by J. Matiegka, head of the Anthropological Bureau of the Bohemian University at Prague, on the subject of neolithic and later prehistoric man in central Europe. Other explorations were carried on by K. Stolyhwo, head of the Anthropological Institute of Warsaw in relation to: (*a*) ancient man of southwestern Russia; (*b*) ancient man of the Yenesei Valley, Siberia. And a journey was made by me for the purpose of making a survey of the various well authenticated ancient skeletal remains of man in the Museums of Europe and of the sites of their discovery.

The direct results of these expeditions were the acquisition of a series of valuable specimens, including a trephined and mineralized neolithic skull in excellent state of preservation, and of valuable data which in part have already been published[1] and in part are in preparation for publication.

The kourgans or burial mounds in northeastern Russia were found to range in age from the neolithic to the Scythic periods. They yielded some exceedingly interesting skeletal material of man as well as that of various animals, some of which are now extinct in these regions. The Yenisei caves, regrettably, gave few skeletal remains, though such could doubtless be obtained by further exploration; but they yielded objects of neolithic culture, which may be of value in future archeological comparisons with the more eastern portions of Asia and possibly even with America. The examination of the ancient skeletal finds in Europe showed, besides a series of more or less doubtful specimens, a wealth of thoroughly authenticated material relating to man's descent, and substantiating his gradual evolution, during the Quarternary, from lower primate forms.

Search in Asia for Traces of the Race that Peopled America. Besides the explorations in the Yenisei caves, above mentioned, two trips were made to northeastern Asia. The first of these, made by me, extended to the upper Yenisei and Selenga river territories and to outer Mongolia, while the second, by St. Poniatowski, head of the recently established Ethnological Institute at Warsaw, penetrated to the living tribes in the Primorskaia Oblast, Siberia. The results of these expeditions comprise anthropological data, a valuable series of photographs, and a collection of rare skeletal material. The evidence indicates that there still exist throughout extensive areas of northern and eastern Asia remnants of former prehistoric (rather than 'paleoasiatic') peoples closely resembling the American Indian in type, mentality and other important particulars. In southern Siberia, Mongolia, and the regions to the eastward, there are also extensive archeological remains in the forms of mounds and occupation sites, ranging in chronology from neolithic to historic times. The exploration of these has as yet scarcely been commenced, and may be expected, when properly entered upon, to reveal details of the greatest consequence to American anthropology, for here somewhere must lie the remains, cultural and skeletal, of the ancestors of those who peopled the American continent.[2]

Comparative Study of Child Growth among Primitive Tribes. It was planned to conduct researches in this especially important field among the aboriginal Australians, the Negrito, the African Pygmies, the Zulu, the Eskimo, and the Chinese; but illness of the available explorers, the war in Europe, and other unforeseen conditions interfered with the proposed excursions to the Australians, Pygmies, and Chinese. The Negrito of the Philippines were studied by P. Newton, demonstrator in Anatomy of Georgetown University; and four hundred individuals, including many children, were measured. Particular care was taken to reach the full-bloods, which involved some perilous journeys. A series of photographs and some interesting skeletal remains were obtained, but casting became impossible, the dampness of the climate (the journey had to be undertaken partly during the rainy season) being such that no plaster could be preserved in dry condition. The Eskimo of St. Lawrence Island, Alaska, were examined by Riley D. Moore, aid in the Division of Physical Anthropology of the National Museum; 180 individuals, including a good proportion of children and adolescents, were measured; 30 facial casts and numerous photographs were made; and an important collection of skeletal material was gathered. The data secured and the collections from this island are of particular interest, as they relate to a group of people intermediary between the American and Asiatic Eskimo. Finally, an expedition, occupying fourteen months, was made by V. Shuck, an able anthropologist of Prague, to the Zulu, the Bushmen, and some British East Africa negroes, resulting in the acquirement of facial casts of 24 Zulus and 20 Bushmen, numerous photographs and other material, and anthropometric observations on upward of 800 children and adolescents of known age. The scientific data have already been partly utilized in charts for the Exposition, and are to be reported on as soon as conditions in Europe permit. They should form an excellent basis for eventual comparisons with observations of similar nature on the American negro, and also serve, of course, for the study of further contrasts or resemblances between the black and the white man. Duplicates of all the African measurements are preserved in the National Museum.

In addition to the above, the first four months of 1913 were spent by me on an expedition to Peru, the objects of which were to conduct a greatly needed extension of former anthropological explorations in that country, and to obtain a collection of material for illustrating prehistoric aboriginal pathology and surgery. Considerable success was met with in both directions. The anthropology of the coast was mapped out for the distance of approximately 600 miles, and some insight was obtained into that of the highlands. It was ascertained that important separate political and cultural coastal groups, such as the Chimu and the Nasca people, were no special units, anthropologically, but belonged to the same physical type as the rest of the coast population. The collections made on this trip, being selections from nearly 5000 burials, are especially valuable. Finally, the exploration made possible rich original exhibits at San Diego, covering practically the whole field of pre-Columbian Indian pathology, to which are added 60 crania showing all the forms of ancient Indian trephining. The general results of this expedition have already been published,[3] but the material collected offers a rich opportunity for further investigation.

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[1] Hrdlička, A., The most ancient skeletal remains of man, *Smithsonian Rept.* for 1914, pp. 491–552, pls. i–xli.

[2] For preliminary reports on this work, see *Smithsonian Inst., Misc. Collect.*, 60 (1912); *Compte-Rendu XIV Cong. Intern. d'Anthropologie et d'Archéologie Préhist.*, Genève, 1913; and *Trans. XVIII Intern. Cong. Americanists*, London, 1914.

[3] Hrdlička, A., Anthropological work in Peru in 1913, with notes on the pathology of the ancient Peruvians. *Smithsonian Inst., Misc. Collect.*, 61, no. 18 (Publication 2264), 1914, pp. i–v, 1–69, 26 pls.

THE SECOND DERIVATIVES OF THE EXTREMAL INTEGRAL FOR A GENERAL CLASS OF PROBLEMS OF THE CALCULUS OF VARIATIONS

By Arnold Dresden

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF WISCONSIN

Presented to the Academy, January 18, 1915

1. In an earlier paper,[1] I obtained expressions for the second derivatives of the extremal-integral for the problem of minimizing the integral

$$\int F\,(x, y, x', y')dt, \tag{1}$$

in terms of fundamental solutions of the Weierstrass form of Jacobi's differential equation for that problem.[2] In the same paper these expressions were used for deriving necessary conditions which must be satisfied by a curve which is to minimize the integral (1), if one or both endpoints are allowed to vary along a curve,[3] or if curves are admitted whose slopes possess a finite number of finite discontinuities.[4]

2. The method of differentiation employed in arriving at these re-

sults had been used before, probably first by Poisson or by Jacobi,[5] in the treatment of the integral (1) and also of more general integrals. These uses had been restricted however either to the derivation of conditions of the first order, or else to the study of special problems of variable endpoints.[6] Now it is evident that the number of special problems of this character increases rapidly as one proceeds to consider problems involving more and more unknown functions. To secure a general mode of treatment for all these special cases, it has seemed desirable to secure formulas for the second derivatives of the extremal-integral in more general problems than the one treated in my paper cited above.

3. The present paper derives such formulae for the integrals

$$\int f(x, y_1, \cdots, y_n; y_1' \cdots, y_n')\,dx, \quad \text{where } y' = dy/dx, \tag{2}$$

and

$$\int F(y_1, \cdots, y_n; y_1', \cdots, y_n')\,dt, \quad \text{where } y' = dy/dt. \tag{3}$$

In the case of the integral (2) the classical theory suffices to carry the work through and one secures expressions for the second derivatives of the extremal-integral in terms of two sets of conjugate solutions of the self-adjoint system of Jacobi differential equations:

Denoting by u_{jk} and v_{jk} systems of solutions of Jacobi's differential equations, which satisfy the initial conditions

$$u_{jk}(x_1) = \delta_{jk},\ u_{jk}(x_2) = 0;\ v_{jk}(x_1) = 0,\ v_{jk}(x_2) = \delta_{jk},$$

and using the current abbreviated notation:

$$f_0 = \frac{\partial f}{\partial x}, f_{n+i} = \frac{\partial f}{\partial y'_i}, \text{ etc.; } f_0{}^{(r)} = f^0(x_r), \text{ etc.; } y'_{ri} = y'_i(x_r) = \frac{d}{dx}\, y_i(x)\big|_{x=x_r},$$

we have

$$\frac{\partial^2 I}{\partial x_1\,\partial x_2} = -\sum_{ijk} f^{(1)}_{n+i,\,n+j}\, y'_{1i} y'_{2k} v'_{kj}(x_1) = \sum_{ijk} f^{(2)}_{n+i,\,n+j}\, y'_{2i} y'_{1k} u'_{kj}(x_2),$$

$$\frac{\partial^2 I}{\partial x_r^2} = (-1)^r [f_0^{(r)} - \sum_i f^{(r)}_{0,\,n+i}\, y'_{ri} - \sum_{ij} f^{(r)}_{n+i,\,n+j}\, y'_{ri} y''_{rj} + \\ \sum_{ijk} f^{(r)}_{n+i,\,n+j}\, y'_{ri} y'_{rk} [\delta_{r1} u'_{kj}(x_r) + \delta_{r2} v'_{kj}(x_r)]],$$

$$\frac{\partial^2 I}{\partial x_r\,\partial y_{ri}} = (-1)^r [f_i^{(r)} - \sum_j f^{(r)}_{n+j,\,i}\, y'_{rj} - \sum_{jk} f^{(r)}_{n+j,\,n+k}\, y'_{rj} [\delta_{r1} u'_{ik}(x_r) + \delta_{r2} v'_{ik}(x_r)]] \\ = (-1)^r [f^{(r)}_{0,\,n+i} + \sum_j f^{(r)}_{n+i,\,n+j}\, y''_{rj} - \sum_{jk} f^{(r)}_{n+i,\,n+j}\, y'_{rk} [\delta_{r1} u'_{kj}(x_r) + \\ \delta_{r2} v'_{kj}(x_r)]],$$

$$\frac{\partial^2 I}{\partial x_r\,\partial y_{si}} = (-1)^{r-1} \sum_{jk} f^{(r)}_{n+j,\,n+k}\, y'_{rj} [\delta_{r1} v'_{ik}(x_r) + \delta_{r2} u'_{ik}(x_r)] \\ = (-1)^r \sum_{jk} f^{(s)}_{n+i,\,n+j}\, y'_{rk} [\delta_{r1} u'_{kj}(x_s) + \delta_{r2} v'_{kj}(x_s)],$$

$$\frac{\partial^2 I}{\partial y_{ri}\,\partial y_{rj}} = (-1)^r [f^{(r)}_{n+i,j} + \sum_k f^{(r)}_{n+i,\,n+k} [\delta_{r1} u'_{jk}(x_r) + \delta_{r2} v'_{jk}(x_r)]],$$

$$\frac{\partial^2 I}{\partial y_{ri}\,\partial y_{sj}} = (-1)^r \sum_k f^{(r)}_{n+i,\,n+k} [\delta_{r1} v'_{jk}(x_r) + \delta_{r2} u'_{jk}(x_r)],$$

where $i, j, k = 1, \ldots, n$; $r = 1, 2$; $r + s = 3$; and $\delta_{jk} \begin{cases} = 1 \text{ when } j = k \\ = 0 \text{ when } j \neq k. \end{cases}$

4. For the integral (3), there is not in the literature a theory of the second variation and of Jacobi's equation analogous to Weierstrass's work for the integral (1). As an aid towards securing such a theory, the author established conditions under which it is possible to reduce a set of n linearly independent linear differential forms in $n + 1$ functions to a set of n such forms of the same order in n linear combinations of these $n + 1$ functions.[7] This reduction is now used to carry through a reduction of the second variation and of Jacobi's differential equations for the integral (3), analogous to Weierstrass's reduction for the integral (1). This reduction is dependent upon the non-vanishing of one of the functions y'_i. In every regular problem one is assured that there is at every point of the interval $(t_1 t_2)$ a function y_i, whose derivative does not vanish at the point, but the function may be a different one at different points. By the use of the Heine-Borel theorem, we can then, in virtue of the continuity conditions on the functions y_i, divide the interval $(t_1 t_2)$ in a finite number of intervals in each of which the reduction is possible. We consider now the system of reduced Jacobi equations which exists in the neighborhood of the point $t = t_0$ and express the second derivatives of our extremal-integral in terms of sets of fundamental solutions of these equations. These solutions are known to exist throughout the entire interval $(t_1 t_2)$, even though the equations which they satisfy may not be valid beyond a sub-interval. In this manner a set of formulae is secured, which may be immediately applied for finding analytical conditions for a minimum of the integral (3), when one of both endpoints are allowed to vary in manifolds of $1, 2 \ldots, n - 1$ dimensions.

It is intended further to develop similar formulas for the problems of minimizing integrals (2) and (3) if the unknown functions are further conditioned so as to satisfy a system of differential equations, or a system consisting of differential and of algebraic equations.

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[1] A. Dresden, *Trans. Amer. Math. Soc.*, **9**, 467 (1908).

[2] See Bolza, *Vorlesungen über Variationsrechnung*, p. 233 and p. 312.

[3] Ibid., p. 316–318 and p. 328–330. These conditions had been known before.

[4] Ibid., p. 372–389.

[5] See Dienger, *Grundriss der Variationsrechnung*, p. 27.

[6] See Mayer, *Leipzig, Ber. Ges. Wiss.*, **36**, 99 (1884) and **48**, 436 (1896); Bliss, *Math. Ann., Leipzig*, **58**, 70 (1903); Erdmann, *Zs. Math., Leipzig*, **23**, 364 (1878).

[7] "On the reduction of a system of linear differential forms of any order," *Annals Mathematics*, **13**, 149 (1912). In a paper entitled On the second variation, Jacobi's equation and Jacobi's theorem, *Ibid.*, **15**, 78 (1913), this reduction was used to unify the Weierstrass theory of the second variation and Jacobi's equation.

GROUPS POSSESSING AT LEAST ONE SET OF INDEPENDENT GENERATORS COMPOSED OF AS MANY OPERATORS AS THERE ARE PRIME FACTORS IN THE ORDER OF THE GROUP

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If $S_1, S_2, \ldots, S_\lambda$ represent a set of operators of the group G such that these λ operators generate G but that no $\lambda-1$ of them generate G, then these λ operators are called a set of independent generators of G. Every subset of a set of independent generators of any group whatever generates a subgroup whose order contains at least as many prime factors as the number of operators in this subset. In particular the number of operators in a set of independent generators never exceeds the number of prime factors in the order of the group except in the trivial case when the group is the identity, unity not being regarded as a prime factor.

If G represents the abelian group of order, p^m and of type $(1, 1, 1, \ldots)$, it is evident that each of its possible sets of independent generators involves as many operators as there are prime factors in the order of G. In what follows we propose to determine some properties possessed by all those groups which have at least one set of independent generators composed of as many operators as there are prime factors in the order of the group. The symbol G will hereafter in the present article be used to represent any one of these groups, and we shall assume in what follows that $S_1, S_2, \ldots, S_\lambda$ always represent a set of independent generators of G such that λ is equal to the number of the prime factors of the order g of G.

Each of the operators $S_1, S_2, \ldots, S_\lambda$ must be of prime order since each subset of these independent generators generates a subgroup whose order has exactly as many prime factors as the number of operators in this subset. A necessary and sufficient condition that the groups generated by two such subsets have only the identity in common is that these two subsets have no common operator. If two of the operators $S_1, S_2, \ldots, S_\lambda$ are of different orders, the larger operator is transformed into a power of itself by the smaller, since these two operators generate a group of order pq, p and q being distinct prime numbers. In particular, if the operators $S_1, S_2, \ldots, S_\lambda$ are not all of the same order those of highest order generate an invariant subgroup of G. If these orders represent more than two prime numbers, the operators of highest order together with those of the next lower order generate an invariant subgroup of G, etc.

If the set of independent generators $S_1, S_2, \ldots, S_\lambda$ contains at least two non-commutating operators of the same order p, let S_1, represent one of these operators which is commutative with the smallest number of the other operators of order p in the given set; and let $S_2, \ldots, S_\gamma$ represent all the operators of order p in the set $S_1, S_2, \ldots, S_\lambda$, which are not commutative with S_1. We proceed to determine the group generater by $S_1, S_2, \ldots, S_\gamma$. Since S_1 and S_2 are non-commutative they generate a group of order p_1p, p_1 being a prime $>p$, which contains p_1 conjugate subgroups of order p. When $\gamma > 2$, S_1 and S_3 will generate a similar group of order p_2p. It may be assumed that $p_1 \geqq p_2$.

The group generated by S_1, S_2 and S_3 is of order p_1p_2p. If $p_1 = p_2$ this group contains a single subgroup of order p_1^2, and each of its remaining operators is of order p since it contains no invariant operator besides the identity. If $p_1 > p_2$, the group in question contains a cyclic subgroup of order p_1p_2 since the quotient group with respect to the invariant subgroup of order p_1, cannot be cyclic. (See O. Hölder, *Göttingen Nachrichten*, 1895, p. 298.) Hence the group generated by S_1, S_2, and S_3 must always contain an abelian subgroup of index p whose order is prime to p, and each of its operators which is not contained in this subgroup must be of order p.

When $\gamma > 3$, we may consider separately the groups generated by S_1, S_2, S_4 and by S_1, S_3, S_4. The operators of order p_3 in the group of order p_3p generated by S_1 and S_4 must therefore be commutative with the operators of orders p_1 and p_2 contained in the group generated by S_1, S_2 and S_3. Hence it results that S_1, S_2, S_3 and S_4 must generate a group having a single abelian subgroup of index p while all its remaining operators are of order p. As this mode of reasoning may clearly be continued until all the operators of the set $S_1, S_2, \ldots, S_\gamma$ have been exhausted, it results that these operators generate a group H having an abelian subgroup of index p whose order is prime to p, and that each of the other operators of H is of order p. In other words, H involves no invariant operator besides the identity, and the prime factors of the order of its subgroup of index p are all of the form $1 + kp$.

Since S_1 was so selected as to be commutative with the smallest possible number of operators of order p in the set $S_1, S_2, \ldots, S_\lambda$ it results that each of the operators $S_1, S_2, \ldots, S_\gamma$ is commutative with every other operator of order p in this set. Each of the operators of H whose order exceeds p must be commutative with every one of the independent generators $S_{\gamma+1}, \ldots, S_\lambda$. In fact, if the order of such an independent generator exceeds p, it must be transformed into a power of itself by all the operators of H, since it is transformed into a power of itself by each of the operators $S_1, S_2, \ldots, S_\gamma$, which generate H. It results therefore from the theorem of Hölder, to which we referred above, that the independent generator in question is commutative with every operator of H whose order exceeds p.

On the other hand, every operator of the set $S_1, S_2, \ldots, S_\lambda$ whose order is less than p, must transform each of the operators $S_1, \ldots, S_\gamma$ into a power of itself. This must be the first power for all of these operators. In fact, if S_α, $1 \overline{\gtrless} \alpha \overline{\gtrless} \gamma$, were not transformed into its first power by such an operator S_β then the group generated by S_α and some other operator of the set $S_1, \ldots, S_\gamma$ would be transformed into itself by S_β. This is clearly impossible in view of the Hölder theorem to which we referred above. Hence it results that every operator of H is commutative with every operator of the group generated by those operators of the set $S_{\gamma+1}, \ldots, S_\lambda$ whose orders do not exceed p.

Some of these results may be expressed in the form of a theorem as follows: *If a set of independent generators, which involves as many operators as there are prime factors in the order of G, includes at least two non-commutative operators of the same order p then all the operators of order p in this set, which are non-commutative with some one of them, generate a group H involving an abelian invariant subgroup of index p under H, and every operator of this invariant subgroup is commutative with every operator of the group generated by all the operators of the given set of independent generators of G, which are not contained in H.* It results also directly from the preceding developments that if H does not include all the operators of order p in the given set of independent generators of G, then these remaining operators of order p may be combined into subsets such that all the operators of each subset generate either an abelian group of order p^m and of type $(1, 1, 1, \ldots)$, or a group having the properties which we proved to apply to H. Hence the theorem.

If a group G contains a set of independent generators composed of as many operators as there are prime factors in the order of G then all the operators of the same order p in this set generate one of the following three groups: (1) *Abelian groups of order p^m and of type* $(1, 1, 1, \ldots)$; (2) *Non-abelian group having an abelian invariant subgroup of index p but* *of order prime to p, while all its other operators are of order p;* (3) *Group which is the direct product of such groups as are defined in* (1) *and* (2).

When no operator in the given set of independent generators of G has an order which exceeds p, it results that the group generated by all the operators of order p in this set is invariant under G. When this set contains non-commutative operators of order p all of these operators which are non-commutative with some one of them generate an invariant subgroup, composed of operators which are commutative with every operator in the group generated by all the operators of the set except those used to generate this invariant subgroup. As an important but very special result we may mention the theorem that *every group which contains a set of independent generators composed of as many operators as there are prime factors in the order of the group is solvable.*

Since each operator in the abelian invariant subgroup of H is commutative with each of the operators of the set $S_1, S_2, \ldots, S_\lambda$, except those which are contained in H, it is easy to see that every Sylow subgroup of G is abelian and of type $(1, 1, 1, \ldots)$. In fact, if all the operators of the same order in the set $S_1, S_2, \ldots, S_\lambda$ are commutative the various subsets composed of the operators of the same order in this set would generate such Sylow subgroups of G. If not all the operators of the same order in $S_1, S_2, \ldots, S_\lambda$ are commutative then these non-commutative operators may be divided into subsets such that each subset generates a group having the properties which have been proved to belong to H. Hence the theorem: *If a group contains at least one set of independent generators composed of as many operators as there are prime factors in the order of the group then all of its Sylow subgroups are abelian and of type* $(1, 1, 1, \ldots)$. As a special case of this theorem we have the known and evident result that when the order of such a group is a power of a prime the group must be abelian and of type $(1, 1, 1, \ldots)$.

Among the important categories of groups which have the property that it is possible to find a set of independent generators composed of as many operators as there are prime factors in the order of the group is the important category of solvable groups composed of all the groups whose order is not divisible by the square of a prime number. If the order of such a group involves more than one prime factor, it is always possible to select a set of independent generators composed of an arbitrary number of operators from 2 to the number of prime factors in the order of the group. This fact can be deduced directly from the theorem due to Hölder to which we referred above. This important category of solvable groups is therefore included in the larger category of such groups considered above.

POINT SETS AND ALLIED CREMONA GROUPS

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It is my purpose to develop, in a series of papers of which the first has been submitted to the *Transactions of the American Mathematical Society*, some aspects of the theory of an ordered set of n discrete points in a linear projective space, S_k, of k dimensions.[1] Such a set, denoted by P_n^k, defines projectively an 'associated' set Q_n^{n-k-2} in S_{n-k-2}, the two sets being mutually related. The study of this type of association, begun by Rosanes and Sturm,[2] is continued here. If $n = 2k + 2$ the associated sets can lie in the same S_k, and in particular they may coincide in order forming thus a 'self-associated' set. In addition to projective constructions for associated and self-associated sets, certain irrational conditions for such types of association which seem to be novel are given.

The invariants of a set P_n^k are formed from the determinants of the matrix of the coördinates of the points. An invariant is required to be homogeneous and of the same degree in the coördinates of each point and to be unaltered under permutations of the points. It appears that associated sets have proportional invariants. This theory of invariants is the natural extension of the theory of binary forms if the form be regarded as the definition of a point set P_n^1. Complete systems for P_6^1 and P_6^2 are derived and the accompanying algebra is utilized to obtain explicit equations not only for the cubic surface mapped from the plane by cubic curves on P_6^2 but also for the tritangent planes and the lines of the surface.

By a certain construction the ordered set P_n^k is mapped upon a point P of a space $\Sigma_{k(n-k-2)}$. The Cremona transformation from one map P to another map P' of the same set is discussed and eventually the construction is simplified so that this transformation is linear. The $n!$ permutations of the points of P_n^k lead to $n!$ points P in Σ which are conjugate under a Cremona group $G_{n!}$ in Σ. When $k = 1$, $G_{n!}$ is the 'cross-ratio group' of Moore.[3] A set of generators of $G_{n!}$ is exhibited. The invariant spreads of $G_{n!}$ are obtained from the invariants of P_n^k. The linear system of spreads of lowest order invariant under $G_{n!}$ is derived from the irrational invariants of P_n^k of lowest weight. Associated sets determine the same group.

The form-problem of the $G_{5!}$ determined by the associated P_5^1 and P_5^2 leads to a solution of the quintic equation; that of the $G_{6!}$ determined by the associated P_6^1 and P_6^3 and by the self-associated P_6^2 to a solution of the sextic equation. These equations have been discussed from this point of view in earlier papers of the writer.[4] The principal novelty here is the use of a connection between the self-associated P_6^2 and the theta modular functions of genus two to solve the related form-problem. In general one may say that the theory of the point set furnishes a common algebraic background not only for the general equations of degree n but also, as will appear later, for such particular equations as are involved in the determination of the lines on a cubic surface and of the double tangents of a plane quartic curve.

By introducing certain conventions it is possible to establish a mutual order for the two sets of singular points of a Cremona transformation in the plane. Since ordinary corresponding point pairs also are mutually ordered the addition of any number of ordinary pairs to the sets of singular points leads to a pair of sets which are said to be 'congruent' under Cremona transformation. The conditions under which congruence can occur are fundamentally important for effective application of the Cremona transformation to geometry or analysis. These conditions are derived up to a certain natural limit. Since two sets congruent to a third are congruent to each other the projectively distinct sets in the plane which are congruent in any order with a given set P_n^2 will be mapped by an aggregate of distinct points in Σ such that the aggregate is determined by any one of its points. If the type of transformation under which the congruence occurs and if the order be given the set is uniquely determined so that the transition from one point in Σ to another is effected by a Cremona transformation. In other words the sets P_n^2 congruent in any order with a given P_n^2 are mapped in Σ by points conjugate under an 'extended' Cremona group $G_{n,2}$ which contains $G_{n!}$ as a subgroup. For $n = 6, 7, 8$ the order of $G_{n,2}$ is $6!.72$, $7!.288$, $8!.2^6.135$ respectively but for larger values of n the $G_{n,2}$ is infinite and discontinuous. The finite groups are isomorphic respectively with those of the lines on a cubic surface, the bitangents of a plane quartic curve, and the tritangent planes of a space sextic of genus four on a quadric cone.

Let us call a Cremona transformation in S_n which can be expressed as a product of the particular transformations obtained by inverting the variables a 'regular' transformation. Obviously the regular transformations are the elements of a 'regular group.' Then the definition of congruence of point sets in S_2 as given above can be generalized to include congruence under regular transformations in S_n. Again the set P_n^k defines as above an extended group $G_{n,k}$ of Cremona transformations in $\Sigma_{k(n-k-2)}$ which in general is infinite and discontinuous. Associated sets define the same group. The only finite types in this doubly infinite (for increasing n and k) series of groups are those defined by the sets mentioned above or by their associated sets. Each $G_{n,k}$ contains $G_{n-1,k}$ as a subgroup.

The Cremona transformations in S_k under which the congruence of sets P_n^k occurs will have a certain effect upon the spreads in S_k. If the order of the spread and its multiplicity at each point of P_n^k be taken as variables,[5] this effect is expressed by a linear transformation on $n + 1$ variables. Thus there is determined a group of linear transformations $g_{n,k}$ in S'_n with integer coefficients which is isomorphic with the Cremona $G_{n,k}$. In S'_n, the $g_{n,k}$ has an invariant S'_{n-1} within which it determines an isomorphic $g'_{n,k}$ of linear transformations whose coefficients are rational numbers. The importance of the finite types in these series of groups would seem to indicate that interesting applications of the infinite types may be expected.

A definite application of the foregoing theory to the determination of the lines on a general cubic surface K^3 will be made.[6] As has been mentioned earlier, cubic curves on P_6^2 map S_2 upon a general cubic surface which appears in Cremona's hexahedral form, C^3. The rational invariants of P_6^2 are rational invariants of the surface after the adjunction of the irrationality which isolates a set of six skew lines of a double six. Such invariants determine, and are determined by, the invariant spreads of $G_{6!}$ in Σ_4. If it be required further that the spreads be invariant under the extended group, $G_{6,2}$ of order 51840 in Σ_4 they determine the invariants of C^3 itself. In this way not only the invariants but also the linear covariants of C^3 are calculated and identified with those of K^3. Now K^3 being given, its invariants and linear covariants are known. The invariants are the known quantities in the form-problem of $G_{6,2}$. The solution of this form-problem furnishes a point P in Σ_4, the map of P_6^2. From P_6^2 as indicated earlier the lines of C^3 as well as the linear covariants of C^3 are derived. The linear transformation obtained from the identification of the linear covariants of C^3 and K^3 transforms the known lines of C^3 into the required lines of K^3. All of the processes outlined here are rational except the solution of the form-problem of $G_{6,2}$. This may be dismissed as a problem in the theory of functions or it may by means of an accessory irrationality be reduced to the form-problem which occurs in the trisection of the periods of the hyperelliptic functions of genus two. The ultimate importance of this latter problem has been pointed out by Klein.[7] It would be more desirable however to utilize directly the theta-modular functions in the solution of the problem of $G_{6,2}$ as has been done in the case of the sextic.

The developments sketched above suggest a number of inquiries which promise results of interest in various directions. Also some of the problems considered such as that of the invariants of P_n^k are worthy of closer study than has been given them in the account here reported.

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[1] This investigation has been pursued under the auspices of the Carnegie Institution of Washington, D. C.

[2] For references see Study, *Math. Ann., Leipzig,* **60**, 348 (footnote).

[3] E. H. Moore, *Amer. J. Math.,* **22**, 279 (1900).

[4] *Trans. Amer. Math. Soc.,* **9**, 396 (1908), and **12**, 311 (1911). The relation of these papers to the earlier work of Klein and others is there set forth at length.

[5] S. Kantor has used this device for an S_2 in his crowned memoir: *Premiers Fondements pour une Théorie des Transformations Périodiques Univoques,* Naples (De Rubertis), 1891.

[6] This will appear as a joint paper by Mr. C. P. Sousley and A. B. Coble.

[7] Letter to Hermite, *J. Math., Paris,* Ser. 4, **4**, 169. See also *Math. Ann., Leipzig,* Witting, **29**, 167; Maschke, **33**, 317, **36**, 190; and Burkhardt, **38**, 161.

THE STRAIGHT LINES ON MODULAR CUBIC SURFACES

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1. In ordinary space a cubic surface without singular points contains exactly 27 straight lines, of which 27, 15, 7, or 3 are real; there are 45 sets of three coplanar lines, the three of no set being concurrent. In modular space, in which the coördinates of points and the coefficients of the equations of lines or surfaces are integers or Galois imaginaries taken modulo 2, it is interesting to notice that *three coplanar lines on a cubic surface may be concurrent* (§2). A point with integral coördinates is called real. A line or surface is called real if the coefficients of its equations are integers. In space with modulus 2, the number of real straight lines on a cubic surface without singular points is 15, 9, 5, 3, 2, 1, or 0.

We shall give here an elementary, self-contained, investigation of some of the most interesting cubic surfaces modulo 2. A complete classification of all such surfaces under real linear transformation will appear in the *Annals of Mathematics*, but without the present investigation of the configuration of their lines.

2. Every real point of space modulo 2 is on the surface[1]

$$xy(x + y) = zw(z + w). \tag{1}$$

since each member is an even integer when x, y, z, w are integers.

Theorem 1. *This surface with exactly 15 real points contains exactly 15 real straight lines. There are exactly 15 sets of three coplanar real lines on the surface, the three of each set being concurrent, while their plane is the tangent to the surface at the point of concurrence.*

The surface has the automorphs[2] (i.e., transformations carrying the surface into itself)

$$(zw);\ \pi = (xz)(yw);\ \beta\colon z^1 = z + w;$$

$$\gamma\colon x^1 = x,\ y^1 = y + w,\ z^1 = z + x,\ w^1 = x + z + w.$$

The fifteen real points of space

$$\begin{array}{llll} 1 = (1000) & 5 = (1100) & 9 = (0101) & 13 = (1011) \\ 2 = (0100) & 6 = (1010) & 10 = (0011) & 14 = (0111) \\ 3 = (0010) & 7 = (1001) & 11 = (1110) & 15 = (1111) \\ 4 = (0001) & 8 = (0110) & 12 = (1101) & \end{array}$$

are permuted transitively by these automorphs, since

$$(zw)\colon (3\ 4)\ (6\ 7)\ (8\ 9)\ (11\ 12),$$

$$\beta\colon (4\ 10)\ (7\ 13)\ (9\ 14)\ (12\ 15),$$

$$\pi\colon (1\ 3)\ (2\ 4)\ (5\ 10)\ (7\ 8)\ (11\ 13)\ (12\ 14),$$

$$\gamma\colon (1\ 13\ 12\ 6)\ (3\ 10\ 8\ 14)\ (4\ 9)\ (5\ 15\ 7\ 11).$$

The tangents to (1) at the fifteen real points give all of the fifteen real planes in space, since the tangent at $(XYZW)$ is

$$Y^2x + X^2y + W^2z + Z^2w = 0.$$

Hence these planes are permuted transitively by the real linear automorphs of the surface. The plane $x = 0$, tangent at 2, contains the concurrent lines

$$A\colon x = z = 0, \qquad B\colon x = w = 0, \qquad C\colon x = 0,\ z = w,$$

evidently on the surface. Hence each of the fifteen real planes contains three (concurrent) lines lying on the surface (1). The only real planes through line A are evidently $x = 0$, $z = 0$, and $x = z$. Hence the number of real lines on (1) is 15.3/3. We have therefore proved Theorem 1.

Since (zw) and β replace A by B and C and since the fifteen real planes are permuted transitively by the automorphs, the fifteen real lines on the surface are permuted transitively by the real linear automorphs.

We next make use of the imaginary automorph I which leaves z and w unaltered, but multiplies x and y by t, where $t^2 + t + 1 = 0$. It replaces the line $w = x$, $y = z$ on (1) by E_{t0} $(w = tx,\ z = ty)$, where

$$E_{ab}\colon w = ax,\ y = bx + a^2z;\ D_{abc}\colon z = ax + bw,\ y = cx + a^2w.$$

The twelve imaginary lines in which b and c are integers, while $a^2 + a + 1 = 0$, are seen to be permuted transitively by (zw), π, β. We now have $15 + 12$ lines on (1). As in the algebraic theory, there are only 27 lines on a cubic surface without singular points.[3] Hence the 27 lines are permuted transitively by I and the real automorphs. To prove Theorem 2 below, it therefore suffices to take A as one of the three coplanar lines. The only planes through A are $z = 0$ and $x = kz$. The former cuts the surface in the lines A, $z = y = 0$, and $z = 0$, $x = y$, which concur at 4. The latter cuts the surface in the lines A and

$$x = kz,\ ky(y + kz) + w(z + w) = 0.$$

This quadratic function has the factor $w + \alpha y + \beta z$ if and only if $\beta^2 = \beta$, $k = \alpha^2$, $\alpha = k^2$. Hence A meets just 10 lines on the surface, the above two and

$$L_{k\beta}\colon\ x = kz,\ w = k^2 y + \beta z \qquad (\beta = 0, 1;\ k^4 = k).$$

The only coplanar sets of three lines, one of which is A, are the above set of three real and the four sets A, L_{k0}, L_{k1}, which meet at $(010k^2)$. In all there are $27.5/3 = 45$ sets of coplanar lines.

Theorem 2. *The 27 straight lines on (1) form 45 sets of three coplanar lines. The three of each set concur and their plane is the tangent to the surface at the point of concurrence.*

3. There are several types of real cubic surfaces the numbers of whose real points and real lines differ from those of (1), but are derivable from (1) by imaginary linear transformation and hence have 45 sets of three coplanar concurrent straight lines.

If in (1) we replace z by $ez + e^2 w$ and w by $e^2 z + e^4 w$, where $e^3 + e + 1 = 0$, and hence replace $z + w$ by $e^4 z + ew$, we evidently get

$$xy(x + y) = z^3 + zw^2 + w^3. \tag{2}$$

For integral values of the variables, the left number is zero modulo 2, whence $z \equiv w \equiv 0$. Thus 1, 2, 5 are the only real points on (2); it contains no real line.

If in (1) we make also the like replacement of x, y, we get

$$x^3 + xy^2 + y^3 = z^3 + zw^2 + w^3. \tag{3}$$

It contains just nine real points and only three real lines: $z = x$, $y = w$; $z = y = x + w$; $w = x = y + z$, no two of which intersect.

If in (1) we replace[4] z by $z + bw$ and w by $z + b^2 w$, where $b^2 + b + 1 = 0$, we get

$$xy(x + y) = w(z^2 + zw + w^2). \tag{4}$$

The only real points on it are evidently the seven in the plane $w = 0$. Hence the only real lines on it are the three having $w = 0$.

If in (2) we replace x by $x + by$ and y by $x + b^2y$, where $b^2 + b + 1 = 0$, we get

$$y(x^2 + xy + y^2) = z^3 + zw^2 + w^3. \qquad (5)$$

Replacing y by $y + z + w$, z by $z + y$, w by $w + y$, we get a surface whose real points are those on the cone $yz + yw + zw = 0$ and hence are 1, 2, 3, 4, 5, 6, 7. Thus the real points are those on the lines $y = z = 0$, $y = w = 0$, $z = w = 0$, meeting at 1.

If in (1), written in Capitals, we set

$$X = x + y,\ \ Y = bx + b^2y + (a^2 + 1)w,\ \ Z = ax + (a + 1)y + z,$$
$$W = w,\ \ b^2 + b + 1 \equiv 0, a^4 + a + 1 \equiv 0,$$

we get the real surface[5]

$$x^3 + y^3 + (x^2 + z^2)w + (y + z)w^2 = 0, \qquad (6)$$

which has only eleven real points and only five real lines

$$w = 0, y = x; \quad z = x + tw, y = x; \quad z = tw, y = x + w \quad (t = 0, 1).$$

4. A surface without singular points and not having every set of three coplanar lines concurrent, as was the case with all the preceding surfaces, is

$$xy(x + y) = w(w + z)(y + z). \qquad (7)$$

It contains thirteen real points, 9 and 12 alone being not on it. We obtain a straight line on (7) by equating to zero one factor of each member. The resulting nine lines will be shown later to be the only real lines on (7). The three in $x = 0$ are not concurrent, likewise the three in $x = y$, while the three in $y = 0$ meet at 1. We obtain a redistribution of these nine lines by beginning with a factor of the second member of (7): the three lines in $w = 0$ meet at 3, the three in $w = z$ meet at 10, the three in $y = z$ meet at 4. There is no plane other than these six which contains three coplaner lines from this set of nine real lines.

Evident automorphs[6] of (7) are

$$(A)\ \ z^1 = z + y + w; \qquad (B)\ \ w^1 = w + z; \qquad (C)\ \ x^1 = x + y.$$

Evident imaginary lines on (7) are

$$(H_a)\quad w = ax, z = ay \qquad (a^3 + a^2 + 1 = 0).$$

Applying the real automorphs to H_a, we get the new lines

$$(D_a)\quad z = ax + w, y = a^2w;$$
$$(E_a)\quad z = ax + w, y = x + (a + a^2)w;$$

$$(F_a)\quad z = ax + a^2w,\ y = a^2w;$$
$$(L_a)\quad w = ax,\ y = (a^2 + 1)x + a^2z;$$
$$(J_a)\quad z = (a + 1)x + (a^2 + a)w,\ y = x + (a^2 + a)w.$$

Those with the same subscript a are permuted as follows:

$$A:\ (DF)(LH)(EJ),\quad B:\ (DL)(HE)(FJ),\quad C:\ (DF)(LJ)(EH).$$

Since the 6.3 imaginary lines and the earlier 9 lines give the 27 possible lines, we now have all the lines on (7), a fact checked otherwise.

The automorphs were seen to replace the three lines H_a by all of the 18 imaginary lines. Hence in studying sets of three coplanar lines at least one of which is imaginary, H_a may be taken as one line. The following are seen to give all such sets:

H_a, F_a and $x = y$, $w = z$, in $w + z = a(x + y)$;
H_a, E_a, $y = z = 0$, in $z = ay$;
H_a, D_{a^2}, J_{a^2+a+1}, in $a\,(w + ax) = ay + z$;
H_a, L_a, $x = w = 0$, in $w = ax$;
H_a, J_{a^2}, D_{a^2+a+1}, in $(a^2 + 1)\,(w + ax) = ay + z$.

Those in the second set alone are concurrent, meeting at (100a). Hence there are $18.2/3 = 12$ sets of three imaginary non-concurrent coplanar lines and $18.3/2 = 27$ sets with two imaginary and one real coplanar lines, in 9 of which sets the lines concur (since A, B, C permute the pairs HE, LJ, DF).

Theorem 3. *Just 9 of the 27 straight lines on* (7) *are real. Of the 45 sets of 3 coplanar lines, the lines in 32 sets are not concurrent and those in 13 sets concur.*

5. We consider briefly certain cubic surfaces with singular points. One for which 11 is the only singular point is

$$x^3 + xz^2 + xw^2 + y^2w + yw^2 + xzw = 0. \tag{8}$$

The[7] only real points not on it are 1 and 5. There are only ten lines on this surface, all being real:

(a) $x = y = z$, (b) $z = x = y + w$, (c) $x = y = z + w$,
(d) $y = z = x + w$, (e) $x = y = 0$, (f) $x = w = 0$, (g) $x = 0$, $y = w$,
(h) $w = 0$, $x = z$, (i) $x = w$, $y = z$, (j) $x = w = y + z$.

Of these, f and h alone are in $w = 0$, h counting as a double line of intersection with (8). The only sets of three coplanar lines are a, b, h, in $x = z$, meeting at 11; c, d, h, in $z = x + w$, meeting at 11; e, f, g, in $x = 0$, meeting at 3; f, i, j, in $w = x$, meeting at 8; together with the sets of non-concurring lines a, d, i, in $y = z$; a, c, e, in $x = y$; b, c, j, in $y = z + w$; b, d, g, in $y = x + w$.

Theorem 4. *There are only ten lines on (8) and all are real. Three coplanar lines concur if and only if one of them belongs to the singular pair f, h.*

In (8) we replace x by y, z by $z + y$, y by $x + y + ew$, where $e^2 + e + 1 = 0$; we obtain the real surface

$$yz(z + w) = w(w^2 + x^2 + xw), \tag{9}$$

having only ten straight lines of which only two are real. The five real points are those for which $yz = w = 0$, viz., 1, 2, 3, 5, 6.

The only type other than (9) having 1, 2, 3, 5, 6 as its only real points and having a singular point is $w^3 + x^2y + xy^2 + yz^2 + yzw = 0$, containing only three straight lines: $y = w = 0$; $y = w$, $z = x + bw$, $b^2 + b + 1 = 0$; these meet at the only singular point 6.

Another type with only five real points is $xw^2 = y^3 + yz^2 + z^2$, the only lines on which are the six in the planes $x = 0$, $w = 0$.

---

[1] And on only the surfaces equivalent to (1) or to $xy(x + y) = 0$ under real linear transformation.

[2] They generate all of the 15.6.4.2 real linear automorphs. For, one leaving point 2 fixed must permute the remaining real points 3, 4, 8, 9, 10, 14 in the tangent plane $x = 0$ at 2. These six are permuted transitively by (zw) and γ, both of which leave 2 fixed. An automorph which leaves 2 and 3 fixed, and hence the point 8 collinear with them, must permute 4, 9, 10, 14. These are permuted transitively by β and $\gamma\beta(zw)$, which leave 2 and 3 fixed. An automorph leaving 2, 3 and 4 fixed is the identity or $y^1 = y + x$, which is the transform of β by $(zw)\pi$.

[3] We readily verify that A, B, C and E_{ab}, D_{abc}, with b and c integers and $a^4 = a$, give all of the 27 lines on (1). We have only to consider first the lines whose two equations are solvable for y,z in terms of x,w, and second the lines one of whose equations is $x = 0$ or $w = ax$.

[4] If we make also the like replacement on x,y, we change the left member of (4) into $y(x^2 + xy + y^2)$. The new surface evidently has its seven real points in $y = w$. Replacing w by $w + y$, x by $x + z$ and then z by $z + y$, we get (4).

[5] Its 16 real linear automorphs are generated by the four: $z^1 = z + w$; $x^1 = y + w$, $y^1 = x + w$; $x^1 = y$, $y^1 = x$, $z^1 = x + y + z$; and $y^1 = y + w$, $z^1 = y + z$.

[6] They generate all the twelve real linear automorphs of (7). For, such a transformation must leave fixed the pair of points 9, 12 and hence the tangents $x = y$, $x = 0$ at those points. These points and planes are interchanged by C. If each is fixed, the transformation is

$$x^1 = x,\ y^1 = y,\ z^1 = by + cz + bw,\ w^1 = fy + gz + (f + 1)w.$$

Since the lines in $x = 0$ are permuted, the case $f = 1$, $g = 0$, is excluded. For $f = g = 0$, we get the identity and A. For $f = 0$, $g = 1$, we get B, BA. For $f = g = 1$, we get AB, BAB.

[7] As shown in the earlier paper, any cubic surface with just 13 real points and having no linear factor is equivalent to (7) or (8).

THE DIFFUSION OF HORSE CULTURE AMONG THE NORTH AMERICAN INDIANS

By Clark Wissler

AMERICAN MUSEUM OF NATURAL HISTORY, NEW YORK CITY

Presented to the Academy, March 17, 1915

Investigation of the horse-culture complex among the American Indians was undertaken to discover the procedure in a concrete case of culture diffusion, an important anthropological problem of the day. One of the most difficult tasks confronting the anthropologist is the elucidation of the precise complexes by which various traits of culture are produced. Since there is on every hand abundant evidence that many traits of culture are borrowed, or diffused, over large areas, the study of typical concrete instances of diffusion are of the first importance. A number of European anthropologists have been so impressed with the significance of diffusion, that they have developed from it a theory to account for the origin of culture traits. This theory is usually known as that of single origin as opposed to the theory of independent invention. The former asserts that all important traits of culture were invented but once and subsequently gradually diffused; the latter, that the same invention was made independently in many parts of the world, whence its diffusion is but apparent. As everyone knows, the discussion of such problems comes to naught unless concrete cases can be investigated.

The horse-culture complex of the American Indian offers an excellent opportunity to study diffusion, because most of the essential facts are obtainable. The horse was introduced by Europeans at an early date and spread ahead of interior exploration. In particular, many of the tribes west of the Mississippi River became horsemen before their discovery by Europeans. The history of their introduction is briefly outlined by me in the *American Anthropologist*, Vol. 16, No. 1, pp. 1–25. The investigation here reported is the intensive study of collections of riding gear and horse-using appliances to be found in anthropological collections. The material available gives us a representative series for each important tribe in the horse-using area. The detailed comparative study of these specimens has developed many interesting points among the most significant of which are:

1. The Indian has shown no originality. He devised no important

appliances for using horses. He manufactured his own saddles, bridles, etc., but followed precisely a few definite patterns. Though these patterns appear to us as Indian, that is because the English colonists brought with them the English saddle. The Indian model is fundamentally like that of Southern Europe and Asia during the period of American colonization and still survives among the tribes of Patagonia. In general, the complete data will show that the greater part of the horse-complex of the North American Indian was borrowed first by the tribes in contact with the Spanish settlements and then diffused as far as the Plains of Canada without loss or essential modification of detail.

The one striking Indian variation is the habit of mounting on the right side of the horse instead of the left as do Americans and Europeans. The comparative data on this point make it clear that if left to their inclinations right-handed people will mount from the right. Historical data show the European method to have been first introduced into cavalry tactics by Vespasian and to have survived to this day because the sword is worn on the left side. The difference, therefore, is not due to motor differences in the Indian but, like most other culture differences, to historical factors.

2. The Indian did not take the cart. Yet the Spanish colonists rarely if ever used the horse, mule, or donkey for anything but riding and packing; their carts were drawn by cattle. (The great abundance of buffalo no doubt prevented the development of an Indian cattle culture.) On the other hand, the Indian of the Plains had developed dog traction by the travois long before the horse came. When he got the horse, he fitted the travois to him. In any event, it is probable that the established use and simplicity of the travois would have inhibited the use of carts. Thus, while in the travois we have an instance of the use of an Indian invention with the horse, the presence of the horse had nothing to do with its origin.

3. The rapidity and completeness of horse-culture diffusion in America is a good illustration of how fully traits of borrowed culture may be assimilated. In this instance we have sufficient data to determine the general lines of diffusion but such is not often the case. For example, maize culture was once diffused over a large part of North and South America, for the wild plant is found only in one area which must have been the place of origin. In the Old World the spread of horse culture was most likely strictly analogous to its diffusion in America. Returning to our problem, it will be seen how if a non-historical people had

brought Old World horse culture to America, we should be puzzled at the similarities observed between these traits on the two hemispheres, but would probably set it down as another case of assumed independent invention. This investigation shows that the invoking of independent invention, to be more than a plea of ignorance, must rest upon specific data.

The final discussion of this subject will appear in full in the Anthropological Papers of the American Museum of Natural History.

DISCOVERY OF ALGONKIAN BACTERIA

By Charles D. Walcott
SMITHSONIAN INSTITUTION, WASHINGTON

Presented to the Academy, April 9, 1915

At a meeting of the Botanical Society of Washington, held April 6, 1915, I spoke upon the subject of 'Prepaleozoic Algal Deposits' and in this connection called attention to the existence of bacteria in association with the algal deposits of the Newland limestone, a formation of the Beltian series of Algonkian rocks in central Montana.

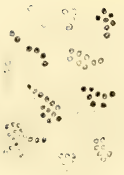
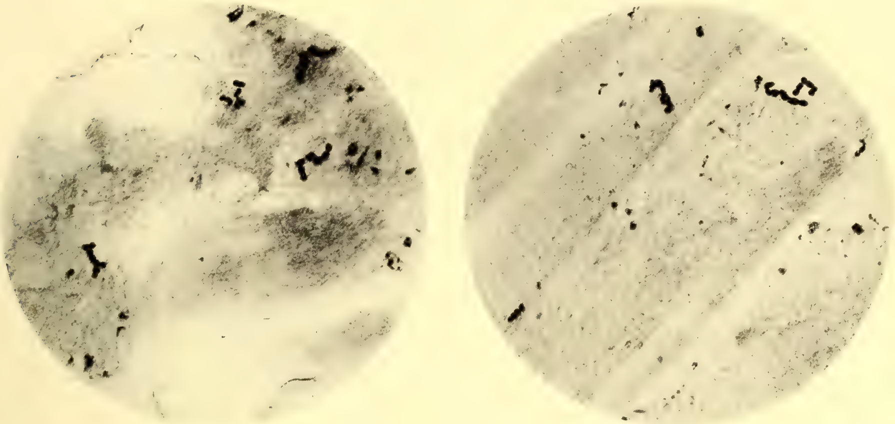


Fig. 1. Characteristic groups of *Micrococcus vaccinae*. (After Cohn.) Very highly magnified. [Encyclopædia Britannica, 11th Ed., Vol. III, p. 160, fig. 5–B.]

In a preliminary publication[1] I stated that it was quite probable that bacteria were the most important factor in the deposition of the Algonkian limestones. At that time no definite bacteria had been discovered. From the collections made during the season of 1914 many thin sections were prepared. These were examined by Dr. Albert Mann, plant morphologist of the Department of Agriculture, assisted by Mr. Charles E. Resser of the National Museum, with the result that bacteria were discovered in three of the sections, which were cut from an algal form included under the generic term *Gallatinia* as defined in the preliminary paper upon the Algonkian Algal Flora.[2]

The bacteria consist of individual cells and apparent chains of cells which correspond in their physical appearance with the cells of Micrococci.[3] For the purpose of comparison in this preliminary notice an illustration is here given of a group of recent forms as shown in the *Encyclopedia Britannica*[3] and of the form shown in the cells in the thin sections cut from the fossil alga of the Newland limestone.



Figs. 2 and 3. *Micrococcus* sp. undt. (× about 1100 diameters.) Average size of Micrococci 0.95 to 1.3 microns in diameter. (Slide D.) From locality 401b, Algonkian: Gallatin formation; north side of East Gallatin River, 5 miles (8 km.) east of Logan, Gallatin County, Montana.

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[1] Walcott, Pre-Cambrian Algonkian Algal Flora, *Smithsonian Misc. Coll.*, **64**, No. 2 (1914).

[2] Idem, p. 116, pl. 23, fig. 1.

[3] *Encyclopaedia Britannica*, 11th ed., vol. 3, p. 160, fig. 5.

A CORRECTION

(Received March 29, 1915)

In my paper on the earth considered as a heat engine, this volume, page 81, the linear expansion of rock forming minerals was inadvertently taken at 100 times its real value. A square area of superficial rock of relatively low diffusivity would really need to be several hundred degrees hotter than the surrounding areas to be shattered by the compressive stresses called into play by mere difference of temperature. Similarly a mean temperature difference of 40° between oceanic and continental columns overlying the level of isostatic compensation, would by itself account for a difference of level of only about 39 metres.

The error committed does not affect the general argument that the subcontinental shell acts as a heat engine, for it is known that several reversible processes such as elastic strain, expansion, liquefaction and volatilization are active in this shell, so that the internal heat does not escape to the surface solely by conduction and must therefore do work. Nor is it impossible that after the final consolidation of the superficial rocks, at from 700° to say 1300°, temperature differences of several hundred degrees were set up on equipotential surfaces. It is a matter of certainty that jointing of the rocks is now universal in continental areas and that it has been prevalent from the Archean onwards That jointing impedes the flow of heat, thus exaggerating any primordial inferiority of diffusivity and tending to increase temperature differences, is equally certain. How primeval jointing was initiated is less clear than my mistake made it seem to me, and offers an interesting problem.

GEORGE F. BECKER.

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SOME PROBLEMS IN STELLAR PHOTOMETRY

Smithsonian Institution
MAY 22 1915
National Museum

By Joel Stebbins
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Read before the Academy, April 20, 1915. Received, April 1, 1915

It is the purpose of this note to point out some of the researches which have been made possible by the development of the selenium and photo-electric photometers. The first advantage of these electrical devices is a considerable increase in accuracy over what is possible by visual and photographic methods. Perhaps the best field for immediate investigation lies in the study of the eclipsing variables, which give us the only direct clue to the actual diameters of stars. Nearly one hundred of these systems are now known, and new ones are continually being found. In the various studies which have been made, there is one line of reasoning which seems to have been overlooked, as follows.

Among the systems discovered, some are constant in light for a large fraction of the period, while others have relatively long times of variation, and are thus easier of discovery and observation. If P be the total period, and D the time of duration of the eclipse, the probability that a star will be faint when observed at any time is D/P. It seems plausible to assume that if we take the number of discovered stars of each class and multiply by P/D we shall get numbers representing the relative occurrence of such objects which are actually visible and might be discovered. For example: suppose that in a random search about the sky we find 5 stars where the eclipse is $\frac{1}{5}$, and 5 where the eclipse is $\frac{1}{10}$ of the period. Then instead of the relative numbers awaiting discovery being equal, it seems much more probable that there are about 25 of the first class to 50 of the second.

There is another consideration that for every double system which exhibits eclipses in our direction, there are others whose orbit planes are inclined so that we miss the eclipse altogether. We may adopt half a stellar magnitude as the ordinary limit above which a star must vary in order to be detected, and if i represents the corresponding inclination the probability of an eclipse of this or greater amount is given by cos i. Conversely, if we have found a certain number of stars with a given type of eclipse, we may multiply by sec i and obtain the corresponding number of such systems with orbit planes in all directions.

Therefore, if we separate the known eclipsing systems into classes according to the ratio P/D and multiply the number in each class by P sec i/D, we get numbers which represent the frequency of occurrence in space of the corresponding systems. From data already brought together by Shapley (*Contributions from the Princeton University Observatory*, No. 3, 82, 1915), the tabulated results have been found. The third line has been derived from the second by the proper multiplication as described, the products being reduced in proportion to make the total of 88.

Value of P/D	2–4	5–7	8–10	11–13	14–16	17–19	20–22
Number of stars observed	12	41	23	7	4	0	1
Relative number existing	3	24	26	16	13	0	6

The interpretation of these results is that there are relatively few systems with components nearly in contact, $P/D = 2$, and there is a distinct preponderance of P/D from 5 to 10, with maximum say at 8. This corresponds to a distance between centers of about 5 times the average radius of the two bodies.

With more data we could derive similar results for each of the spectral classes. The systems with spectra of classes G and K seem to be quite different from the others, and omitting these we find for stars of B, A, and F spectra a close connection between the relative length of eclipses and the average periods.

Value of P/D	2–4	5–7	8–10	11–13	14–20
Number of stars	9	38	22	6	4
Average period, days	1.0	2.4	5.0	11	15

The increase of period with increasing P/D seems to mean, what has been pointed out before, that the stars which are farther apart move more slowly not only on account of the greater distance, but also because these systems are less dense. However, as the density seems to be a marked function of the spectral class, a further discussion with more material is essential.

From these and other simple considerations, we may conclude: *The preponderant type of close binary with components of the same order of size and of equal or unequal brightness, consists of bodies whose distance* *between centers is approximately 5 times their average radius, whose period of revolution is about 4 days, and whose mean density is 1/20 that of the sun. Systems of greater or less separation are not so numerous, or we should find more of them among the eclipsing variables.*

The foregoing discussion is based upon the variables which have been found by visual and photographic methods, but there is abundant field for work in the same lines for the new forms of photometer. Here instead of examining thousands of stars and picking up a variable now and then, it will be better to make a special study of a few stars which from spectroscopic results are known to have large companions. I have made a start in this testing of spectroscopic binaries, and the first two stars observed were found to be variables. Further observations yielded 4 variables out of 11 stars, and among the hundreds of spectroscopic binaries already known, there are no doubt dozens of variables awaiting discovery. The point to emphasize is that not only will systematic studies of stars which vary in light give us direct information, but indirectly we can draw far reaching conclusions about stars which do not vary.

In addition to the eclipses, other causes of light-variation of binaries may be mentioned, such as ellipsoidal figure, orbital eccentricity producing changes in the interaction of radiation of the components, tidal phenomena, and the effects of resisting media. There is a class of objects like *Polaris*, whose spectral variations are accompanied by synchronous light changes, but there is the possibility that these are not double systems. Some day we shall probably be able to assert that all stars with rapid changes in their spectra, whether these be in the shift or the character of the lines, must vary in light, and it will become a practical problem to find really constant stars to be used as standards of comparison. These may be found only by trial, and we shall have to observe stars in groups so as to eliminate the variables. No doubt we shall find a difference in the average amount of variation as we pass along the spectral series.

This takes us to the case of our sun, which, according to Abbot, is a variable star. Of the variation there can not be the slightest doubt, for a single sunspot is enough to change the total light, the only question is how much. In direct measures of the sun's radiation, the chief difficulty lies in the proper allowance for the atmospheric absorption, but this trouble may be eliminated by comparing the reflected light of one of the planets with the light of a number of stars. Probably *Saturn* is a good object for this purpose, as there are few markings on the surface, but *Uranus* would be still better on account of its slower motion, and the greater number of suitable comparison stars which could be found for it.

Another advantage of the selenium and photo-electric methods comes from the fact that the light of a body like a comet, nebula, or star cluster may be compared directly with a point source like a star. In addition to photometric studies of diffuse objects, it is not too much to expect that we shall be able to get a very good measure of the general light of the sky, i.e., measures of the total light of the stars both in and out of the Milky Way.

The photometry of lunar and solar eclipses also presents problems which can be attacked with success. The total light of the corona as well as its distribution will be easy to determine. Measures of the zodiacal light can be made, and also any absorption of star light by these particles near the sun may be detected. This last would require very accurate observations, but would be feasible at a mountain station.

The problems here presented are some of those for which we already have the means of attack, and which will lead to definite results. Besides what may be predicted in a way there is always the probability of something unexpected, and it is evident that only a beginning has been made in the use of the new methods.

THE COMPOSITION OF BRACHIOPOD SHELLS

By F. W. Clarke and W. C. Wheeler

UNITED STATES GEOLOGICAL SURVEY, WASHINGTON

Presented to the Academy, April 8, 1915

The present communication[1] is part of a much larger research relative to the inorganic constituents of marine invertebrates. The object of the investigation is to determine, more definitely than has been done hitherto, just what substances are contributed by each group of organisms to the marine sediments, and therefore to the formation of marine limestones, with especial reference to their magnesian and phosphatic varieties. A few analyses of brachipod shells are already on record; and they show that they fall into two chemically distinct groups—one calcareous, the other highly phosphatic. This conclusion is supported and emphasized by the new data obtained by us, which, however, bring out some minor peculiarities that seem not to have been previously observed. For our material we are indebted to the kindness of Dr. W. H. Dall, who selected typical specimens from among the duplicates in the collection of the U. S. National Museum. First in order come five brachiopods representing as many genera in the calcareous group. The species analysed are as follows:

1. *Terebratula cubensis*, Pourtalès. Coast of Florida.
2. *Terebratulina septentrionalis*, Gray. Eastport, Maine.
3. *Laqueus californicus*, Koch. Esteros Bay, California.
4. *Rhynchonella psittacea*, Gmelin. Shetland Islands.
5. *Crania anomala*, Müller. Coast of Norway.

Actual analyses

	1	2	3	4	5
SiO_2	0.06	0.50	0.18	0.14	0.21
$(Al, Fe)_2O_3$	0.04	0.14	0.47	0.23	0.26
MgO	0.44	0.62	0.32	0.23	3.90
CaO	54.96	51.79	54.48	53.76	48.67
SO_3	0.21	0.66	0.21	0.31	0.97
P_2O_5	trace	trace	trace	0.17	0.25
Loss on ignition	44.35	45.28	44.46	44.81	45.38
	100.06	98.99	100.12	99.65	99.64
CO_2 calculated	43.42	40.55	42.91	42.16	40.88
Organic matter*	0.93	4.73	1.55	2.65	3.52

* Approximate. Loss on ignition minus CO_2.

Rejecting organic matter and recalculating to 100%, the analyses assume the following rational form.

Revised analyses

	1	2	3	4	5
SiO_2	0.06	0.52	0.18	0.15	0.22
$(Al, Fe)_2O_3$	0.04	0.15	0.48	0.23	0.27
$MgCO_3$	0.93	1.37	0.68	0.49	8.63
$CaCO_3$	98.61	96.78	98.30	98.20	88.59
$CaSO_4$	0.36	1.18	0.36	0.55	1.72
$Ca_3P_2O_8$	trace	trace	trace	0.38	0.57
	100.00	100.00	100.00	100.00	100.00

For comparison, the analyses of the following calcareous brachiopods, made elsewhere, are significant:

6. *Terebratula* sp. Collected by Pourtalès between Florida and Cuba. S. P. Sharples analyst.[2]

7. *Terebratulina caput serpentis.* Locality not given. F. Kunckell analyst.[3]

8. *Crania anomala*, Müller. Locality not given. Kunckell, analyst.

9. *Waldheimia cranium*, Müller. Locality not given. Kunckell, analyst.

10. *Waldheimia cranium*, Müller. Collected by Norwegian North Sea Expedition, Station 255, latitude 68°.12′ N., longitude 15°.40′ E., in 624 metres depth, and at bottom temperature 6°.5 C.

11. *Waldheimia cranium.* Lofoten Islands. Analyses 10, 11, by L. Schmelck.[4]

Older analyses of calcareous brachiopods

	6	7	8	9	10	11
SiO_2	....	....	....	....	0.60	?
Fe_2O_3	trace	....	....	....	0.40	0.15
$MgCO_3$	trace	1.05	3.40	....	1.20	1.40
$CaCO_3$	98.39	94.60	87.80	96.20	96.20	95.58
$CaSO_4$	....	2.40	2.15	0.90	0.85	?
$Ca_3P_2O_8$	0.61	....	0.28	0.18	....	....
SrO	....	....	....	....	....	trace
CaO	....	....	....	0.30	....	....
MgO	....	....	1.80	0.60	....	....
P_2O_5	....	....	....	....	trace	0.12
Organic matter	1.00	2.55	4.30	2.00	1.24	1.99
	100.00	100.60	99.73	100.18	100.49	99.64

With these analyses ours agree in a broad, general way, although the older ones vary much as regards completeness. Kunckell's analyses, showing free lime and magnesia are suspicious, but only in this detail; otherwise they have confirmatory value. All the analyses show that this group of brachiopods have shells in which calcium carbonate is the principal constituent, and the proportion of organic matter is low. The only aberrant one is *Crania*, which is noteworthy on account of its high percentage of magnesia; in which respect if recalculated to a common basis, Kunckell's analysis agrees approximately with ours. *Rhynchonella* is also interesting for the reason that an analysis by Hilger of shells purporting to belong to this genus are made to be phosphatic, and practically identical in composition with those of *Lingula.* The authenticity of Hilger's material is questionable; and his analysis will not be reproduced here.

In the group of phosphatic brachiopods four analyses have been made by us. On account of the large amount of organic matter in these shells, which possibly varies with the age or maturity of the animal, we prefer to report our results, as others before us have done, in proximate rather than ultimate form.

1. *Lingula anatina*, Gmelin. Coast of Higo Province, Japan. Organic matter, rejected, 40%.

2. *Lingula anatina*, Iloilo, Philippine Islands. Organic matter, rejected, 39.5%.

3. *Discinisca lamellosa*, Broderip. Coast of Peru. Organic matter, rejected, 25%.

4. *Glottidia* (formerly *Lingula*) *pyramidata*, Stimpson. Coast of North Carolina. Organic matter about 37%. Analysis incomplete for lack of sufficient material.

Phosphatic brachiopods

	1	2	3	4
SiO_2	0.91	0.50	0.85	0.49
$(Al, Fe)_2O_3$	0.54	0.29	0.58	1.16
$MgCO_3$	2.70	0.79	6.68	1.71
$CaCO_3$	1.18	4.25	8.35	?
$CaSO_4$	2.93	4.18	8.37	?
$Ca_3P_2O_8$	91.74	89.99	75.17	74.73
	100.00	100.00	100.00	

These analyses are noteworthy, on account of the unusual proportion of calcium sulphate reported in them. *Discinisca* is especially remarkable in this respect, and also in its percentage of magnesium carbonate. Small amounts of sulphates have often been found in mollusks and corals as well as in the calcareous brachiopods, but in nothing like the proportion given here. A new analysis of *Discinisca,* made upon fresh material, is much to be desired.

In the older analyses of this group the sulphate seems to have been ignored, or at least to have escaped attention. The figures are as follows:

5. *Lingula ovalis.* Hawaiian Islands. T. S. Hunt, analyst.[5]

6, 7. *Lingula ovalis.* Locality not given. A. Hilger, analyst.[6]

8. *Lingula anatina.* S. Cloëz, analyst.[7] Recalculated to 100% after rejecting 42.6% of organic matter.

Older analyses of phosphatic brachiopods

	5	6	7	8
SiO_2	....	0.18	0.17	trace
$MgCO_3$	....	2.94	3.13	....
$CaCO_3$	11.75	10.76	10.86	12.19
$Ca_3P_2O_8$	85.79	84.94	85.24	77.17
$Mg_3P_2O_8$	....	....	....	7.03
$FePO_4$	....	0.77	0.76	3.61
MgO(?)	2.80	....	....	....
	100.34	99.59	100.16	100.00

In this table the relatively high figures for $CaCO_3$ are doubtless due to the neglect of the sulphate determinations. The analysis by Cloëz differs from the others principally in form, that is, in its mode of calculation. If the phosphoric oxide in it is entirely assigned to the lime, then the proportion of calcium phosphate becomes 88.6%, and well in line with the other figures. The amount of calcium carbonate would be correspondingly reduced.

Now to sum up. The brachiopods, as stated at the beginning of this paper, are to be divided into two distinct groups; the shells of one consisting mainly of calcium carbonate with little organic matter; and of the other predominating calcium phosphate with much organic matter. The two groups, although they may be alike structurally, are physiologically quite dissimilar, the chemical reactions involved in building the shells being of two different orders. Such a distinction ought to be significant to biologists, and it is for them to determine what it means. Geologically, however, we can see that the phosphatic brachiopods have probably played some part in the formation of phosphatic sediments, a function which is shared by vertebrate animals and some crustaceans.

[1] Published by permission of the Director of the U. S. Geological Survey.

[2] Sharples, S. P., *Amer. J. Sci.*, ser. 3, **1**, 168 (1871).

[3] Kunckell, F., *J. prakt. Chem.*, ser. 2, **59**, 102 (1899).

[4] Schmelck, L., *Norske Nordhavs Expedition*, No. 28, p. 129 (1901).

[5] Logan, W. E. and Hunt, T. S., *Amer. J. Sci.*, ser. 2, **17**, 237 (1854).

[6] Hilger, A., *J. prakt. Chem.*, **102**, 418 (1867).

[7] Cloëz, S., *Jahrber. Chem.*, 1859, p. 642. From *l'Institut*, 1859, p. 240.

ON THE OCCURRENCE OF THE LINE 4686 A AND THE RELATED SERIES OF LINES IN THE SPECTRA OF THE PLANETARY NEBULAE

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The line 4686 A and the related series of lines known as the ζ *Puppis* series, or 'sharp series' of hydrogen, were first observed in the spectra of the heavenly bodies. In 1912 Fowler succeeded in exciting them in a laboratory source.[1] More recently they have played an important rôle in certain theories of the constitution of the atom.[2]

The lines in question exist in the spectra of the planetary nebulae, and a number of facts concerning their occurrence have been brought out in the course of an investigation, now being undertaken, of these spectra. This study, while not yet complete, has led to some tentative generalizations on the behavior of certain of the nebular lines, prominent among them the one at 4686, and it may add to the significance of some of the observations of this line if a brief account is given of these. In what follows I shall attempt to state in general terms what appear to me to be the most significant facts gathered from an examination of about a dozen planetary nebulae, and shall make only occasional references to individual objects. A detailed account of all of the observations will appear on the completion of the work. The conclusions arrived at here are, of course, subject to revision in the light of additional data.

In another publication reference has been made to the fact that the nuclei of the planetary nebulae are in general Class O, or to use an older term, Wolf-Rayet stars.[3] The inquiry into the relationship between the spectra of the nebulae and their nuclei has necessitated a study of the distribution of light in the nebulae, line by line. This has been pursued by various means, only one of which need be referred to here. If the slit of the spectroscope be placed across the image of one of these usually symmetrical nebulae, those lines which are emitted throughout its full extent will be longer than those given out say only close to the center. In this way the length of a line may be taken as the measure of the extent of its occurrence in the source, just as is the case in laboratory studies of the arc, spark and other terrestial sources.

It has been found that in the majority of cases the nebulae are not homogeneous,[4] that is, lines observed in the above described manner have different lengths, and in general there is a certain order of length for the different lines. Selecting a few from the sixty or more nebular lines we may say that 4686 is always among the shortest, followed by the lines of helium, 'nebulium' and hydrogen in the order of length. An interpretation of this phenomenon on the assumption that the nebulae are in the process of condensation into stars is that the lines shrink into the nucleus in this order. We have examples of nebulae in many stages of the process. The present purpose however is not to enter into a speculative discussion of the development of stars, but to sketch the behavior of the line 4686 and the members of the related series.

The line 4686 is found in two forms: (*a*) as a narrow line in the nebulosity of some of these objects, and (*b*) as a broad band confined to the nuclei of certain others. Its occurrence in both forms in the same nebula is rare, one case only of this sort having been observed.

(*a*) *Occurring in the first form.* The line presents the appearance of a monochromatic radiation, under the strongest dispersion used, that of three dense prisms. It is generally (perhaps always) accompanied by very faint ζ *Puppis* lines (5411, 4541, 4200) of a similar character. It is to be looked for in those nebulae in which the lines 5007 and 4959 have their greatest strength relatively to the hydrogen lines. A case in point is the nebula N. G. C. 7027, the brightest known planetary. In this object the position of the line, derived from four photographs secured with a three prism spectrograph, is 4685.93, (R), a quantity in satisfactory agreement with Fowler's laboratory determination of 4685.98 (Rowland). The wave-lengths of the three related lines were measured with a dispersion of one prism. They are 5411.4, 4541.7, and 4202.2. The last value is not very trustworthy and all three are subject to revision with further observation. The agreement with the values calculated by theory: 5411.74, 4541.79, and 4200.02, is good. N. G. C. 7027 appears to be an unusually homogeneous nebula, 4686 being probably coextensive with the helium and hydrogen lines.

In addition to the ζ *Puppis* series of lines, another series lying in the ultraviolet was found by Fowler to be associated with 4686. The first member of this series, 3203, has been searched for with a quartz spectrograph attached to the Crossley reflector. It could not be photographed with a prolonged exposure, but this does not prove its nonexistence, as the reflectivity of silver is very low at 3200. It should be looked for with more suitable apparatus.

(*b*) *Occurrence as a nucleus line.* As has been remarked, the line when confined to the nucleus is broadened out into a band, which is frequently bordered on the edge of shorter wave-length by a broad dark line. The maximum of this band is commonly displaced toward the red end of the spectrum, the amount of the shift being of the order of an angstrom. The lines of the ζ *Puppis* series are present, in planetary nuclei, in some instances as bright and in others as dark bands. In this respect they parallel their behavior in Class O stars in general, as described by the observers of the Harvard College Observatory.[5] As an example we may cite the nebula B. D. + 30°3639. The nucleus of this object is a well known Class O star, and contains numerous bright bands in addition to that at 4686. Nearly all of these are of the same width, about 10 A. A very few bands have widths of about a quarter of this amount, and among them are the members of the ζ *Puppis* series. These, with 4686, are bordered on the violet edge by broad dark lines.

In only one instance has 4686 been found to exist in both nebula and nucleus. In N. G. C. 2392 the narrow nebular line contains a knot where the continuous spectrum of the nucleus crosses it. From the apparent rarity of this condition it seems probable that the transition from one form of radiation to the other, in any progressive change which may be going on in these objects, takes place with comparative rapidity, once it has begun.

It is an unsettled question among physicists whether the lines here discussed are due to hydrogen or helium. On this phase of the matter the nebulae appear to shed no light, as they afford no indication that the radiations belong to either of these elements. Fowler however noticed certain peculiarities of their localization in vacuum tubes, and it is possible that their behavior in this respect in the planetary nebulae may lead to some understanding of the conditions obtaining in these objects.

The interesting theory of radiation proposed by Bohr, to which reference has already been made, predicts lines separated by about two angstroms from the members of the Balmer series. Their verification in the spectra of the nebulae would be a matter of great importance, but the problem presents unusual difficulties which I have not yet been able to overcome. It is possible that future efforts will meet with greater success.

An attempt has recently been made, by an excellent observer, to classify the nebulae on the basis of the intensity of 4686. The line is common to both the nebulae and the Class O stars, and in the proposed system of classification it is assumed that the brighter a line in a nebula the closer the latter approaches to the stellar type. While I believe that the line in question, with others, will eventually serve as the basis of such a classification, I am strongly of the opinion that the matter of the intensity of the line alone affords no clue to the position of the spectrum in the scale of development unless indeed it be in a sense entirely opposite to the one suggested. It is not until the line settles into the nucleus, and is practically eliminated from the nebula proper, that the Class O stage is approached. In such objects as N. G. C. 2d Index 418, N. G. C. 6826 and N. G. C. 6572, which have Class O nuclei, the line is confined to the nucleus, and is a very inconspicuous affair. In general the more prominent this line in the spectrum of a nebula the further removed is the object from the stellar stage. The nebula N. G. C. 7027 apparently has no Class O characteristics, but 4686 is among its strongest lines.

[1] *London, Mon. Not. R. Ast. Soc.*, **73**, 62 (1913); E. J. Evans, *Nature, London*, **92**, 5 (1913).

[2] N. Bohr, *Phil. Mag.*, *London*, ser. 6, **26**, 1 and 476 (1913); A. Fowler, *London*, *Phil. Trans. R. Soc.*, **214**, 225 (1914).

[3] *Astrophys. J.*, December, 1914.

[4] A number of cases of non-homogeneity of nebulae have been reported by previous observers. Campbell found the hydrogen lines relatively strong in the outer parts of the *Orion* and Trifid nebulae and also of N. G. C. 2d Index 418 = S. D. M. − 12° 1172. See also Hartmann, *Astrophys. J.*, **21**, 389 (1905). See also Burns's observations of the Ring nebula, *Lick Obs. Bul.*, **6**, 92 (1910). Wolf has observed the Ring nebula in *Lyra*, by a method identical with that described in this paper, and finds 4686 to be a short line. *Sitz. Heid. Akad. Wiss.*, **27**, 1 (1911).

[5] Miss A. J. Cannon, *Ann. Obs. Harvard Coll.*, **56**, 66 (1912).

THE NATURE OF NERVE CONDUCTION IN CASSIOPEA

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Presented to the Academy, March 24, 1915

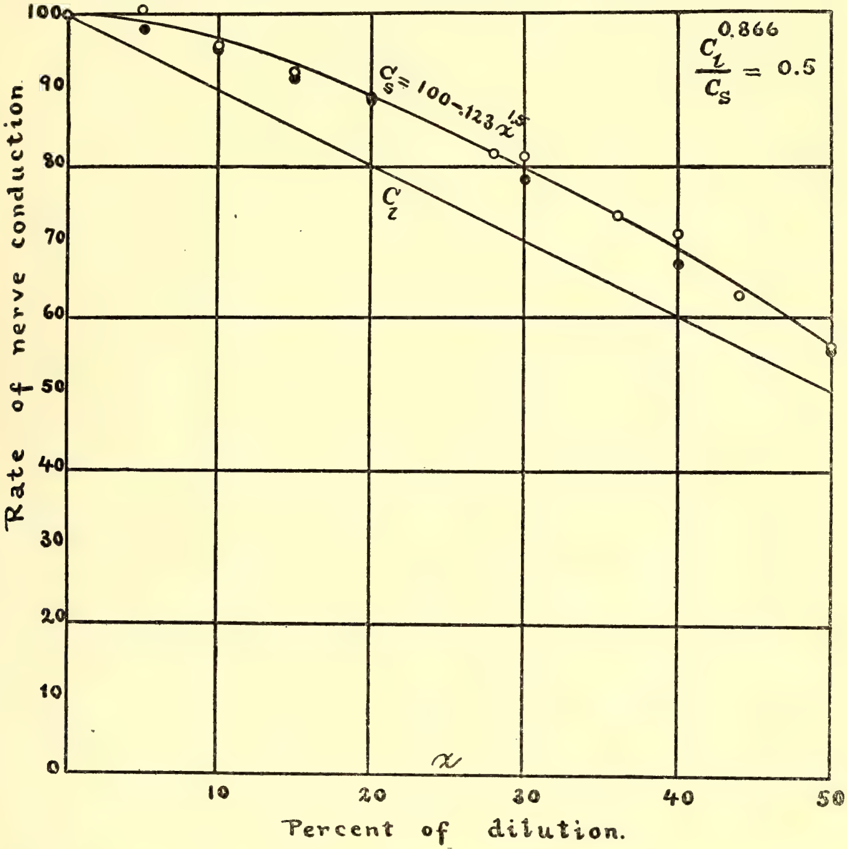
The experimental side of this research was conducted at the Marine Laboratory of the Carnegie Institution at Tortugas, Florida, and the kymograph records were studied under most advantageous surroundings kindly offered in Guyot Hall, Princeton University, by Profs. Edwin G. Conklin, Ulric Dahlgren, and Louis R. Cary. Ring-shaped strips of subumbrella tissue of the scyphomedusa *Cassiopea xamachana* were cut so as to remove the marginal sense organs, thus paralyzing the ring. Then a contraction wave was entrapped in the circuit, and the ring was placed in sea water diluted either with distilled water, or with 0.415 molecular magnesium chloride. The decline in rate was almost identical in similar concentrations of both these solutions although the magnesium appears to be slightly more depressant than the distilled water.

TABLE I

COMPOSITION OF THE SOLUTION	AVERAGE RATE OF NERVE CONDUCTION IN SEA WATER DILUTED WITH DISTILLED WATER	AVERAGE RATE OF NERVE CONDUCTION IN SEA WATER DILUTED WITH 0.415 MOLECULAR $MgCl_2$	RATE CALCULATED FROM THE FORMULA $C_s = 100 - 0.123\,x^{1.5}$ WHERE C_s IS THE RATE CORRESPONDING TO ANY PERCENT OF DILUENT, x
Natural sea water....................	100.0	100.0	100.00
95.0 volumes sea water + 5 of diluent	100.5 ± 0.67	97.9 ± 0.60	98.63
90.0 “ “ + 10 “	95.89 ± 0.825	95.3 ± 1.23	96.11
85.0 “ “ + 15 “	92.3 ± 1.21	91.6 ± 0.75	92.85
80.0 “ “ + 20 “	88.3 ± 1.19	88.9 ± 1.20	89.00
72.0 “ “ + 28 “	81.94 ± 0.48		81.78
70.0 “ “ + 30 “	81.4 ± 1.49	78.1 ± 0.74	79.79
64.0 “ “ + 36 “	73.91 ± 0.68		73.43
60.0 “ “ + 40 “	71.1 ± 1.29	67.2 ± 0.65	68.88
56.0 “ “ + 44 “	64.55 ± 1.26		64.10
50.0 “ “ + 50 “	56.31 ± 1.62	55.4 ± 0.76	56.50
33.3 “ “ + 66.7 “	29.0 ± 1		33.05

Experiments made in 1913 showed that the decline in rate of nerve conduction is the same in sea water diluted with 0.9 molecular dextrose as in sea water diluted with distilled water, and thus it is evident that down to 50% concentration the decline in rate is due solely to the change in concentration of the cations sodium, calcium and potassium, and not to the reduced osmotic pressure, or to magnesium.

Table I shows the results of the observations of 1914, the probable errors being expressed as ± following the mean. Thus 100.5 ± 0.67 means that the average rate was 100.5 with a probable error of ± 0.67. As the table is based upon 373 observations, the probable errors are on the whole low for physiological results and are not greater than 1.62%.



If C_s be the velocity of nerve conduction corresponding to any percentage of diluent x (thus in 75 volumes of sea water mixed with 25 volumes of distilled water, $x = 25$), then

$$C_s = 100 - 0.123\ x^{1.5}$$

But a more interesting relation exists which may throw new light upon the nature of nerve conduction, for it appears that the curve representing the velocity of nerve conduction is identical in form with that for adsorption.

Past experiments show that the rate of nerve conduction in *Cassiopea* depends upon the concentration of the cations sodium, calcium, and potassium, and not upon osmotic pressure or upon magnesium. This being so, and if nerve conduction be a chemical phenomenon, we should expect according to Wilhelmy's law that its rate would be proportional to the concentration of these three cations which take part in the reaction.

Thus if the particles of the colloidal nerve substance carry a negative charge they would attract these positive cations to their surfaces, in accordance with the well known formula $C_s = \beta C_l^{1/p}$ where C_s is the concentration of the sodium, calcium, and potassium cations in the colloidal nerve substance; C_l is the concentration of these cations in the surrounding sea water; and β and p are constants.

TABLE II

COMPOSITION OF THE SOLUTION	C_l RELATIVE CONCENTRATION OF THE CATIONS $Na^{\cdot}$, $Ca^{\cdot\cdot}$, AND $K^{\cdot}$ IN THE DILUTED SEA WATER	C_s OBSERVED RATE OF NERVE CONDUCTION	$\frac{C_l^{0.866}}{C_s}$
Natural sea water..................................	100.0	100.00	
95.0 volumes of sea water + 5.0 of distilled water....	95.2	100.50	0.51
90.0 " " + 10.0 " "	90.5	95.89	0.51
85.0 " " + 15.0 " "	85.5	92.30	0.51
80.0 " " + 20.0 " "	80.8	88.30	0.50
72.0 " " + 28.0 " "	73.3	81.94	0.50
70.0 " " + 30.0 " "	71.4	81.40	0.49
64.0 " " + 36.0 " "	66.0	73.91	0.52
60.0 " " + 40.0 " "	62.2	71.10	0.50
56.0 " " + 44.0 " "	58.4	64.55	0.52
50.0 " " + 50.0 " "	52.8	56.31	0.55
33.3 " " + 66.7 " "	37.5	29.00	0.79*

* Injurious osmotic effects are produced by this extreme dilution.

As appears in table II the conditions of this hypothesis are fulfilled down to concentrations of sea water diluted with an equal volume of distilled water, the exact formula being

$$C_s = 2.0\, C_l^{\frac{1}{1.154}}. \quad \text{Hence } \frac{C_l^{0.866}}{C_s} = 0.5.$$

As the rate of nerve conduction must according to Wilhelmy's law be proportional to C_s, we may substitute it for C_s.

Thus apparently the velocity of nerve conduction is proportional to the degree of concentration of the cations sodium, calcium and potassium which are drawn from the surrounding sea water and concentrated by adsorption upon the surface of some undetermined negatively charged colloidal particles of the nerve. In fact the nerve stimulus which produces pulsation appears to be conducted by these cations themselves, taking part in some reversible chemical reaction accelerated by an enzyme.

The presence of these positively charged cations must tend to reduce the surface tension of the colloidal particles, and with it the negative potential. In fact the sudden augmentation of the negative charge upon any one of the colloidal particles would attract other positive cations from the surfaces of neighboring particles and initiate a local negative potential which would travel through the nerve.

It is more probable however that, as the velocity of nerve conduction is proportional to the degree of concentration of the adsorbed sodium, calcium, and potassium cations, these cations themselves initiate the reaction and form some reversible chemical compound with some proteid substance, thus temporarily neutralizing their positive charges and unmasking the negative charges upon the colloidal particles to the surfaces of which they were attracted by adsorption.

These various hypotheses are suggested not so much as possible explanations of the phenomenon of nerve conduction, but more to stimulate interest in the problem and to suggest directions for further research. It is perhaps a step in advance to know that, whatever its precise nature, nerve conduction is some chemical reaction involving the adsorbed sodium, calcium, and potassium cations, and thus its rate is proportional to the concentration of these adsorbed ions.

The almost instantaneous recovery observed when the medusa is taken out of dilute sea water or magnesium solutions and replaced in natural sea water, and the very short time required to assume a constant rate of nerve conduction when the medusa is placed in concentrated or diluted sea water is readily explained by the theory that nerve conduction is dependent upon the adsorption of the conducting cations by negatively charged colloidal particles and is thus a surface effect.

The previous work of Meltzer and Auer, Mines, and Mayer has shown that it is probable that the calcium enters into combination with the sodium, possibly forming an ion-proteid. It now seems probable that this intimate association between calcium and sodium takes place upon the surfaces of the particles of the colloidal nerve fluid or some other colloidal elements of the nerve.

In this brief summary it may be inexpedient to review the literature of the subject, but the bearing of this research upon the observations of Macallum, 1913, upon changes in surface tension in active cell membranes, and of Tashiro, 1914, upon the production of carbon dioxide by active nerves (see these *Proceedings*, **1**, 110), is apparent, and the contention of R. S. Lillie that cell surfaces play the important rôle in stimulation is strongly supported.

It is a great pleasure to express my sense of indebtedness to Dr. E. Newton Harvey, Prof. George Augustus Hulett, and Dr. Stewart Paton of Princeton University for timely advice and aid.

A NEW CANONICAL FORM OF THE ELLIPTIC INTEGRAL

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The elliptic norm curve E_n in space S_{n-1} admits a group G_{2n^2} of collineations, and in fact there is a single infinity of such curves which admit the same group. A particular E_n of the family is distinguished by a value of the parameter τ, itself an elliptic modular function defined by the modular group congruent to identity (mod n).

In the group G_{2n^2} there are certain involutory collineations with two fixed spaces. If E_n is projected from one fixed space upon the other, a family of rational curves R_m mapping the family of E_n's, is obtained. The quadratic irrationality separating involutory points on E_n involves the modulus τ and the parameter t of the R_m. When the genus of the modular group is zero and $n = 3, 4, 5$, the irrationality can be used to define the elliptic parameter

$$u_1 = \int \frac{(t\,dt)}{(t\,\tau)\ \alpha_\tau^{r-3}\ \alpha_t^3},$$

where α_t^r is the tetrahedral, octahedral, or icosahedral form. This is in contrast to Klein's form[1] as developed by Bianchi,[2] for there the normal elliptic integral is a rational curvilinear integral along an elliptic curve.

A comparison of the two integrals is more illuminating if it is carried out for a special case. Let E_n be E_5 in S_4. In Bianchi's notation the five quadrics having E_5 as their common intersection are

$$\varphi_i : a\,x_i^2 + a^2\,x_{i+2}\,x_{i+3} - x_{i+1}\,x_{i+4} = 0,\ (x_{i+5} \equiv x_i),\ (i = 0, \cdots 4),$$

where a is the modulus. If a transformation of coordinates is made in order to bring into evidence the fixed spaces of the involutory collineation used in the projection, then the icosahedral form which appears in the irrationality is

$$\alpha_t^{12} = t_1\,t_2\,(t_1^{10} + 11\,t_1^5\,t_2^5 - t_2^{10}).$$

The integral u_1 involving $\tau = a$ explicitly in a rather simple form is uniquely defined. Moreover it is invariant under all cogredient transformations of t and τ, which leave the form α_x^{12} unaltered, i.e., the sixty transformations of the icosahedral group applied simultaneously to t and τ, the parameter of the doubly-covered conic R_2 and the modulus of the elliptic quintic curve E_5, leave u_1 unaltered.

Consider now Bianchi's integral. It is defined as

$$U = C \int \frac{(u\,dv - v\,du)}{(\varphi_0\,\varphi_1\,\varphi_2\,u\,v)},$$

where C is a constant, u and v any two expressions linear in x, and the denominator is the functional determinant of φ_0, φ_1, φ_2, u and v. For a particular choice of u and v the integral assumes the simple form

$$U = C \int \frac{(x_0\,dx_1 - x_1\,dx_0)}{5\,a^3\,x_2\,x_4 - (2\,a^5 + 1)\,x_0\,x_1},$$

where the x's are subject to the relations φ_i. Different expressions for U can be obtained by making different choices for u and v. Hence there is no unique form for U as there is for u_1. The integral U assumes various conjugate forms under the Group G_{50} of collineations on the x's, and also under the transformations of a.

So the integral u_1 seems to have an advantage over U in its simplicity of form, its uniqueness, and its invariancy under transformations.

By a study of the integral u_1 itself some interesting results are derived. The modular equation connecting τ and J, the absolute invariant of u_1, can be deduced as the result of the binary syzygy of lowest weight connecting the concomitants of α_x^r. The requirement that the Riemann surface attached to the modular equation be regular leads to the modular equations associated with the regular bodies. It is then possible to eliminate the more tedious individual proofs used by Bianchi in the discussion of the moduli of E_3 and E_5 to show that these moduli are the tetrahedral and icosahedral irrationalities respectively. In fact the algebraic discussion carried out once for α_x^r is complete for factor groups of genus zero, which have been discussed by Klein,[3] i.e., those isomorphic with the groups associated with the regular bodies, namely, the one dihedral group G_6 and the tetrahedral, octahedral, and icosahedral groups.

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[1] Klein-Fricke, *Vorlesungen über die Theorie der Elliptischen Modulfunctionen,* Bd. 2, Abschnitt 5.

[2] Bianchi, Über die Normalformen dritter und fünfter Stufe des elliptischen Integrals erster Gattung, *Math. Ann., Leipzig,* 17, 234–262, (1880).

[3] Klein-Fricke, loc. cit., vol. 1, pp. 339 ff.

THE STRUCTURE OF COMPLEX ATOMS AND THE CHANGES OF MASS AND WEIGHT INVOLVED IN THEIR FORMATION

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A study of the atomic weights referred to hydrogen as unity shows that for the series of elements from helium to fluorine the average deviation from a whole number is 0.09 units, which is smaller than would be expected if the atomic weights were wholly the result of accident. However, the average deviation for the first 27 elements (0.21) is almost as great as would be expected as the result of chance (0.25 units). This, on first sight, does not seem to accord well with the numerical form of Prout's hypothesis, that these atomic weights should be whole numbers. If now the atomic weights referred to oxygen are considered, the remarkable relation is found that the numbers are exceedingly close to whole numbers as a rule. In order to save space these relations may be considered in the form of the results of calculations of probability. The chance that the sum of the deviations from whole numbers for the 27 lighter elements should be as small as it is when hydrogen is taken as 1 is found to be one chance in ten, when the method used for the calculation is of the form known as De Moivre's problem, and when the probable errors in the atomic weights are taken into account. In contrast with this it is found that there is only about one chance in fifteen million that the sum of the deviations from whole numbers of the atomic weights on the oxygen basis should be as small as it is.

This might seem to indicate that the atoms other than hydrogen are built up from units different from hydrogen and of a weight 1.000, while hydrogen has a weight 1.0078 (on the oxygen basis); but this would involve the existence of two kinds of particles with masses very nearly the same, and gives the additional difficulty that the atomic weights are not exact multiples of 1, which indicates that if the particle from which the atoms are built has a weight of nearly 1, this weight must vary somewhat from atom to atom. Attempts have been made to explain the discrepancy as the result of a variation in the number of negative electrons, but this method is neither in accord with recent theories of atomic structure, nor does it seem to agree with the fact that the atoms are neutral electrically.

If the assumption is made that the other atoms are hydrogen complexes, then it is found that in the formation of the complex there must result a change of weight which is in general negative in sign. The percentage decrease in weight may be called the packing effect, and the average value of this effect for the 27 lighter elements in 0.77%. The magnitude of this effect is nearly constant, as is shown by the value 0.77 for helium, while for the six elements from boron to sodium the packing effects are 0.77, 0.77, 0.70, 0.77, 0.77, and 0.77%. Since the value of the packing effect is -0.77% for oxygen, and this is the average packing effect of the 27 elements as well as the value of the effect for a considerable number of the different atoms, it is seen that the use of a whole number as the atomic weight of oxygen makes allowance for this effect and so gives to the other elements atomic weights which are whole numbers or very nearly whole numbers.

In order to determine a cause for this decrease in weight which must result if the other atoms are built up from the nuclei of hydrogen atoms as units, it will be well to consider if this effect may not be due to the overlapping of the fields of the charged particles which unite to form the *nucleus* of the complex atom. Recent work shows that this nucleus is extremely small. Rutherford, from data on the scattering of alpha particles in passing through thin gold leaf, has calculated the *upper* limit of the nucleus of the gold atom as 3.4×10^{-12} cm., and Darwin found the similar value for hydrogen to be about 0.8×10^{-13} cm., which is of the order of size of the negative electron as usually considered. If the gold nucleus is built up of hydrogen nuclei this would mean that it would contain about 197 hydrogen nuclei and 118 negative electrons in its extremely minute volume, if the idea of van den Broek and Moseley[1] that the positive charge on the nucleus is equal to the atomic number is used for the calculation. Thus over 300 particles would be contained in a volume which according to Rutherford's calculation has a radius of only 3.4×10^{-12} cm. as its upper limit of value. It must be emphasized that in this paper only the electrons contained in the nucleus of the atom are the ones which are considered, and that the distance of the external electrons, sometimes considered as of the order of the assumed radius of the atom as about 10^{-8} cm., has absolutely nothing to do with the packing effect.

The simplest case for an actual nucleus which could be calculated, is that of helium, which may be assumed to contain four positive particles and two negative electrons. However, the arrangement of these particles is not known, so the solution will be given for the case of a single positive and a single negative electron. That a change in mass will result if these two particles are brought very close together, is suggested by Lorentz,[2] but he calculates neither the sign nor the magnitude of the effect, nor does he indicate how the calculation may be made. The symbols used are **G** = electromagnetic momentum per unit volume, **H** = magnetic intensity vector, **E** = electric intensity vector, **S** = Poynting's vector for magnetic flux, c is the velocity of light, and [**AB**] represents the vector product.

Now $\mathbf{G} = [\mathbf{EH}]/c = \mathbf{S}/c^2$, and for the field due to a system of charges

$$\frac{[\mathbf{E}\mathbf{H}]}{c} = \frac{\left[\left(\sum_i \mathbf{E}_i\right)\left(\sum_j \mathbf{H}_j\right)\right]}{c} = \frac{\sum_i [\mathbf{E}_i\mathbf{H}_i]}{c} + \frac{\sum_{(ij)}[\mathbf{E}_i\mathbf{H}_j]}{c}$$

where the summation $\Sigma_{(ij)}$ is the vector product of each i with each j. The first summation gives the electromagnetic momentum which would be due to the particles if their fields did not overlap, and the second term gives the effect of the overlapping of the fields. This may be called the "Mutual electromagnetic momentum," and is designated by $\overline{\mathbf{G}}$.

For point charges $E_1 = \dfrac{(1 - u^2)e_1}{4\pi r^2(1 - u^2 \sin^2 \theta_1)^{3/2}}$. Let $(1 - u^2) = K^2$ and $(1 - u^2 \sin^2 \theta_1) = \beta_1^2$. The transverse component of E due to the two particles 1 and 2 is

$$E_t = \frac{K^2 e}{4\pi}\left\{\frac{\sin \theta_1}{r_1^2 \beta_1^3} \pm \frac{\sin \theta_2}{r_2^2 \beta_2^3}\right\}$$

where the sign is positive if the charges have the same sign, and *negative if they are of opposite sign.* As only the longitudinal component of the vector **G** is desired, only the transverse component of **E** is needed. Then $H = uE \sin \phi/c$, where ϕ = the angle between E and the direction of u.

If E_t is used, $\phi = 90°$, and $H = u\,(E_1 \sin \theta_1 \pm E_2 \sin \theta_2)/c$. Hence

$$\mathbf{G} = \frac{[\mathbf{E}_t\mathbf{H}]}{c} = \frac{u}{c^2}(E_1 \sin \theta_1 \pm E_2 \sin \theta_2)\,(E_1 \sin \theta_1 \pm E_2 \sin \theta_2),$$

$$\text{and } \overline{\mathbf{G}} = \pm\frac{2u}{c^2}\int E_1E_2 \sin \theta_1 \sin \theta_2 d\tau = \frac{2uK^4e^2}{c^2(4\pi)^2}\int\frac{\sin \theta_1 \sin \theta_2}{r_1^2r_2^2\beta_1^3\beta_2^3}\,d\tau.$$

Now $r^2\beta^2 = r^2 - u^2\,(r^2 \sin^2 \theta)$ and $r^2 \sin^2 \theta = y^2$. Let $a = \frac{1}{2}$ distance between e_1 and e_2. Neglecting all terms in u^2, and placing $d\tau = 2\pi y dy dx$, we have

$$\overline{\mathbf{G}} = \pm\frac{uK^4e^2}{8c^2\pi^2}4\pi\int_0^\infty\int_0^\infty\frac{y^3 dy dx}{\sqrt{\{[(x - a)^2 + y^2]\,[(x + a)^2 + y^2]\}^3}}$$

which is obtained by making use of the symmetry of the equation. The evaluation of this integral gives the value $1/2a$ so $\overline{\mathbf{G}} = \pm uK^4e^2/(4\pi ac^2)$. The mass represented by this value of $\overline{\mathbf{G}}$ is $\Delta m = \pm e^2/(4\pi ac^2)$ where Δm is the change of mass due to the overlapping of the fields of the electrons. Now the longitudinal mass is $m_1 = e^2/(6\pi c^2R)$ where R = radius of the electron. By division $\Delta m/m_1 = 3R/2a$, where a equals one-half the distance apart of the electrons.

Taking R as the radius of the positive electron, and -0.77% as the packing effect, it is seen that in this simple case the distance apart of the positive and the negative electrons would be 400 times the radius of the positive electron. However, the system used for the calculation does not correspond to any actual atom, but if it is considered that the gold nucleus is *smaller* than corresponds to a radius of 3.4×10^{-12} cm., as Rutherford calculates, then when it is considered that the large number of positive and negative electrons contained in this space must make up a very complex system, undoubtedly with a special structure, it is evident that the distances of the electrons as calculated to give the observed packing effect, is of the right order of magnitude to give a gold nucleus of the kind supposed by Rutherford's theory.

The Hydrogen-Helium System. Fajans,[3] Soddy,[4] Russell,[5] von Hevesy,[6] and Fleck,[7] have proved that when a radioactive element ejects an alpha particle, found by Rutherford to have a mass of 4 units, and to give ordinary helium gas, the new substance produced has different properties and a different valence from the parent material. The change is such that the new element lies two groups to the left in the periodic table and therefore has an atomic number and a valence with values two less than before. Now that this relation has been found to apply to elements of high atomic weights, the question arises as to whether the same relations hold for the lighter atoms which have not been found to give an appreciable alpha disintegration. If they do, then the atomic weights of the elements of even atomic number could best be found by beginning with helium, and adding a weight of 4 for each step of two atomic numbers, and by proceeding in the same way beginning with lithium for the odd numbered elements. This gives:

Atomic number even = 4 8 12 16 20 24 28 32
Atomic number odd = 7 11 15 19 23 27 31 35

which are on the whole the correct atomic weights. The system obtained in this way may be best represented in the form of a periodic table as in the table.

When given in this way it is seen that the atomic weights not only follow the helium system derived from the behavior of the radioactive elements, but in addition, after this has been applied it is seen that there is a great regularity in the number of hydrogen atoms, H_3, which must be included to give the atomic weights of the elements of odd atomic number.

The table gives an explanation of the fact that the atomic weight of argon is higher than that of potassium while its atomic number is lower. This is seen to be due to the general tendency for the increment between the third and fourth series to take the value 5He, and in this sense it is the potassium and calcium which are irregular in behavior, and not the argon. In this part of the table there is seen to be a tendency to add 2He instead of 1He for one step between elements of even atomic number.

SYMBOLIC REPRESENTATION OF THE ATOMIC WEIGHTS ACCORDING TO THE HELIUM SYSTEM DERIVED FROM THE BEHAVIOR OF THE RADIOACTIVE ELEMENTS IN THEIR ALPHA DISINTEGRATIONS

H DETD. = 1.0078

GROUP	0	1	2	3	4	5	6	7	8	
Series 2	He	Li	Be	B	C	N	O	F		
	He	He+H_3	2 He+H	2 He+H_3	3 He	3 He+2 H	4 He	4 He+H_3		
Calc.		7.	9.	11.	12.	14.	16.	19.		
Detd.	4.	6.94	9.1	11.	12.	14.01	16.	19.		
Series 3	Ne	Na	Mg	Al	Si	P	S	Cl		
	5 He	5 He+H_3	6 He	6 He+H_3	7 He	7 He+H_3	8 He	8 He+H_3		
Calc.	20.	23.	24.	27.	28.	31.	32.	35.		
Detd.	20.	23.	24.3	27.1	28.3	31.02	32.07	35.46		
Series 4	A	K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co
	10 He	9 He+H_3	10 He	11 He	12 He	12 He+H_3	13 He	13 He+H_3	14 He	14 He+H_3
Calc.	40.	39.	40.	44.	48.	51.	52.	55.	56.	59.
Detd.	39.88	39.1	40.07	44.1	48.1	51.	52.	54.93	55.84	58.97

Increment from series 2 to series 3 = 4 He.
Increment from series 3 to series 4 = 5 He. (For K and Ca = 4 He.)
Increment from series 4 to series 5 = 6 He.

If a weight of 4 is added for each increase of 2 in the atomic number, then the average increase of the atomic weight per atomic number should be 2, and that this is in accord with the facts is shown by the atomic weights of neon and calcium. These elements have the atomic numbers 10 and 20, and the atomic weights, $10 \times 2 = 20$, and $20 \times 2 = 40$. The equation[8] which gives the atomic weights of the lighter elements is $W = 2n + \frac{1}{2} + \frac{1}{2}(-1)^{n-1}$, where n is the atomic number. In order to include the heavier elements it is necessary to insert another term, the meaning of which will be considered in a later paper, to account for the tendency of the increment of weight to become greater as the atomic weight increases, as follows: $W = 2(n + n') + \frac{1}{2} + \frac{1}{2}(-1)^{n-1}$.

That no system can explain the atomic weights of the heavier elements, unless account is taken of the fact that different series of elements exist, is evident when it is considered that the isotopes of lead, radium B, and lead from radium, differ in atomic weight by eight, or by 4%, and even in the case of some of the lighter elements that a part of the irregularity may be due to the existence of isotopic forms is indicated by the discovery by Thomson and Aston that neon, formerly supposed to have an atomic weight of 20.2, consists of two isotopes of atomic weights 20 and 22. It has been proved by Fajans and others, as has been mentioned, that the helium system holds for the heavy radioactive elements; in this paper it is shown that it also holds for the 27 lighter elements, which would seem to make it almost certain that it holds for the other elements when the proper allowances are made for variations in packing effect, and differences of series.

For the elements from helium to iron the average value of the packing effect is 0.78 units, which is, so far as the accuracy of the atomic weights allows us to determine, the same as the decrease in weight which would occur if four hydrogen atoms were to be transformed into one of helium. This means that if any element of higher atomic weight is built up entirely of helium atoms, then, on the average, the decrease in mass is practically entirely due to the primary formation of the helium atoms, and not at all to the aggregation of these into the heavier atoms. From this point of view an atom composed entirely of helium units would have extreme instability in relation to its disintegration into helium units, in comparison with its instability with reference to a hydrogen decomposition. Such an atom in a radioactive transformation should lose alpha particles very much more readily than hydrogen nuclei, in fact, if it is remembered that even the alpha decomposition does not occur appreciably for many of the known elements, it will be seen that it is doubtful if such an atom would ever give a detectable hydrogen disintegration.

If the elements of higher atomic weight are built up exactly according to the special system presented in the table, according to which the members of the even numbered groups are in general aggregates of helium alone, then since all of the radioactive elements which are *now known* to give a simple alpha disintegration, belong to even numbered groups, they could not be expected to give hydrogen upon disintegration. Thus one of the chief objections to the theory that the atoms are hydrogen complexes, which is based on the fact that up to the present time no hydrogen has been detected as the product of any radioactive change, is seen to be not contrary to, but rather in accord with the theory as here presented. However, the value of this idea is lessened by the fact that there is no evidence that the exact relations between the composition of the odd and even groups as found for the elements of smaller atomic weight hold so far into the system as the radioactive elements. The general form of the system indicates at least that the heavy atoms contain more helium than independent hydrogen units, and this seems in accord with the fact that uranium loses alpha particles in eight steps without any apparent loss of a hydrogen nucleus.

The stability with which the hydrogen nuclei which are not contained in helium groups, but which generally occur in threes (H_3 in the table), are built into the complex atom is not in general indicated with any degree of accuracy, but in the case of lithium it seems to be great, for lithium shows the extremely large packing effect equal to 1.57%, which might seem doubtful if it were not for the extreme care taken by Richards and Willard[9] in the determination of the atomic weight of lithium. The atomic weight of hydrogen used in these papers (1.0078) is probably not in error[10] by so much as one part in 5000.

The hydrogen-helium system here presented is entirely in accord with, but independent of, the astronomical theory that the order in which the elements appear in the stars is first nebulium, hydrogen and helium, then such of the lighter elements as calcium, magnesium, oxygen, and nitrogen, and finally iron, and the other heavy metals, although in the present system it has not been found necessary to include nebulium. Some of the nebulae give bright line spectra of nebulium, hydrogen and helium; such Orion stars as those of the Trapezium give lines for hydrogen and helium; while those that are more developed show magnesium, silicon, oxygen and nitrogen, and some of the other low atomic weight elements in addition. Bluish white stars such as Sirius give narrow and faint lines for iron, sodium, and magnesium, and the solar stars give a much weaker hydrogen spectrum, and many more and stronger lines for iron and the heavy metals.[11] The astronomical theory that the heavier elements are thus formed from those of smaller atomic weight is of extreme interest, but the evidence for it is somewhat uncertain, since it is possible that it is the difference in the density of the different elements which is the effective factor in causing the spectra to appear in the order in which they are found to occur. The relative brightness of the different lines varies greatly, such lines as the calcium H and K lines being extremely strong, and this also interferes with the determination of the order of the appearance of the elements in the stars. On the other hand the evidence presented in these papers seem to give some support to the theory of the evolution of the heavier atoms from those which are lighter, but the hydrogen-helium system here presented has the support of much better evidence than that for atomic evolution from the standpoint of astronomy alone.

We wish to thank Professor A. C. Lunn of the Department of Mathematics, for outlining for us the method for determining the distances between the positive and negative electrons.

[1] Moseley, *Phil Mag., London,* **26,** 1024; **27,** 703.
[2] Lorentz, *The Theory of Electrons,* p. 47.
[3] K. Fajans, *Physik. Zs., Leipzig,* **14,** 131 and 136.
[4] F. Soddy, *Chem. News, London,* **107,** 97; *Jahrb. Radioakt.,* **10,** 188.
[5] Russell, *Chem. News, London,* **107,** 49.
[6] Von Hevesy, *Physik. Zs., Leipzig,* **14,** 49.
[7] Fleck, *Trans. Chem. Scc.,* **103,** 381 and 1052.
[8] Rydberg, *Zs. anorg. Chem., Hamburg,* **14,** 86–7.
[9] Richards and Willard, *J. Amer. Chem. Soc.,* **32,** 4 (1910).
[10] W. A. Noyes, *Textbook of Chemistry,* p. 72.
[11] Hale, *The Study of Stellar Evolution.*

HUNTINGTON'S CHOREA IN RELATION TO HEREDITY AND EUGENICS

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It is now generally conceded that predisposition plays an important rôle in mental disorder but the views as to the nature of its part are very diverse and hazy. Some investigators conclude that there is an inheritance of a particular type of mental trouble, while others conceive that only a general psychopathic constitution is inherited. Again a difference of view maintains as to the nature of the recognized mental 'diseases'—by some these are regarded as distinct 'entities,' manifestations of a single, indivisible, disturbing factor. Others conceive of them as syndromes or chance associations of symptoms. Even those who hold the latter view would perhaps except as the one clear case of a neuropathic entity the condition known as 'Huntington's Chorea.' This chorea is defined by the following traits: (1) persistent tremors of the head, appendages and trunk; (2) the onset of such tremors in middle or late life; (3) the progressive nature of the tremors; and (4) progressive mental deterioration. These characters are frequently found together; is their association a necessary one?

A study of four family complexes in eastern Long Island, south-western Connecticut, south-central Connecticut and eastern Massachusetts which show nearly a thousand cases of Huntington's chorea yields the remarkable result that practically all can be traced back to some half-dozen individuals, including three (probable) brothers who migrated to America during the 17th century. But, already, numerous 'biotypes' having specific and differential hereditary behavior have appeared. Thus there is a biotype in which the tremors are absent but mental deterioration present; a biotype in which the tremors are not accompanied by mental deterioration; a biotype in which the chorea does not progress; and a biotype in which the onset of the choreic movements is in early life. In general, the symptomatology of chronic chorea is dissimilar in different strains of families. The age of onset, the degree of muscular involvement, the extent of mental deterioration all show family differences and enable us to recognize various species, or biotypes, of the disease. These biotypes are less striking than they would be were it not for the extensive hybridization that is taking place between biotypes in random human matings.

The method of inheritance of some of the elements of Huntington's chorea has been worked out. In general, the choreic movements never skip a generation and in other respects show themselves clearly to be a dominant trait. The mental disorder is usually of the hyperkinetic or manic type and this also shows itself as a dominant. The age of onset *apparently* tends to diminish in successive generations—'law of anticipation'—but this is partly, if not wholly, illusory and is due to the fact that in comparing the age of onset in grandfathers with that in grandchildren we are not comparing on the same basis, for the grandparents are a selected lot (selected on the basis of late onset—at least late enough for them to become parents), while grandchildren include those in whom the onset is so early in life that they will never marry. If instead of comparing the average age of onset in successive generations, one compares the age of onset in a number of choreic parents, *their* parents and their grandparents, then the evidence for anticipation vanishes. Eight such series give for average age of onset of the propositus 35.5 years, parent 38.8, grandparents 36.9. In this series we can see no evidence of anticipation.

Among the 3000 odd relatives of the 962 choreics studied many nervous traits are recorded. Thus epilepsy is recorded 39 times, infantile convulsions 19 times, meningial inflammations and brain fever 51 times, hydrocephaly 41 times, feeblemindedness 73 times, Sydenham's chorea 11 times, and tics 9 times, mostly in one small family. This incidence, which would seem high for an unselected population, suggests that chorea occurs in families characterized by a general liability to nervous and mental troubles.

Though it can be shown that the 962 cases of chorea originated from 6 or 7 ancestors and that the tendency has been handed down almost without a break through the generations and that for generations there have been individuals who recognized the hereditary nature of the disease and were influenced in marriage accordingly; nevertheless, there is no clear evidence that persons belonging to the choreic lines voluntarily abstain to any marked degree from, or are selected against, in marriage.

THE ALCYONARIA AS A FACTOR IN REEF LIMESTONE FORMATION

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The early students of coral reef formation based their conclusions concerning the depth at which reef formation could take place and the rate at which material was added to such reefs upon the known bathymetrical distribution and growth rate of stony corals (Madreporaria). The results obtained from the examination of the cores from borings made at the Island of Funafuti in the South Pacific showed that, in this particular region at least, calcareous algae of the genus *Halimeda* were a very important factor in reef limestone formation. Among the other group of lime secreting animals, those species of Alcyonaria, which form a massive skeleton have received proper recognition as contributing to reef formation. The remaining representatives of this group have been neglected as reef formers because their limy secretion is laid down in the form of minute spicules which are set free at the time of the disintegration of the living tissues of the colony leaving no recognizable skeleton.

On most coral reefs in all parts of the world the Alcyonaria with free spicules are much more abundant than those forming a massive skeleton while in many regions they constitute by far the greatest part of all lime secreting organisms living permanently attached to the bottom. On many reef areas in the Florida-Antillean region the areas occupied by stony corals and calcareous algae are relatively small while flexible alcyonaria or the family Gorgonaceae make up the most characteristic feature of the fauna. The spicules of these organisms are found in practically all bottom samples both from the crests of the reefs and in the soft mud from the channels between the reefs so it is evident that they may be incorporated into the reef limestones before they have undergone marked erosion.

In order to determine the amount of material contributed to reef formation, during any given time, by the gorgonians, three factors must be taken into consideration: first, the amount of lime held as spicules in the tissues of these colonies; second, the bulk of the gorgonians present on any reef area; and third, the number of colonies which will set free their spicules through the death and subsequent disintegration of their coenenchyma.

1. *Spicule content.* In making the determination of the amount of spicules for any species the colony was removed from the reef without injury to any portion except the expanded base, which was always discarded as it was difficult to separate from it fragments of limestone from other sources. The colony was then weighed while still wet, cut into small pieces and the living tissue destroyed by treatment with a cold 25% solution of caustic soda. When the organic material had been dissolved the spicules were washed in rain water until no trace of organic matter remained, collected on a weighed filter, dried in a water bath kept at 100°C. and after cooling in a dessicator to room temperature, carefully weighed. By this procedure the proportion of spicules to the fresh weight of the colony could be estimated for any collection of gorgonians by simply separating the several species and determining the weight of each one.

As previous studies upon the growth rate and ecology of the gorgonians of this region had shown that at least nine-tenths of the bulk of these organisms on any reef is made up of not more than twelve species, the spicule determinations were consequently restricted to these forms. The results of analyses of the twelve most common species is shown in the table in which the figures given represent the average of five determinations for each species.

AVERAGE WEIGHT IN POUNDS AND PERCENTAGE OF SPICULES IN GORGONIAN COLONIES

Species of gorgonian	*Average weight of colonies*	*Percent spicules*	*Species of gorgonian*	*Average weight of colonies*	*Percent spicules*
Briareum sp.	2.00	26.66	Plexaurella dichotoma	0.75	35.86
Eunecia rousseaui	1.00	35.60	Plexaurella sp.	0.75	24.92
Eunecia crassa	0.30	22.66	Gorgonia flabellum	1.00	22.33
Plexaura flexuosa	1.00	30.66	Gorgonia acerosa	3.00	19.75
Plexaura homomalla	1.50	27.41	Gorgonia citrina	0.25	35.05
Pseudoplexaura crassa	2.50	21.48	Xiphigorgia anceps	0.25	25.83

Average proportion of spicules for all species 27.40%

The determinations given above were made primarily to afford a basis from which the spicule content of large masses of gorgonians could be calculated and in order to have a check for these computations a series of twenty analyses were made of the gorgonians growing upon a square yard of the surface of widely separated reefs. The bulk of the gorgonians varied from 1.5 pounds to 25.0 pounds, the spicule content from 0.45 pounds to 6.94 pounds, for the different squares. The average for all determinations was 7.746 pounds of gorgonian colonies containing 2.1225 pounds of spicules.

2. *Distribution of gorgonians on the reefs about Tortugas.* To determine the distribution of the species of gorgonians included in the foregoing analyses on the reefs about Tortugas a series of lines were laid out extending over a number of the most important reefs and along these lines counts of the number of gorgonians occurring on a square yard were made at regular intervals, with the following results:

	Squares	*Average number of colonies*	*Number barren squares*
Line No. 1	45	5.72	8
Line No. 2	150	8.97	14
Line No. 3	30	10.86	3
Line No. 4	25	7.62	1
Line No. 5	40	13.27	7
Line No. 6	36	5.86	5

In these counts the number of colonies was alone recorded for each square yard. The weight of the mass of gorgonians on any square was estimated upon the basis of the determinations shown in the second column of the first table in which the figures represent the average weight for twenty medium sized colonies of each species.

Computed upon this basis the average weight of the gorgonians from each square along line No. 1, as given above, is 7.32 pounds. Estimating the spicule contents on the basis of the figures obtained from the analyses recorded in the table it is shown that there would be two pounds of spicules in the gorgonians from any square yard of reef area along this line. This result approaches very closely the amount found by actual analyses of the colonies from twenty representative squares. The number of colonies to the square along this line was the smallest found in the counts on any of the reefs so that when estimated upon the same basis the spicule content of the gorgonians on the squares along the lines on any of the other reefs would be greater than that found by actual analyses for the twenty squares previously mentioned. Using the smaller figure, which is well below that obtained by careful observation, the amount of lime held as spicules in the tissues of living gorgonians on a representative acre of reef area is 5.28 tons.

3. *Disintegration of the gorgonian colonies and the addition of their spicules to the reef building materials.* The spicules held in the tissues

of Gorgonian colonies represent only a potential contribution to reef formation. The actual number of gorgonians destroyed on any reef area in a given year can not be determined save by actual observation extending over the entire period, a method that had been impossible to carry out. A considerable number of observations bearing directly upon this phase of the subject has, however, been accumulated in the course of ecological studies on the gorgonians of the Tortugas regions, extending over a period of five years.

These observations have shown that when a gorgonian colony is removed from its place of attachment on the reef and allowed to lie on the bottom where it will be moved by the action of tidal currents death follows within a short time. The amount of time necessary for the complete disintegration of the coenenchyma of a colony of each of the several species which make up the most important elements in the gorgonian fauna about Tortugas was determined. This series of observations showed that when torn from the bottom, the most resistant form—Gorgonia flabellum—would be disintegrated within 120 hours, while a colony of Pseudoplexaura crassa would be broken down in 18 hours. Under normal conditions of the reefs the greatest number of the axial skeletons of dead colonies are found in positions which indicate that the tearing of the colony from its normal attachment by wave action is the cause of the greatest mortality among these animals. When destroyed in this manner the spicules of any gorgonians would be added to the limestone forming materials on the reef within a few days, at most, from the time when the colony was torn from its place of attachment and consequently before the spicules had undergone any noticeable erosion.

Next to the destruction by wave action (storms) the greatest mortality of the gorgonians is brought about by the overgrowth of their tissues by other organisms. The most important of these agents are the hydrocoralline *Millipora alcicornis* and some encrusting bryozoa. Under the influence of both these agents the tissues of the gorgonian are disintegrated very slowly and the spicules set free a few at a time. While the *Millipora* will eventually cause the complete destruction of a colony the bryozoa were never found to extend for more than a few inches above the base of the colony. The actions of the two sorts of organisms is markedly different in that the Millipora causes the death of the gorgonian tissues by some chemical action and itself encrusts only the axial skeleton of the colony; while the bryozoa form a cylindrical tube surrounding the base of the gorgonian and cause the death of its tissues by the exclusion of food and oxygen.

There is no evidence that gorgonian colonies ever die from old age and to the agencies mentioned above is to be attributed the destruction of all those colonies the skeletons of which are found on the reefs. The fascies of the gorgonian fauna on any reef remains practically unchanged from year to year except for the introduction of the unusual factor of extensive destruction of the colonies by severe storms. The determination of the number of skeletons of dead colonies on any reef area will, therefore, afford the most practicable method of obtaining the death-rate of these organisms. This method unfortunately has the serious drawback that the results will always be too low because when the colonies are torn from their attachment on the reef by severe storms some of them will be carried so far from their original location that they will not be included in a count of skeletons on any given area. The percentage of axial skeletons to living colonies on a small reef where the entire number of colonies has been counted each year since 1910 has varied from 11.67% to 24.80% with an average of 17.22% for the six counts. On other reefs in the Tortugas region the destruction of gorgonians was nearly complete in the hurricane of October 17, 1910. In some restricted areas the amount of spicules set free from disintegrating gorgonian colonies was as much as 25 pounds to the square yard. On the basis of all the recorded observations it seems probable that, at a conservative estimate, about one-fifth of the gorgonian colonies on any reef area will be destroyed annually.

Since the fascies of the gorgonian fauna on the reefs remains relatively constant, and it is estimated that about one-fifth of the number of colonies are destroyed annually, it is important to determine the growth rate of the several species of these organisms which make up the most important elements in the gorgonian fauna.

Growth records for the species entering into this study have been kept for a period sufficiently long to cover the period from the time of their attachment as planulae to the attainment of the normal average size of the colony. The time necessary for this growth varies from two years (Gorgonia citrina), to five years (Gorgonia acerosa). The time necessary for any of these colonies to reach the average size falls well within the limit marked by the observed rate of destruction which would be necessary to maintain the constancy of the gorgonian fauna of any reef. Some few of these species appear to keep growing for an indefinite period after the medium size for the colony has been reached, but in such instances the growth is very slow after this time and would not greatly influence the general results.

TRANSFORMATIONS OF CONJUGATE SYSTEMS WITH EQUAL INVARIANTS

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Recently we considered at length two types of transformations of conjugate systems of curves on surfaces, which we called[1,2] *transformations K* and *transformations Ω*. It is the purpose of this note to show that there is a fundamental relation connecting these transformations.

If x, y, z are the cartesian coördinates of a surface S, referred to a conjugate system with equal point invariants, it is necessary and sufficient that x, y, z are solutions of an equation of the form

$$\frac{\partial^2\theta}{\partial u\partial v}+\frac{\partial\log\sqrt{\rho}}{\partial v}\frac{\partial\theta}{\partial u}+\frac{\log\sqrt{\rho}}{\partial u}\frac{\partial\theta}{\partial v}=0, \tag{1}$$

where ρ is a function of u and v in general. If θ_1 is any solution of (1) and λ_1 is the function defined by the consistent equations

$$\frac{\partial\lambda_1}{\partial u}=-\rho\frac{\partial\theta_1}{\partial u},\quad \frac{\partial\lambda_1}{\partial v}=\rho\frac{\partial\theta_1}{\partial v}, \tag{2}$$

the equations

$$\frac{\partial}{\partial u}(\lambda_1 x_1)=-\rho\left(x\frac{\partial\theta_1}{\partial u}-\theta_1\frac{\partial x}{\partial u}\right),\quad \frac{\partial}{\partial v}(\lambda_1 x_1)=\rho\left(x\frac{\partial\theta_1}{\partial v}-\theta_1\frac{\partial x}{\partial v}\right) \tag{3}$$

are consistent, and x_1 satisfies an equation of the form (1) when ρ is replaced by ρ_1, given by

$$\rho_1=\lambda_1^2\big/\rho\theta_1^2. \tag{4}$$

The three functions x_1, y_1, z_1, given by (3) and similar equations in the y's and z's, are the cartesian coördinates of a surface S_1, which by definition is in the relation of a transformation K with the surface S. If M and M_1 are corresponding points on these surfaces, the developables of the congruence G of the lines MM_1 cut S and S_1 in the parametric curves and the focal points on the line MM_1 are harmonic to M and M_1. Conversely, when two surfaces S and S_1 are so related that the congruence of lines joining corresponding points meet S and S_1 in conjugate systems and the points of S and S_1 are harmonic to the focal points of the congruence, they are in the relation of a transformation K, as defined analytically by (3) [cf. M_1, p. 403].

Whenever any two surfaces are in a one-to-one correspondence such that the developables of the congruence of lines joining corresponding points cut the surfaces in conjugate systems, the lines of intersection of corresponding tangent planes to these surfaces form a congruence G' whose developables correspond to the developables of the former congruence.[3] Evidently two surfaces in the relation of a transformation K possess this property, but it is not a characteristic property.

When any surface S is subjected to a polar transformation with respect to a quadric, points and tangent planes are transformed into tangent planes and points respectively of the new surface Σ. Since straight lines go into straight lines, it is readily shown that conjugate directions on S go into conjugate directions on Σ. Also a congruence of lines is transformed into a congruence of lines, and the developable surfaces of the two congruences correspond; furthermore to the focal points on a line correspond the focal planes through the corresponding line of the other congruence.

We consider the effect of applying a polar transformation to two surfaces S and S_1 in the relation of a transformation K. If the new surfaces be denoted by Σ and Σ_1, the lines joining corresponding points $\overline{M}$ and $\overline{M}_1$ on these surfaces form a congruence $\overline{G}'$ whose developables meet Σ and Σ_1 in conjugate systems, and the tangent planes to Σ and Σ_1 meet in the lines of a congruence $\overline{G}$ whose developables correspond to the developables of the congruence $\overline{G}'$; moreover, the focal planes of the congruence $\overline{G}$ are harmonic to the tangent planes to Σ and Σ_1. These properties of the surfaces Σ and Σ_1 are possessed likewise by a pair of surfaces in the relation of a transformation[2] Ω. It is our purpose to show that Σ and Σ_1 are in the relation of a transformation Ω, and that the properties just mentioned are characteristic of transformations Ω.

The equation of any quadric may be put in the form

$$ax^2 + by^2 + cz^2 + 2\,dx + 2\,ey + 2\,fz + g = 0. \tag{5}$$

The equation of the polar plane of the point (x, y, z) with respect to this quadric is

$$Xx' + Yy' + Zz' = W. \tag{6}$$

where x', y', z' are the current rectangular coördinates and

$$X = \frac{ax+d}{\sqrt{\sigma}}, \quad Y = \frac{by+e}{\sqrt{\sigma}}, \quad Z = \frac{cz+f}{\sqrt{\sigma}}, \quad W = -\frac{dx+ey+fz+g}{\sqrt{\sigma}},$$

$$\sigma = (ax+d)^2 + (by+e)^2 + (cz+f)^2. \tag{7}$$

Hence X, Y, Z are the direction-cosines of the plane (6), and if x, y, z are the cartesian coördinates of a surface S, then X, Y, Z and W are the tangential coördinates of its transform Σ. These four coördinates are solutions of the equation

$$\frac{\partial^2\varphi}{\partial u\partial v}+\frac{\partial\log\sqrt{\bar{\rho}}}{\partial v}\,\frac{\partial\varphi}{\partial u}+\frac{\partial\log\sqrt{\bar{\rho}}}{\partial u}\,\frac{\partial\varphi}{\partial v}+F\varphi=0, \tag{8}$$

where

$$\sqrt{\bar{\rho}}=\sqrt{\rho\sigma},\quad F=\frac{1}{\sqrt{\sigma}}\,\frac{\partial^2\sqrt{\sigma}}{\partial u\partial v}+\frac{\partial\log\sqrt{\rho}}{\partial v}\,\frac{\partial\log\sqrt{\sigma}}{\partial u}+\frac{\partial\log\sqrt{\rho}}{\partial u}\,\frac{\partial\log\sqrt{\sigma}}{\partial v}. \tag{9}$$

Consequently the parametric conjugate system on Σ has equal tangential invariants.

When the same polar transformation is applied to S_1, the tangential coördinates of Σ_1, namely X_1, Y_1, Z_1, W_1, are obtained from (7) on replacing x, y, z by x_1, y_1, z_1 respectively. These functions satisfy an equation of the form (8), obtained on replacing ρ and F by ρ_1 and F_1, where in consequence of (4),

$$\sqrt{\bar{\rho}_1}=-\lambda_1\sqrt{\sigma_1}\Big/\sqrt{\rho}\,\theta_1,\quad \sigma_1=(ax_1+d)^2+(by_1+e)^2+(cz_1+f)^2, \tag{10}$$

and F_1 is analogous to F.

Because of equations (3), the functions X_1, Y_1, Z_1, W_1 are the respective transforms of X, Y, Z, W by means of the equations

$$\frac{\partial}{\partial u}(\lambda_1\sqrt{\sigma_1}\varphi_1)=-\rho\sigma\left(\varphi\frac{\partial w_1}{\partial u}-w_1\frac{\partial\varphi}{\partial u}\right),\ \frac{\partial}{\partial v}(\lambda_1\sqrt{\sigma_1}\varphi_1)=\rho\sigma\left(\varphi\frac{\partial w_1}{\partial v}-w_1\frac{\partial\varphi}{\partial v}\right), \tag{11}$$

where

$$w_1=\theta_1/\sqrt{\sigma}. \tag{12}$$

Consequently w_1 is a solution of (8). From (9) and (10) it follows that

$$\lambda_1\sqrt{\sigma_1}=-\sqrt{\bar{\rho}\bar{\rho}_1}\,w_1. \tag{13}$$

Hence equations (11) are equivalent to those of a transformation[2] Ω.

We are now in a position to prove the theorem:

If Σ and Σ_1 are so related that for the congruence of lines of intersection of corresponding tangent planes π and π_1 to Σ and Σ_1 respectively the focal planes of the congruence are harmonic to π and π_1, and the developables of the congruence correspond to conjugate systems on Σ and Σ_1, the latter systems have equal tangential invariants; and Σ and Σ_1 are in the relation of a transformation Ω. For if we apply the polar transformation to the surfaces Σ and Σ_1, the resulting surfaces are related in the manner which we have stated to be characteristic of a transformation K. But as we have just shown, the surfaces Σ and Σ_1, being polar transforms of two surfaces in the relation of a transformation K, are themselves in the relation of a transformation Ω. Hence we have proved the above theorem and also the following:

When two surfaces S and S_1 are in the relation of a transformation K, their polar transforms are in the relation of a transformation Ω; and conversely.

Because of the dual relation between these two types of transformations, we are enabled to add to Theorems 4 and 6 of memoir M_1 the dual of the last part of the theorem of §4 of memoir M_2, and thus have the following theorem of permutability of the transformations K: *If S_1 and S_2 are two surfaces arising from S by transformations K, there can be found by quadratures an infinity of surfaces S', each of which is in the relation of transformations K with both S_1 and S_2. If M, M_1 and M_2 denote corresponding points on S, S_1 and S_2, the corresponding points M' on the surfaces S' lie on a line through M and in the plane π of the points M, M_1, M_2. The corresponding tangent planes to the surfaces S' envelope a quadric cone to which are tangent the tangent planes to S, S_1, S_2 at M, M_1, M_2. Moreover, the plane π touches its envelope at the point of intersection P of the lines MM' and M_1M_2; and the parametric curves on the envelope form a conjugate system whose tangents are harmonic to the lines MM' and M_1M_2, and contain the focal points of the lines MM_1, MM_2, $M'M_1$, $M'M_2$.* An analogous theorem of permutability of transformations Ω follows from the above in accordance with the principle of duality.

The relation between the two types of transformations is likewise helpful in interpreting the significance of certain evident forms of the transforming functions θ_1 and w_1. Thus we have shown (M_1, p. 401) that the necessary and sufficient condition that for two surfaces S and S_1 in the relation of a transformation K the corresponding tangent planes be parallel is that θ_1 be constant. In this case S and S_1 are associate surfaces, that is not only are the tangent planes parallel but also to asymptotic lines on either surface corresponds a conjugate system on the other. Moreover, any two associate surfaces are in this special kind of relation of a transformation K. Let S and S_1 be two associate surfaces and apply to them the polar transformation with respect to the quadric (5), where a, b, c are different from zero. Since corresponding tangent planes to S and S_1 meet in a line in the plane at infinity, the lines joining corresponding points on Σ and Σ_1 meet in a point—the pole of the plane at infinity with respect to the quadric. From (7) it is seen that the coördinates of this point are $-d/a$, $-e/b$, $-f/c$. Conversely, if the lines joining corresponding points on two surfaces Σ and Σ_1, in the relation of a transformation Ω, meet in a point M, the surfaces S and S_1 arising from Σ and Σ_1 by a polar transformation are so placed that the lines of intersection of corresponding tangent planes to S and S_1 lie in a plane, the polar plane of M. When in particular, the fundamental quadric of the transformation is chosen so that M is the pole of the plane of infinity (which can always be done) S and S_1 are associate surfaces.

When θ_1 is a constant, it follows from (12) that w_1 is equal to $1/\sqrt{\sigma}$ to within a constant multiplier, which is unessential, as is evident from (11) and (13). From (7) we have

$$W + \frac{d}{a}X + \frac{e}{b}Y + \frac{f}{c}Z = \frac{1}{\sqrt{\sigma}}\left(\frac{d^2}{a} + \frac{e^2}{b} + \frac{f^2}{c} - g\right). \qquad (14)$$

We choose the quadric so that the coefficient in (14) of $1/\sqrt{\sigma}$ is not equal to zero. Hence w_1 is a homogeneous linear function of X, Y, Z and W, when the lines joining corresponding points on Σ and Σ_1 are concurrent. Conversely, suppose that w_1 is a homogeneous linear function of the form of the left-hand member of (14). Apply to Σ the polar transformation with respect to the quadric (5) and let S be the transform of Σ. Take an associate surface of S, say S_1. When now the transformation with respect to the quadric (5) is applied to S_1, we get a surface Σ_1 in the relation of a transformation Ω to Σ, the function w_1 differing by a constant factor at most from the left-hand member of (14). Combining this result with the observations made in the preceding paragraph, we have the theorem:

When the function w_1 determining a transformation Ω of a surface Σ is equal to W plus a homogeneous linear function of the direction-cosines of the normal to Σ, ths lines joining corresponding points on Σ and its transform Σ_1 are concurrent; and conversely.

If in (5) we put $c = 0$, we have in place of (14) the equation $Z = f/\sqrt{\sigma}$. Consequently w_1 differs from Z by a constant factor at most, which is unessential. Since the z coördinate of the plane at infinity with respect to the quadric (5) is infinite, the lines joining corresponding points on Σ and Σ_1 are parallel. By reasoning analogous to the preceding we arrive at the theorem:

When the function w_1 determining a transformation Ω of a surface Σ is a homogeneous linear function of the direction-cosines of the normal to Σ, the lines joining the points on Σ and its transform Σ_1 are parallel; and conversely.

If θ_1 is a linear function of x, y, z, it is possible to choose the quadric (5) so that by means of (7) and (14) $\theta_1/\sqrt{\sigma}$ is expressible as a homogeneous linear function of X, Y, Z, W, involving W at least. Hence:

When the function θ_1 determining a transformation K of a surface S is a linear function of the cartesian coördinates of S, the corresponding tangent planes to S and its transform meet in line of a fixed plane.

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[1] Transformations of conjugate systems with equal point invariants, *Trans. Amer. Math. Soc.*, **15**, 397–430 (1914). This will be referred to as memoir M_1.

[2] Conjugate systems with equal tangential invariants and the transformation of Moutard, *Palermo, Rend. Circ. Mat.*, **39**, (1915). This will be referred to as memoir M_2.

[3] Guichard, *Ann. sci. Ec. norm., Paris*, Ser. 3, **14**, 492 (1897).

ON THE POLE EFFECT IN THE IRON ARC

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In a communication to the Academy which appeared in the PROCEEDINGS for March 1915, we reported some results of our investigations on the pole effect in the iron arc under normal conditions. We have found between two and three hundred lines whose wave-lengths at the negative pole are distinctly longer than when the light is taken from a cross section at the center of the arc.

Aside from the theoretical interest in such changes in wave-length, reference may be made to the following points:

1. A number of these lines are included among the international standards of the second order. Their wave-lengths depend upon interferometer measurements made by three independent observers, the means of which have been adopted as standards by the International Union for Coöperation in Solar Research.

2. There is a region of the iron spectrum extending from λ 5500 to λ 6000 in which no other class of lines is available for standards.

3. In various laboratories there are in progress redeterminations, based upon the iron standards, of the wave-lengths in international units of the lines of many elements. In these redeterminations the instrument most frequently used is the concave grating in the usual Rowland mounting. In ordinary practice, the slit of the spectrograph is parallel to the axis of the arc and includes the major portion of its length. We have found that the pole effect appears at a considerable distance from the negative pole and that for high precision

the light should not be taken from a point less than 2 mm. from the pole of the arc. The astigmatism of the concave grating in the usual mounting introduces more or less pole effect and to that degree vitiates results involving the lines under consideration. The practice of reversing the current in the arc in order to overcome the tendency to produce wedge-shaped lines, when the slit and the axis of the arc are parallel, obscures but does not eliminate the pole effect. Since the redeterminations aim at a precision of 0.002 to 0.003 angstrom, it is important to take the pole effect into consideration.

4. Lines of the type considered are not limited to iron but occur in the spectra of other elements, the detailed investigation of which is necessary before safe deductions can be made from their use in astrophysical investigations.

Bearing upon the explanation of the pole effect, our recent investigations yield the following results. The absence of a general increase in pressure at the negative pole, evidenced by the unchanged wave-length of whole classes of lines known to be affected by pressure, led us to suggest an increased density of the radiating vapor as a possible cause, but our measurements of furnace spectra taken with a 10 fold increase in the quantity of iron vaporized show no change of wave-length for these lines. Furthermore, the lines of manganese appearing as an impurity in the iron poles show displacements of the same order as neighboring iron lines; it is difficult to imagine a high density, even at the pole, for manganese vapor appearing only as a trace.

The effect of temperature was examined by comparing the wave-lengths of these lines at temperatures as widely different as is practicable in the furnace (2100°–2600°C.). No evidence was found of any dependence of wave-length upon temperature.

As the result of a study of the behavior of these lines in an arc *in vacuo*, it was found that the pole effect in general does not occur at pressures below 10 cm. of mercury. The arc which we employ *in vacuo* is of the same type and length and carries the same current as that used at normal pressure, but in appearance it is strikingly different. Under normal pressure the vapor mainly concerned in the production of these lines issues from a point source on the electrode, is highly luminous and is confined to a comparatively limited volume; *in vacuo* the luminosity covers the entire surface of the pole more uniformly and is much less intense, while definite structure is for the most part lacking. The disappearance of the pole effect indicates that the electrical conditions play a subordinate rôle, if any, in producing the displacements, but a more definitive investigation is about to be undertaken

In view of the absence of a general increase in pressure at the negative pole, it is probable that some agency other than pressure is involved, but there is the possibility of a local increase in pressure affecting only the innermost portion of the vapor in the vicinity of the pole. To obtain light upon this point, a determination of the pressure shift per atmosphere has been made for a large number of the lines under consideration. Their wave-lengths *in vacuo* and at normal pressure have been compared, the light in each case being taken from the central section of the arc. These conditions are well suited to a measurement of the pressure shift for lines of this type, as they are of good quality in both spectra. In a paper by St. John and Ware[1] they said:

> Neither the small pressure-changes of about one-fifth of an atmosphere taken advantage of in this investigation, nor the high pressures used by Gale and Adams are well adapted to the study of lines of this type, and it is purposed to examine *in vacuo* and under normal pressure the behavior of an extended list of lines belonging to groups *d* and *e*.

If the pole effect is due to pressure alone, the connection between it and wave-length should be similar to that relating pressure shift and wave-length. A brief summary of our results is given in the accompanying table.

POLE EFFECT AND PRESSURE SHIFT RELATIVE TO WAVE-LENGTH

Group	*No. of Lines*	*Mean λ*	*Pressure Shift per Atm.*	*Pole Effect*
d	25	4085.14	+0.0048A	+0.0096A
c5	16	4766.41	+0.0093	+0.0119
d	12	5528.44	+0.0089	+0.0206
d	5	6350.74	+0.0160	+0.0185

The three sections of group *d* show pressure displacements which vary as the cube of the wave-length, a result in harmony with the observations of Gale and Adams.[2] On the other hand there is no relation apparent between pole effect and wave-length. If we attempt to determine the difference in pressure between the center of the arc and a point in the core near the negative electrode by comparing the last two columns in the table, we find 2.0; 1.3; 2.3; and 1.1 atmospheres, numbers whose discrepancies exceed the errors of observation. Such a treatment of the data assumes that the pressure in the core of the arc at the negative pole is the same for all wave-lengths. The increase of pressure, if it exists, must be produced by rapid vaporization and the almost explosive expulsion of the emission centers, which are not necessarily the same for all wave-lengths. Investigations are now to be undertaken in order to obtain some measure of the velocities of the emitting particles.

Summary. 1. Emphasis has been placed upon the necessity of considering the pole effect in the redetermination of wave-lengths in international units.

2. The wave-lengths of these sensitive lines are not affected by a wide variation of density of the radiating vapor.

3. Their wave-lengths are independent of changes in temperature over the range of our observations.

4. For the lines considered the pole effect does not occur *in vacuo*, and in so far appears independent of electrical conditions.

5. The observed pole effect does not vary with wave-length in the same way as pressure shift, and cannot be explained as a pure pressure effect unless the pressure changes in certain definite ways with the wave-length.

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[1] *Mt. Wilson Contr.* No. 61; *Astrophys. J.* **36**, 37 (1912).

[2] *Mt. Wilson Contr.* No. 58; *Astrophys. J.* **35**, 10 (1912).

INHERITANCE IN THE ASEXUAL REPRODUCTION OF HYDRA VIRIDIS

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Do heritable variations commonly occur among the offspring of a single individual multiplying asexually? May selection among such offspring produce strains differing in hereditary characters? The investigation here resumed is designed to contribute data toward the answer to these much debated questions.

A number of specimens of Hydra viridis, taken at random from wild populations, gave rise by asexual reproduction to clones differing from one another in their average number of tentacles and in other characters. As a test of whether such differences are the result of internal factors or of environmental differences two clones were bred in large numbers for a period of five months, during which time the members of the two clones were kept under environmental conditions as nearly as possible the same. Each polyp was kept in a separate culture dish and food was distributed uniformly to all. The number of tentacles was recorded at the time when the polyps began their independent life after separation from the parents (the initial number of tentacles), and records were kept, also, of changes in the number of tentacles of parents.

The two clones kept under parallel conditions gave the following results. The average number of tentacles of 1353 members of one clone (A) bred during the first three months was 6.463 ± 0.013; the average number of tentacles of 1395 members of the other (D) was 5.793 ± 0.011; the difference between these averages is 0.724 ± 0.017. At the end of three months a single polyp was taken from each clone and used to found another clone. The average number of tentacles of the subordinate clone obtained in this way from clone A was 6.907 ± 0.026; that of the clone derived from D was 5.844 ± 0.029; the difference is 1.063 ± 0.039, in the same direction as before. At different times during the history of the clones the number of tentacles of the buds fluctuated considerably but the difference between the averages of the buds produced at the same time by the two remained fairly constant. The minimum difference in any one week of cultivation was 0.449 ± 0.042, the maximum difference, 1.063 ± 0.039. Small groups of polyps from the two clones were kept in mass cultures under partly controlled environmental conditions, such as reduced food supply. Under such conditions the difference persisted so long as the two clones were kept under the same environment.

These clones differed in other respects besides the average number of tentacles. The polyps of clone A were, on the average, more than twice as large as those of clone D, the average for the two being: A = 0.869 ± 0.021 cu. mm., D = 0.322 ± 0.022 cu. mm: the average difference is 0.547 ± 0.023 cu. mm. Polyps of clone A began to produce buds at an average age of 3.74 ± 0.074 days, those of clone D at 4.81 ± 0.10 days; a difference of 1.06 ± 0.13 days in the average age at reproductive maturity.

At no time during their history did the two clones show an identity of characters. There were no constant differences in the environmental conditions under which they were cultivated and it is certain that the differences between the clones were the result of some internal factors. The clones represented hereditarily diverse races. Other diverse races showing somewhat less well marked differences have been found and seem to be of rather frequent occurrence in wild populations. No direct evidence upon the origin of such diverse races or their relation to sexual reproduction has been obtained.

The inheritance of variations in the number of tentacles within the clone was studied by statistical methods and by the continued selection of variates. A comparison of the variations in the initial number of tentacles of parent and offspring by the use of the coefficient of correlation shows no significant resemblance between parent and offspring. The coefficients obtained are such as the following:

No. of Parents	*No. of Offspring*	*Coefficient of Correlation*
251	1395	0.0038 ± 0.018
78	439	−0.0342 ± 0.032
164	859	0.0011 ± 0.023
28	204	0.0314 ± 0.047
18	153	−0.2420 ± 0.051
51	154	−0.0750 ± 0.054

As is well known, the number of tentacles of Hydra changes during the life of the individual. When the number of tentacles of each bud is compared with the number born by the parent at the time when the bud was produced there is a slight correlation in the variations of parents and offspring. For the first clone recorded above this is 0.096 ± 0.016. There is also a slight positive correlation in the variations of the buds produced by a single parent, as is shown by the following:

No. of Pairings	*Fraternal Coefficient of Correlation*
12099	0.161 ± 0.004
10766	0.077 ± 0.006

A study of the relation of variations in the number of tentacles to environmental changes shows that unfavorable conditions tend to reduce, or to prevent increase in, the number of tentacles of parents and at the same time lead to a reduction in the number of tentacles of the offspring of these parents. Thus, wherever diversities of environment occur in cultures of Hydra there should be produced a likeness between parent and offspring that is not the result of heredity. To test this, the period of cultivation of the two clones giving the above fraternal correlations was divided arbitrarily into five-day periods and the buds produced within each of these periods were compared. From this comparison it appears that unrelated buds produced under like conditions of cultivation resemble each other as much as do the offspring from a single parent. The coefficients of correlation between the unrelated buds produced at the same time were:

No. of Pairings	*Coefficiont of Correlation*
95141	0.0774 ± 0.0015
101872	0.1313 ± 0.0014

It thus appears that the slight resemblance found between parent and offspring and between members of the same fraternity within the clone may be due either to an inheritance of variations or to the like action of environment upon individuals produced at nearly the same time. Statistical methods do not suffice to distinguish between the two possibilities.

Twenty-five variates from a single clone were selected for seven or more tentacles, and 25 for six or less, the mean of the clone lying between six and seven. Selection was continued for six or more generations. At the end of this time records were kept of all buds produced by the last selected generation. Those produced by parents from the series selected for few tentacles were found to have somewhat fewer tentacles than those from parents of the other series, but the difference appeared only in the first six buds and did not persist in the later buds produced by the same parents. The average number of tentacles of the first six and of later buds produced by the last selected generation is shown in the following table.

Buds of the Last Selected Generation

	AVERAGE OF ALL	FIRST SIX	LATER BUDS
Selected for many tentacles	6.695 ± 0.023	6.677 ± 0.029	6.712 ± 0.030
Selected for few tentacles	6.605 ± 0.026	6.460 ± 0.034	6.782 ± 0.037
Difference	0.095 ± 0.035	0.217 ± 0.044	−0.070 ± 0.047

Continued selection at first seemed to have produced a change in the hereditary character of the two groups but this did not persist even through a single generation. Complete regression appeared as soon as the polyps reached maturity.

An almost identical result was obtained in an earlier experiment on the effects of injury. Polyps which had regenerated the mouth and tentacles showed a marked reduction in the number of tentacles of the buds which they produced immediately after regeneration, but the average number of tentacles of successive buds formed after regeneration increased until at the end of two weeks, when four to eight buds had been produced, the polyps had returned to the normal condition of the clone from which they were derived.

The similarity of these results makes it certain that the only effect of selection was a temporary change in the vigor of the selected polyps (the selection of individuals with few tentacles involving the selection of weaker polyps) and that there is no cumulative inheritance of variations in the number of tentacles within the clone. Races of Hydra differing in their hereditary number of tentacles exist but individual variations do not involve changes in the hereditary constitution of such races.

Some evidence that the same conclusions apply to the inheritance of size was obtained, but the relation of variations in size to environmental changes has not been investigated thoroughly.

ON THE MONTICELLITE-LIKE MINERAL IN METEORITES, AND ON OLDHAMITE AS A METEORIC CONSTITUENT*

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Presented to the Academy, March 23, 1915

It may be remembered that in 1883 Tschermak[1] mentioned the occurrence in many chondritic meteorites of an almost colorless constituent occurring with very irregular outlines, in the groundmass, and exceeding in size the prevailing granules. These showed only traces of cleavages, were doubly refracting and optically biaxial though never showing good interference figures. The polarization colors were weak. It was found impossible to determine absolutely the mineral nature of the constituent, but it was suggested that it might be monticellite. Its presence was noted in the Alfianello, Mocs, Knyahinya, and Mezo Madaras stones. The occurrence in the Knyahinya stone was later figured by him on plate 14, figure 3, of his *Die Microskopische Beschaffenheit der Meteoriten.* Since Tschermak's writing the mineral has been noted by various authors. Lacroix[2] describes in some detail what is unquestionably the same mineral in the stone of St. Christophe la Chartreuse. He finds it possessing two easy but interrupted cleavages, cutting at an angle of about 60°, with a birefringence less than 0.003, which he notes is less than that of olivine and superior to that of feldspar, with which it is often found in contact. He also was unable to determine its exact nature optically, or to separate it by means of heavy solution, but noted that it could not be monticellite on account of its low birefringence. He further noted its occurrence in the meteorites of Chantonnay, d'Aumale, de Barbotan, de Salles, de Berlanguillas, and Lancon. The present writer recognized the same mineral in sections of the meteorites from Rich Mountain, N. C.,[3] and Felix, Alabama,[4] and though recognizing its biaxial nature and weak birefringence was likewise unable to identify it absolutely. Still later, Borgström recognized it in the meteorite of St. Michel,[5] and was able to determine the positive character of the acute bisectrix.

In connection with his work on the minor constituents of meteorites, the present writer had opportunity for examining numerous occurrences of the mineral, and it was decided if possible to determine its nature or at least to carry a knowledge of its properties one step farther. Unable by the means at his command to fully satisfy himself regarding all its optical properties further than given in the paper referred to above, sections of both the Alfianello and Rich Mountain stones were submitted to Dr. F. E. Wright of the Geophysical Laboratory, who reported the mineral to be weakly birefracting with interference colors not exceeding gray white of the first order, and the refractive indices of which were $\alpha = 1.623 \pm 0.002$ and $\gamma = 1.627 \pm 0.002$, the birefringence being less than 0.005. He also verified previous determinations of its biaxial and doubtfully positive character. With these additional data at hand reference was made to E. S. Larsen's tables,[6] where it was found that the only known mineral possessing the properties found by the various observers was the phosphate *francolite*, which is reported, however, as optically negative. Nevertheless, with this much for a guide, an attempt was then made at ascertaining its chemical nature. The small size of the particles and their sporadic occurrence rendered it impossible to isolate them either by heavy solution or by an electro-magnet. Recourse was then made to micro-chemical methods. After repeated trials, a small particle, less than 0.5 mm. in diameter, was removed from an uncovered section of the Alfianello stone, and transferred to a clean slide where it was covered with a drop of strong hydrochloric acid and allowed to stand over night, during which time the acid evaporated to dryness. On taking it up again in a fresh drop of acid the mineral was found to have been strongly attacked though not completely dissolved. A drop of the solution was then transferred to still another slide and placed in contact with a drop of sulphuric acid. Abundant acicular and plumose crystals of gypsum shortly began to make their appearance around the margin of the drop. The liquid remaining on the first slide was then evaporated to dryness over an alcohol lamp and the residue taken up with nitric acid, a drop of ammonium molybdate solution added, and the slide slightly warmed on the steam radiator for a few moments, when abundant characteristic globules of the phospho-molybdate of ammonium separated out. As checks upon this, independent tests were made in slides from the Alfianello, Dhurmsala, Felix, Mocs, and Rich Mountain, meteorites by painting around the mineral with vaseline and treating the exposed portion with a drop of nitric acid. The mineral was found to be completely and easily soluble in this acid and in all cases results confirmatory of the first were obtained. It may be added that the solution in nitric acid goes on so rapidly that its progress may readily be observed under the microscope. In making this test, nitric acid has the advantage in that it does not so readily attack the olivine and there is hence no trouble from the obscuring reactions by gelatinous silica. The tests for other constituents, such as alumina, yielded no satisfactory returns. As a last resort a 12-gram fragment of the Alfianello stone was submitted to Dr. Whitfield with the request that it be pulverized, and, after removal of the metallic portion, boiled in water to remove any possible oldhamite or its alteration products, and the residue then boiled in dilute hydrochloric acid. Disregarding as a matter of necessity the iron and magnesia which might come from the olivine, the results obtained were as follows: The water solution yielded 0.03% CaO and 0.013% S, the equivalent of 0.030% CaS, or oldhamite. The acid solution yielded 0.344% CaO and 0.08% P_2O_5, with traces of alumina.

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Satisfactory results from such tests could scarcely be anticipated but are nevertheless not without interest. It will be recalled that the ratio of CaO to P_2O_5 in francolite as given by Schaller[7] is 10 to 3. The results obtained by Whitfield by the method described were 9 to 2.

The accompanying figures, drawn free hand, directly from the thin sections, will serve to give an idea of the general appearance and optical properties of the mineral. No attempt has been made to outline other of the constituents excepting where noted as enclosures. The actual size of the largest forms, as those of Alfianello and Rich Mountain (figs. 1 and 2), is not above 0.5 mm. in diameter. In figure 1, from a slide of the Alfianello stone, the mineral in question, left colorless and unshaded, is traversed by wavy cracks or cleavage lines extending from the upper right to the lower left. The dotted areas are olivine and those with broken parallel lines at the right are enstatite. The completely black are metal and metallic sulphide. In the upper portion the mineral reaches its maximum extinction in the position indicated by the cross. In the lower right, which remains almost completely dark during an entire revolution of the stage, there appears an indistinct, slightly curved black bar, indicating the emergence of an optic axis of a biaxial mineral.

Figure 2, from a slide of the Rich Mountain stone, shows likewise a characteristic occurrence, with near the center an enclosure of olivine and other scattered enclosures of the same mineral and metallic particles. The entire area extinguishes as a unit and in the position indicated by the cross, though the lower portion shows indications of two cleavages and an indistinct black brush sweeps across the field as the stage is revolved between crossed nicols with the occular removed.

Figures 3 and 4 show the same mineral in the Pultusk and Dhurmsala stones, respectively. The position of maximum extinction is again shown by the black cross. In figure 4, which extinguishes as a unit, two easy but interrupted cleavages are very evident at the left and bot-

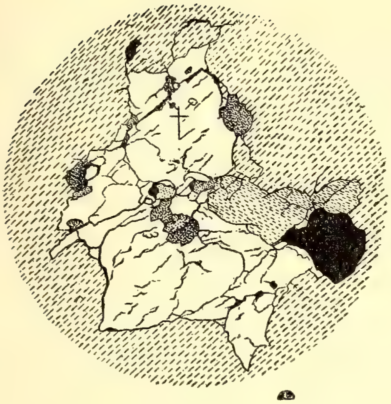

Fig. 1

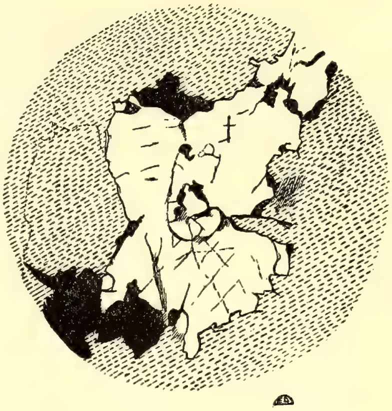

Fig. 2

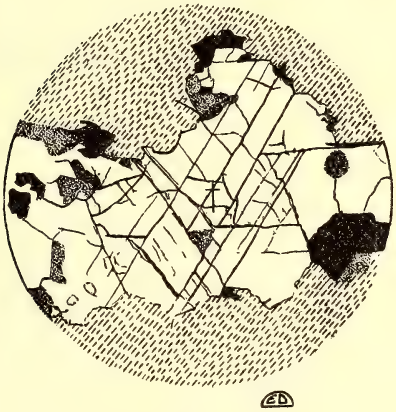

Fig. 5

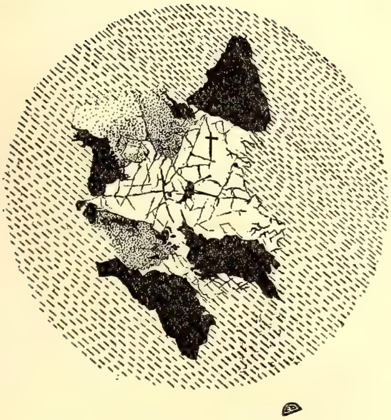

Fig. 3

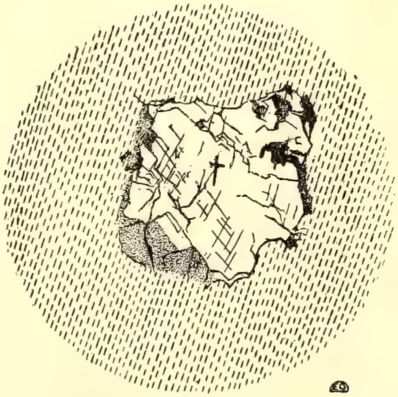

Fig. 4

tom of the section. No satisfactory interference figure is obtainable from this section, though a very indistinct dark brush sweeping across the field indicates that the plane of the optic axes cuts the obtuse angle of cleavage.

In figure 5 is shown an occurrence in the Mocs stone concerning which there was considerable doubt, not merely on account of the perfection of the cleavage but from the fact that it showed an indistinct interference figure in which the axial angle was so small that it was at first thought to be uniaxial: it was, moreover, optically negative, and more nearly resembles normal apatite than do the other occurrences under discussion. To settle the matter the cover was removed from the slide and the mineral treated with a drop of nitric acid as in previous cases. Within an hour from the time the drop was applied the mineral was entirely eaten out and the solution, as before, yielded lime and phosphoric acid, with the proper reagents.

The accumulated determinative characteristics of the doubtful mineral so far as made out by the various workers are then as follows:

Colorless, with but few inclosures, two cleavages, imperfect and interrupted, cutting at an acute angle of 60°; optically biaxial and positive (?), with large 2E; birefringence weak, less than 0.005, refractive indices, as determined by Dr. Wright, $\alpha = 1.623 \pm 0.002$ and $\gamma = 1.627 \pm 0.002$; no pleochroism, and often with undulatory extinction; polarization in light and dark colors and at times almost completely dark during a revolution of the stage between cross nicols; easily soluble in cold nitric acid, giving solutions reacting for lime and phosphoric acid; very brittle and occurrence sporadic.

There is apparently no question then but that we have a heretofore unrecognized form of mineral phosphate as a fairly common constituent of meteoric stones. From normal apatite it differs in its low birefringence as well as cleavage and general optical characters. From francolite it would appear to differ in that so far as determined it is optically positive, while the latter is negative. Just how much weight is to be attached to this last characteristic, is yet to be determined. One has a natural hesitation in relegating it to francolite for genetic reasons, since francolite, so far as known, is a secondary mineral and a deposit from aqueous solution. It is to be noted, however, that the mineral I have been describing occurs with outlines suggestive of its secondary origin and filling drusy cavities formed at the time of the original consolidation of the stone in which it occurs. Whether secondary or not, as the term is usually employed, it is certainly a product of the latest period of crystallization. Until the material can be found in such quantity as to be available for chemical analysis, apparently this is the best that can be done. Attention may, however, be called to the fact that phosphorous in the form of apatite has thus far been definitely recognized in meteorites only by Berwerth,[8] although a small amount of phosphorous is almost invariably reported either as P or P_2O_5 from the soluble (silicate) portion of the stony meteorites. It would seem probable, therefore, that this soluble salt is not in the form of apatite but that rather it owes its presence to the monticellite-like mineral which is now relegated doubtfully to the species francolite.

On the assumption that the doubtfully monticellite-like mineral referred to by the various workers is in all cases to be relegated to the same species, it has now been recognized in the stones listed below.

Alfianello,
Aumale,
Barbotan,
Berlanguillas,
Chantonnay,
Dhurmsala,
Felix,
Knyahinya,
Lancon,
Mezo Madaras,
Mocs,
Pultusk,
Rich Mountain,
Saint Christophe,
Saint Michel,
Salles.

OLDHAMITE AS A METEORIC CONSTITUENT

The presence of the calcium sulphide oldhamite in a meteorite was first made known by Maskelyne who found it in granules macroscopically developed in the Busti stone.[9] He also noted its probable presence in that of Bishopville, and suggested an even wider distribution. Since Maskelyne's writing but few new occurrences have been noted, though its probable presence has been more than once suggested. The present writer found a lime salt soluble in water in the Morristown meteorite which he surmised might be gypsum, resulting from the alteration of the sulphide.[10] He also found abundant chemical evidence of its one-time presence in the stone of Cullison,[11] and was able to verify Lacroix's microscopic determination of its presence in the Indarch stone.[12] Borgström has also shown its presence in the stone from Hvittis,[13] Cohen in that of St. Marks,[14] and Tassin by chemical tests has pointed out its probable occurrence in that of Allegan.[15]

These facts, considered in connection with my work on the minor constituents of meteorites, have led me to make a most careful search for the mineral, but with the exception of that of Indarch none of the stones examined have yielded absolute ocular proof of its occurrence. The known solubility of the mineral in water has, however, afforded opportunity for a safe presumption regarding its presence or absence.

Quantities of a gram or more from each of the stones listed below were finely pulverized and boiled for an hour or so in distilled water, the solutions then filtered and tested for lime with ammonium oxalate.

In alphabetical order the stones tested are as follows:

Alfianello, positive lime reaction
Beaver Creek, negative lime reaction
Bishopville, positive lime reaction
Cullison, positive lime reaction
Dhurmsala, positive lime reaction
Dores dos Campos, positive lime reaction
Estherville, positive lime reaction
Farmington, faint positive lime reaction
Fayette, positive lime reaction
Felix, positive lime reaction
Forest City, positive lime reaction
Hessle, faint positive lime reaction
Holbrook, positive lime reaction
Homestead, positive lime reaction
Knyahinya, negative lime reaction
L'Aigle, positive lime reaction
Mocs, positive lime reaction
Monroe, negative lime reaction
New Concord, doubtful lime reaction
Parnallee, faint positive lime reaction
Pultusk, positive lime reaction
Quenggouk, positive lime reaction
Tennasilm, positive lime reaction

The mere presence of a soluble calcium compound does not necessarily prove the presence of oldhamite, but in cases where the boiling is accompanied by a liberation of H_2S, it would seem safe to assume its presence in this form, and in the cases of the stones of Alfianello, Bishopville, and Indarch we have been able from the lime in the water solution to calculate its approximate percentage amount.

That the mineral occurs more or less sporadically is suggested by Maskelyne's figure and description of the Busti stone, and has been demonstrated by our own researches, certain individuals from the Pultusk fall giving abundant indications of its presence, while others gave no sign. It is evident, however, that the mineral is a common and widespread constituent, and the examination of no stone meteorite should be regarded as complete unless it has been sought for by both optical and chemical means. A failure to find it in thin sections may be due to the breaking out of the granules in the process of grinding, or more likely, particularly in the case of stones that have lain long in the ground, to its having undergone alteration into an earthy, gypseous, and quite unrecognizable product.

[1] *Sitz. Wien. Akad.*, 88, No. 1, p. 355 (1883).

[2] *Bull. Soc. Sci. Nat. Ouest*, ser. 2, 6, 81–112 (1906).

[3] *Proc. U. S. Nat. Mus.*, 32, 243 (1907).

[4] Ibid., 24, 196 (1901).

[5] *Bull. de la Com. Geol. de Finlande*, No. 34, 1912, p. 43.

[6] *Indices of Refraction of Minerals*, in MS.

[7] *Bulletin* 509, *U. S. Geol. Survey*, p. 599.

[8] *Min. u. Pet. Mittheil.*, 25, 188 (1906).

[9] *Phil. Trans. R. Soc. London*, 160, 189–214 (1870).

[10] *Amer. J. Sci.*, 11, 149 (1896).

[11] *Proc. U. S. Nat. Mus.*, 44, 330 (1913).

[12] *Proc. U. S. Nat. Mus.*, 1915.

[13] *Die Meteoriten von Hvittis in Marjalahti*, 1903.

[14] *Ann. S. African Mus.*, 5, 1–16 (1906).

[15] *Proc. U. S. Nat. Mus.*, 34, 433 (1908).

ABSOLUTE SCALES OF PHOTOGRAPHIC AND PHOTOVISUAL MAGNITUDE

By Frederick H. Seares

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Presented to the Academy, April 14, 1915

The first extensive photometric investigation undertaken with the 60-inch reflector has been the determination of absolute scales of photographic and photovisual magnitude for stars near the North Pole. Although the instrument is best adapted for the observation of objects fainter than the tenth magnitude, it can also be used for the brighter stars; and it thus becomes possible to secure, on a uniform system of color, results covering the entire range of stellar brightness at present known.

In any photometric problem involving the use of photographic methods, there is serious difficulty in evaluating the functional relation connecting the intensity of the light acting on the plate with the observed photographic effect. The photographic process is too complicated and too sensitive to slight fluctuations in the conditions which determine its action to permit of the application of anything like a general relation; and it is necessary to standardize or calibrate the images appearing on each plate which is to contribute toward an absolute scale.

Besides photographic difficulties, there is another which is serious, namely, that encountered in all physical observations involving the comparison of quantities which, relatively, are very large and very small. In the present case, the range of brightness actually covered in the determination of the photographic scale is about $17\frac{1}{2}$ mags.; the intensity ratio for the brightest and the faintest of the stars observed is therefore of the order of 1 to 10,000,000, and consequently the opportunity for an accumulation of error in passing from the one extreme of brightness to the other is great.

The methods commonly employed for the standardization of the photographic effect in terms of intensity have as their principle the production of two series of images with a known variation of the intensity between the exposures. This in itself is not easily accomplished, at least with assurance of freedom from systematic error. A variation of the total energy by a change in the exposure time immediately introduces the photographic difficulties that we wish to avoid; a change in the intensity by a reduction of the aperture changes the diffraction pattern of the optical image, and it is not a priori certain that the same quantity of energy distributed over different patterns produces the same photographic result; the use of a coarse objective grating or screen of wire gauze is unobjectionable from the standpoint of diffraction, but requires a troublesome laboratory investigation for the determination of its constant; the diversion of a certain fraction of the light into a secondary image by means of a prism of small angle is subject to disturbances by diffraction and absorption. Other methods are subject to similar uncertainties; and, in addition, all which involve successive exposures upon the same plate, like the majority of those just enumerated, have to contend with irregularities in the atmospheric conditions.

In arranging the program of observations it was considered that the latter could be disregarded, because of the favorable situation of Mount Wilson and the moderate exposure times which would be required. Moreover, the considerable accumulation of material necessary for the investigation would render atmospheric disturbances wholly accidental. Other difficulties, it was hoped, would be sufficiently controlled by the use of a wire gauze screen and a series of circular diaphragms covering a wide range of aperture. For the latter, the following diameters in inches were adopted: 40, 32, 14, 9, 8, and 6. Two large screens of wire gauze were used, one of single thickness and one of double thickness, the wires of the latter being crossed at an angle of 45°; smaller screens for use over the diaphragms of 14 inches and less were also available. The constants depend upon laboratory determinations of the absorption for point sources, controlled by the measurement of the absorption for surfaces.

It was found convenient to separate the investigation into three parts, with a corresponding variation in the treatment: (*a*) bright stars, including objects brighter than the tenth magnitude; (*b*) intermediate stars, from the tenth to the eighteenth magnitude; (*c*) faint stars, fainter than the eighteenth magnitude. The limits refer to the photographic scale.

For both scales the work was begun with the second group—the intermediate stars. The details of observation, measurement, and reduction have been described elsewhere.[1] For the photographic scale, to which the following paragraphs refer, the exposure times fall into two groups, one of 11 minutes or less, the other ranging between 30 minutes and 60 minutes. Forty-two separate determinations of the scale were derived from the data on 15 plates of shorter exposure; from those of longer exposure there were fourteen determinations, from an equal number of plates. The former series involved the use of the single-thickness screen, 60 inches in diameter, and the 32, 14, 8, and 6-inch diaphragms, besides two other arrangements of less importance.

For the interval 10.5–17.6, the mean of 17 scales with the 32-inch diaphragm is practically identical with that of 10 found with the 14-inch diaphragm. The 5 scales with the two smaller diaphragms show a deviation of about 1% from the mean of all, and there is a similar divergence between the results for the wire gauze screen and the mean for all the diaphragms. The agreement is therefore such as to leave little doubt as to the negligible character of the diffraction effect and the satisfactory elimination of other errors.

The longer exposure plates, which in several cases reach the nineteenth magnitude, were all obtained with diaphragms of either 32 or 14 inches, and are likewise satisfactorily accordant. These plates also give results for the brightest of the intermediate stars; in other words, they overlap completely the region covered by the plates of shorter exposure. The average difference between the mean scales from the two series, derived from nine groups of stars between 10.6 and 16.8, is 0.015 mag. There seem, therefore, to be no systematic errors which depend upon the exposure time.

The determination of the scale for the two remaining groups—faint stars and bright stars—presupposes a knowledge of that for the intermediate group. Here again the methods have been fully described elsewhere,[2] and only a very brief statement need be included here.

The extension to the fainter objects was by means of plates which, with one exception, received two exposures, both with the full aperture of 60 inches, but with different exposure times; the longer exposure was four or five hours, the shorter approximately half an hour. The reduction was based upon the empirical relation established by Kron,[3] which expresses the law of photographic action. Strictly speaking, the process involves an extrapolation, but one which is fairly reliable. The accordance of the six plates for the faint stars is scarcely less satisfactory than that of the long-exposure diaphragm plates used for the intermediate stars. The limiting photographic magnitude thus reached is about 20, which may be taken as the faintest attainable, under favorable conditions, with an exposure of four hours.

Owing to the small field of the reflector, and the relatively wide distribution of the bright stars, these objects had generally to be observed individually. They were photographed with screens or diaphragms producing images comparable with those of stars between the tenth and fifteenth magnitudes, obtained with the same exposure but without screen or diaphragm. The following is a typical arrangement of the exposures, which were 2 minutes each: (1) intermediate stars, full aperture; (2) bright star with reduction of intensity, usually three or four exposures with different arrangements of diaphragms and screens; (3) intermediate stars, full aperture. The magnitudes of the reduced-intensity images were found by comparison with those of the intermediate stars, which had already been investigated; the subtraction of the reduction constants then gave values for the real magnitudes.

The method possesses many advantages and, when applied with sufficient elaboration, apparently gives a scale of high precision. For example, suppose that a screen absorbing 6 mags. is used. Since standards between the tenth and fifteenth magnitudes are presupposed, bright stars between magnitudes 4 and 9 are within reach; and since in each case the same constant is subtracted from the magnitude of the reduced-intensity image, the slope of the resulting scale will be independent of any error affecting the constant. The process is, in effect, a transference of the adopted scale for the intermediate stars to the region of the brighter objects.

To provide the necessary checks, it is desirable that all possible arrangements of apertures, covering a wide range of reduction constants, should be used. There will thus be accumulated a number of separate, but overlapping, determinations of the scale, each of which should be homogeneous with that of the intermediate stars; their intercomparison controls the errors of the constants, and their mean is the final result. When the data are extensive, as is here assumed, the reductions can be modified so that the result is no longer dependent upon that for the intermediate stars.

To keep the labor within reasonable limits, this part of the investigation was restricted to the stars of the North Polar Sequence, the brightest of which is Polaris; but in order that there might be a satisfactory connection with the intermediate group, a number of the latter were reobserved by the bright-star method. These considerations led to the selection of 32 objects distributed between magnitudes 2.5 and 12.3, photographic. More or less complete series of observations were obtained with 10 different arrangements of the aperture, besides partial series for several others, the constants ranging from 1 to 11 mags. The overlapping scales, which are based upon 213 plates and 662 individual magnitudes, are in good agreement; not only are they sensibly parallel, but the constant systematic differences are small, which indicates that the reduction constants are well determined.

With the completion of the investigation of the bright stars, it became possible to reduce the entire series of results to the international zero point. This was accomplished by making the mean brightness of the white stars near the sixth magnitude equal to the mean of their visual magnitudes in *Harvard Circular*, No. 170, proper allowance for spectrum being made for those objects not of type AO.

The resulting catalogue of photographic magnitudes includes 617 objects, the great majority of which are fainter than the tenth magnitude. All of the latter are close to the Pole of 1900, and among them are included the stars of the Polar Sequence. To these special attention has been given, although a large number of other stars have well determined magnitudes, based on ten or more individual values. The very faint objects, which are relatively numerous, are naturally subject to greater uncertainty.

The treatment of the data for the photovisual scale was exactly the same as that outlined above for the photographic scale. Although only 6 plates were used for the intermediate and faint stars, the precision is much greater than one would thus be led to expect. Owing to the better definition of the images on the isochromatic plates, the probable error of a photovisual magnitude is only two-thirds that of a photographic result. The photovisual scale for the bright stars, which depends upon 122 plates and 367 individual magnitudes, is probably quite as reliable as the photographic scale in the same region. Final photovisual magnitudes were obtained for 339 stars between magnitudes 2.1 and 17.5.

The average deviation of a single photographic magnitude, including the scale error of a plate over a range of 8 or 9 magnitudes is 0.125 mag.; the corresponding photovisual result is 0.087 mag.

About 300 stars have had both their photographic and photovisual magnitudes determined, and, in consequence, color indices for these are available. These confirm a result announced at an earlier stage of the work, namely, that the faint stars are all relatively red. For the bright stars, we have small, and even negative, color indices in large numbers. At the seventeenth photovisual magnitude the smallest value is 0.6 or 0.7 mag., the lower limit gradually increasing as the fainter stars are approached. This phenomenon is undoubtedly of very great significance, but its interpretation cannot at present be given with certainty. It is evident, however, that the determination of star colors will form an important part of stellar investigations in the future.

This last consideration only emphasizes the necessity of precision in the magnitude scales, for any relative error enters to its full amount into the color index, and thus vitiates the results of comparisons involving stars of different degrees of brightness. In the present case a supplementary investigation provided an important control.

Hertzsprung has determined by an entirely different method—that of effective wave-lengths—the colors of nearly 200 stars in the cluster N. G. C. 1647. These can be expressed as color indices and compared with similar results derived directly from photographic and photovisual magnitudes. The agreement or disagreement of the two series of values immediately checks the relative errors of the scales.

To execute this comparison, the polar scales were transferred to the region of the cluster by 10 plates of duplicate exposure—5 for each scale. All possible care was taken to avoid errors in the zero points, for uncertainties here also enter to their full amount into the color indices.

In this manner color values were obtained for about 50 stars between magnitudes 11 and 14.5, which had also been observed by Hertzsprung. There is some uncertainty involved in the transformation of the effective wave-lengths into color indices; but it is not likely that the mean difference in the two series of values exceeds a tenth of a magnitude, and there is some evidence that it is even less. The relative errors of the scales between the sixth magnitude, which defines the international zero point, and the thirteenth or fourteenth magnitude can therefore scarcely exceed this limit.

Comparisons of the individual scales for the Pole with the results of other observers are also important in their indications. The only other investigation covering the entire range of brightness is that of the photographic scale carried out at the Harvard Observatory. With proper allowance for color, the Harvard and Mount Wilson results are in close agreement, that is, the scales are parallel, from the tenth to the fifteenth magnitude. For the fainter stars there is a large divergence, which has arisen from differences in the method of reduction (the Harvard scale for the fainter stars is based upon Mount Wilson plates). For the bright stars there is also a divergence, amounting to about 6%, whose origin is not yet fully explained.

Other investigations of the photographic scale have been made at Greenwich and Potsdam. These extend from the ninth to the sixteenth or seventeenth magnitude; after correction for color to reduce to the system of the reflector, both show a close accordance with the Mount Wilson results. In addition, the bright stars between the fourth and eighth magnitudes have been observed at Göttingen and at the Yerkes Observatory. These results are also in close agreement with the Mount Wilson scale.

The material for the photovisual scale is less abundant. Visual magnitudes of the Polar Sequence stars have been determined at Harvard as far as the fourteenth magnitude. This scale intersects the Mount Wilson photovisual scale at the sixth and the twelfth magnitudes, but at other points there are important differences, some of which are obviously due to color. Further, the *Yerkes Actinometry* contains photovisual magnitudes as far as 7.5. The Mount Wilson results agree satisfactorily with these, and there is also good accordance with the visual magnitudes of Müller and Kempf. Beyond this no results have as yet been published, although others are in preparation. A detailed comparison with all these various investigations will be included in forthcoming papers in the *Astrophysical Journal*,[4] which will also give fuller details and a summary of the numerical results for the Mount Wilson scales. The complete discussion will appear as Volume 3 of *Papers of the Mount Wilson Solar Observatory*.[5]

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[1] *Mt. Wilson Contr.* No. 80; *Astrophys. J.*, **39**, 307 (1914).

[2] *Ibid; Mt. Wilson Contr.* No. 70; *Astrophys. J.*, **38**, 241 (1913).

[3] *Potsdam Publ.* No. 67.

[4] *Mt. Wilson Contr.* Nos. 97, 98, 102; *Astrophys. J.* (In press.)

[5] *Publications of the Carnegie Institution of Washington.*

MITOSIS IN TRICHOMONAS

By Charles Atwood Kofoid and Olive Swezy
ZOOLOGICAL LABORATORY, UNIVERSITY OF CALIFORNIA

Presented to the Academy, March 16, 1915

The process of cell-division in the simpler Protozoa is significant in its relation to the evolution of nuclear and extra-nuclear structures. The behavior of the extra-nuclear organelles in unicellular organisms during the process of binary fission is significant as to the origin and relationships of such structures as flagella, blepharoplast, and axostyle. One of the distinctions between the Ciliata and the Flagellata has been held to be the direction of the plane of division, transverse in the former, longitudinal in the latter; therefore all reported cases of transverse division in Flagellata should be critically inspected.

Observations on mitosis in *Trichomonas augusta*, a flagellate parasitic in the digestive tract of amphibians (*Diemyctylus torosus Rana boylei*, and *Bufo halophilus* from California, and *Rana pipiens* from Chicago) enable us to make a fuller correlation of mitosis in the Flagellata with that in the Metazoa, to correct or supplement the observations of others regarding mitosis in trichomonad flagellates, and to establish on ample morphological grounds the essentially longitudinal nature of their division. Our conclusions have been verified in all essential features, in eleven species of seven genera parasitic in vertebrates. The conclusions are based on hundreds of preparations made by the wet Schaudinn iron-haematoxylin, or by the Giemsa method, and controlled by observations on living forms in sealed culture slides, or on mixed pure cultures in sterilized media.

The vegetative phase of *Trichomonas augusta* (fig. 1) presents an elongated pyriform body with a hyaline axial rod, the axostyle (*ax.*), enlarged anteriorly and projecting posteriorly in a sharp point. A pale spheroidal nucleus (*n.*) lies in the anterior end. It contains one or more deeply staining karyosomes and a faint chromatin network.

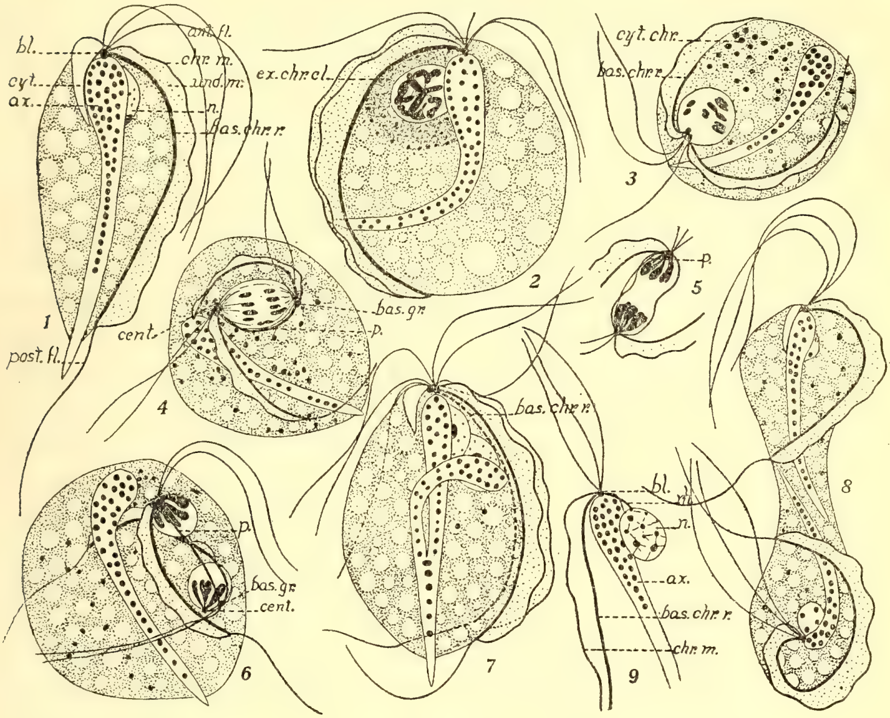
The spheroidal deeply staining blepharoplast (*bl.*, figs. 1–3) lies close to the anterior surface attached to the anterior end of the axostyle. From it pass anteriorly the three long lightly staining anterior flagella (*ant. fl.*), and posteriorly the undulating membrane (*und. m.*) consisting of a waving protoplasmic film in whose margin lies a deeply staining chromatic thread or margin (*chr. m.*), and at whose base is a heavier basal chromatic rod (*bas. chr. r.*). Posteriorly the two chromatic structures unite at the point of emergence in the lightly staining posterior flagellum (*post. fl.*).

From the blepharoplast the axostyle (*ax.*), a large hyaline club-shaped flexible and mobile structure, passes posteriorly in an axial position. It is enlarged anteriorly near the nucleus and contains a varying number of fairly uniform axostylar chromidia (*ax. chr.*). In macerated specimens (fig. 9) the nucleus is seen to be attached to the head of the axostyle by a persistent and but slightly chromatic strand or rhizoplast (*rh.*). These extra-nuclear structures together with the nucleus survive as a coherent unit the maceration of the surrounding cytoplasm.

The prophase of the process of mitosis is initiated in rather large, somewhat spheroidal individuals (fig. 2), by the apparent splitting of the wavy chromatic margin of the undulating membrane distally from the blepharoplast. At the same time the homogeneous intra-nuclear chromidial cloud, which diffusely fills the nucleus as mitosis approaches, emerges from it and forms the extra-nuclear cloud (*ex. chr. cl.*), a diffusely staining halo about the nucleus, in which minute chromidia soon appear (fig. 2), spread into the surrounding cytoplasm, and increase in the axostyle throughout its length.

As the nucleus clears up there emerges in it a distinct (fig. 2), often clearly continuous chromatin skein, which speedily breaks up, finally into five chromatin masses, or chromosomes, each of which splits longitudinally into two chromosomes (fig. 3) within the intact nuclear membrane, prior to any arrangement in an equatorial plate. There are differences in these chromosomes observable in most, if not all, of the mitotic figures (figs. 3–6). There are two of medium size, one large one, and two small ones, one of which lags (fig. 4) in the metaphase. After splitting the sister chromosomes swing into an end-to-end position (figs. 3, 4) as they move into the equatorial plate.

The blepharoplast has in the meantime (fig. 3) divided into two, to



Mitosis in *Trichomonas augusta* Alexeieff. × 1500; *ant. fl.*, anterior flagella; *ax.*, axostyle; *ax. chr.*, axostylar chromidia; *bas. chr. r.*, basal chromatic rod; *bas. gr.*, basal granule; *bl.*, blepharoplast; *cent.*, centrosome; *chr. m.*, chromatic margin; *cyt.*, cytostome; *cyt. gr.*, cytoplasmic granule; *ex. chr. cl.*, extra-nuclear chromidial cloud; *n.*, nucleus; *p.*, paradesmose; *post. fl.*, posterior flagellum; *rh.*, rhizoplast; *und. m.*, undulating membrane.

Fig. 1. Active motile form prior to division.

Fig. 2. Prophase, with chromatin skein, split border and sprouting flagellum.

Fig. 3. Later prophase with five pairs of split chromosomes.

Fig. 4. Late metaphase with blepharoplasts at poles, each divided into centrosome and basal granule.

Fig. 5. Late anaphase with blepharoplast not divided.

Fig. 6. Early telophase. Nuclei still connected by paradesmose.

Fig. 7. Late telophase. Paradesmose seen end to end and therefore foreshortened, axostyle splitting.

Fig. 8. Mitosis completed, cytoplasmic division approaching.

Fig. 9. Nucleus and extra-nuclear motor apparatus after maceration.

one of which are attached two of the anterior flagella, while the other takes the remaining one and a new anterior flagellum which grows out from the blepharoplast. We have not found the blepharoplast to consist of four basal granules at the roots of the flagella as indicated by Martin and Robertson[3] for *Trichomonas gallinarum*.

The new chromatic basal rod grows out from the blepharoplast (figs. 3, 6), apparently independently of the old, as a new structure in the membrane below one of the chromatic marginal filaments (fig. 3) and distally extends in a new posterior flagellum, thus completing the division of the undulating membrane.

The metaphase is approximated in figure 4 in which, however, the nuclear membrane remains intact, as in fact it does throughout mitosis. The two blepharoplasts have migrated to the two poles of the pointed ellipsoidal nucleus, and each has divided into a centrosome (*cent.*), at the apex of the spindle, and the adjacent basal granule (*bas. gr.*) to which the flagella remain attached. In some instances (fig. 5) this division of the blepharoplast is not apparent. No astral rays are evident. Within the nuclear membrane faint spindle fibres connect the parting chromosomes to the centrosomes at the poles. Connecting the two blepharoplasts as they migrate to the polar position (figs. 3–6) is a heavy chromatic thread which lies *outside* of the nuclear membrane. This we name the paradesmose, though in origin it may seem to be homologous to the central spindle of the metazoan mitotic figure. It is this structure which according to Dobell[1] gives rise to the new axostyles of the daughter cells. It later disappears (fig. 7) without giving rise to any structures. As the split chromosomes swing in to the equatorial plate they assume the end-to-end position like that described by Montgomery[5] for the spermatocytes of *Euschistus*. Spindle fibres play no part in their splitting. Slight inequality in an "x"-"y" relation between the daughter chromosomes is generally evident in the case of the large chromosome. There is no evidence, however, that any of the divisions here described are maturation divisions, or that their end-to-end position is a telosynapsis.

The anaphase is accomplished by the movement of the chromosomes to the poles of the spindle into contact with the polar centrosomes and by the constriction of the nucleus into a dumbbell shape (figs. 5–6).

The telophase (fig. 6) results from the final constriction and separation of the daughter nuclei which immediately assume a spheroidal form and move apart drawing out the paradesmose between their basal granules. This is perfectly distinct as an extra-nuclear chromatic thread joining the two blepharoplasts or the two basal granules after each blepharoplast parts into centrosome and basal granule (figs. 4–6). The paradesmose later fades out after the karyosomes and chromatin network reappear within the daughter nuclei (fig. 7).

At the close of the telophase (fig. 7) division of the axostyle proceeds by longitudinal splitting from the anterior end posteriorly. There is a suggestion that axostylar chromidia are also divided, at least in the distal region (fig. 7). Actual division of individual chromidia has not been seen though in some dividing axostyles (fig. 7) their relative dimensions in parent and daughter axostyles strongly suggest their division. The numbers of chromidia vary so that they do not afford critical evidence for use on this point.

The proofs of division of the axostyle in our material are ample though their detection requires well-prepared slides and persistent search. Dobell[1] and Prowazek[2] describe the origin of these axostyles from the "centrodesmose" and the former concludes that it is therefore the homologue of the central spindle, and Doflein in his recent *Lehrbucher der Protozoenkunde* accepts this origin. This strand throughout all our material is everywhere extra-nuclear and we therefore propose for it the name paradesmose since it cannot occupy the position of the central spindle or "centrodesmose." Kuczynski[4] has shown that Dobell's and Prowazek's conclusions are invalid since the paradesmose persists *till after the new axostyles appear.* This author, however, falls into the error of concluding that the daughter axostyles arise as new outgrowths from certain end granules of the old axostyle originally connected with the blepharoplast. He finds that the old axostyle fades out and the daughter axostyles arise as new outgrowths. In our material it has always been possible to detect in all individuals which have passed the anaphase of mitosis one of three conditions, either (1) the single undivided axostyle or (2) this axostyle in process of splitting longitudinally, or (3) two complete daughter axostyles. The presence of the axostylar chromidia has been of great assistance in following the longitudinal splitting of this organ in *Trichomonas augusta.* A similar splitting of the axostyle occurs in *Lamblia muris* and in two or more species of *Hexamitus.*

Wenyon[6] states that the axostyle of *Trichomonas intestinalis* of the mouse divides longitudinally after the division of the nucleus and blepharoplast. His figure, however, does not support his statement. It shows two axostyles rather than a dividing one.

Shortly after the axostyle splits the two nuclei and axostyles move farther apart, the body elongates, the anterior ends of the two daughters come to lie at opposite poles (fig. 7) and the cytoplasmic central bridge progressively narrows down until the two daughter organisms separate. The final plane of constriction is transverse to the main axis of each cell and of the common elongating mass.

An inspection of our figures and still more of our preparations reveals a great variety of positions of the organelles of the cell. Observations on living cells under high magnifications in culture slides convince us that this variability in position is normal and not due to artefacts.

These conditions have led Martin and Robertson[3] to state that division in *Trichomonas gallinarum* may occur in any one of three planes and that "these divisions may be termed respectively longitudinal, transverse, and oblique."

During the period of mitosis the body is somewhat rounded up, and is constantly in more or less violent motion. In these activities not only the flagella and undulating membranes share but also the axostyle. This bends at right angles (fig. 2), curves (fig. 6), and twists about very energetically almost as a flagellum lashing about in the viscid cytoplasm. Its activity is accompanied by a considerable mobility of the cytoplasm with the result that the positions of the organelles, of the nuclei, of the blepharoplasts with their accompanying flagella and undulating membrane, and even of the axostyle itself, are subject to incessant readjustments in their relations. They may even become somewhat widely detached from one another (figs. 4–7). These conditions which Martin and Robertson[3] have interpreted as indicating other types of division than longitudinal are thus due to the protean activity of the organisms. When, however, we seek to define the plane of division on morphological grounds and accept the premitotic cell (fig. 1) as indicating the normal orientation of the organelles within the body, we find that, if the daughter blepharoplasts migrate equally 90°, as they normally do in mitosis, the nucleus of *Trichomonas* would be divided in a plane which in the premitotic arrangement is approximately longitudinal. The chromatic margin of the undulating membrane, the membrane itself, and the axostyle all split lengthwise, that is, longitudinally. *On morphological grounds the division of this flagellate is therefore solely longitudinal.* The interpretation of any other plane rests purely on variable and temporary positions of organelles which become movable during the *later phases* of mitosis.

This process of division has been interpreted, with emphasis, in *Trichomonas gallinarum* by these same investigators[3] as purely amitotic. The differences between their figures and our own are not very great, and it is quite possible that under certain pathological conditions nuclear appearances simulating amitosis may occur. We are inclined to the view that division in this species also is mitotic and that Martin and Robertson's[3] figures are not wholly incompatible with this interpretation. A division in which chromatin masses (chromosomes) of definite number and regular forms are differentiated, split longitudinally, and move to the poles of a spindle at each of which a centrosome formed by the division and polar migration of the parent organelle is found, is essentially mitotic rather than amitotic.

The conclusions drawn are:

(1) Cell division in trichomonad flagellates is a true mitosis with differentiated chromosomes (five in *Trichomonas augusta*), which split longitudinally prior to their location in the equatorial plate.

(2) The nuclear membrane persists throughout mitosis. The blepharoplast which is connected to the flagella, rhizoplast, chromatic margin and basal rod of the undulating membrane, and to the axostyle, after division usually gives rise by division to a polar centrosome and an adjacent basal granule which alone retains connection with the extranuclear apparatus including the paradesmose. These two fuse again to form the daughter blepharoplasts.

(3) The paradesmose between the migrating blepharoplasts is extranuclear at all times, disappears after nuclear division, and does not give rise to the axostyle. Since this connecting strand lies outside of the nuclear membrane and connects basal granules and not centrosomes, it should not be called centrodesmose or central spindle, but rather the paradesmose.

(4) The axostyle splits longitudinally and thus forms the two daughter axostyles. These do not grow out as new structures in *Trichomonas augusta*. It and the chromatic basal rod (homologous with the parabasal chromatic structures of Janicki[7]) may be regarded as derivatives of the blepharoplast, homologous with flagella but intra-cytoplasmic.

[1] Dobell, C. C., Researches on the intestinal Protozoa of frogs and toads. *Q. J. Microsc. Sci., London,* 53, 201–277, pls. 2–5 (1908).

[2] Prowazek, S., Untersuchungen über einige parasitische Flagellaten. *Arb. Gesundhtsamt., Berlin,* 21, 1–41, pls. 1–4 (1904).

[3] Martin, C. H. and Robertson, M., Further observations on the caecal parasites of fowls. Part I. *Q. J. Microsc. Sci., London,* 57, 53–81, pls. 10–14 (1911).

[4] Kuczynski, M. H., Untersuchungen an Trichomonaden. *Arch. Prot.,* 33, 119–204, pls. 11–16 (1914).

[5] Montgomery, T. H., The spermatogenesis of an hemipteron. *J. Morph., Boston,* 22, 731–815, pls. 1–5 (1911).

[6] Wenyon, C. M., Observations on Protozoa in the intestine of mice. *Arch. Prot.,* Suppl. v. 1, 169–201, pls. 10–12 (1907).

[7] Janicki, C., Bemerkungen zum Kernteilungsvorgang bei Flagellaten namentlich bei parasitischen Formen. *Verh. Nat. Ges. Basel,* 23, 82–111, 8 figs. (1912).

REPORT OF THE ANNUAL MEETING

Prepared by the Home Secretary

The Annual Meeting of the Academy was held in the Oak Room of the Raleigh Hotel and in the National Museum at Washington, D. C., on April 19, 20, and 21, 1915.

Sixty-one members were present, as follows: Messrs. Abel, Becker, Bell, Boltwood, Britton, Bumstead, Cattell, Chamberlin, Chittenden, W. B. Clark, F. W. Clarke, J. M. Clarke, Conklin, Coulter, Cross, Dall, Davenport, Davis, Day, Donaldson, Fewkes, Frost, Hague, Hale, Harper, Harrison, Hayford, Hillebrand, Holmes, Howell, Jennings, Loeb, Mall, Meltzer, Mendel, Merriam, Michelson, Moore, Morgan, Morley, E. L. Nichols, A. A. Noyes, W. A. Noyes, H. F. Osborn, T. B. Osborne, Parker, Pickering, Pirsson, Ransome, Reid, Remsen, Schuchert, Scott, Erwin F. Smith, Walcott, Webster, Welch, Wheeler, White, R. W. Wood, Woodward.

BUSINESS SESSIONS

The President announced that the preparation of Biographical Memoirs of deceased members had been assigned as follows:

Memoir of	*Assigned to*
Bowditch, Henry P.	Cannon, Walter B.
Gould, Benjamin Apthorp	Comstock, George C.
Mitchell, Henry	Hayford, John F.
Mitchell, Silas Weir	Welch, William H.
Chandler, Seth Carlo	Elkin, William L.
Peirce, Benjamin Osgood	Hall, Edwin H.
Holden, Edward Singleton	Campbell, William W.
Hill, George William	Brown, E. W.
Peirce, Charles Sanders	Cattell, J. McKeen.
Gill, Theodore Nicholas	Dall, William H.
Minot, Charles Sedgwick	Donaldson, Henry H.
Billings, John S.	Garrison, Fielding H.

REPORTS FROM OFFICERS OF THE ACADEMY

The reports of the President and of the Treasurer for 1914 as transmitted to the Senate of the United States by the President of the Academy were presented in their printed form and approved.

The report of the Home Secretary was presented as follows:

The President of the National Academy of Sciences.

Sir: I have the honor to present the annual report of the home secretary of the National Academy of Sciences for the year ending April 21, 1915.

Of the Scientific Memoirs of the National Academy of Sciences, volume 12, part 1, bearing the title, "Monograph of the Bombycine Moths of North America," by A. S. Packard, edited by T. D. A. Cockerell, has been published and distributed to the members, foreign associates, institutions, and reference libraries. Volume 12, part 3, of the Memoirs, entitled, "The Turquoise," by Joseph E. Pogue, has also been published and distributed to the members. Part 2 of this same volume entitled, "Variations and Ecological Distribution of the Snails of the Genus Io," by Charles C. Adams, has received final consideration, and is now waiting to be bound at the Government Printing Office. The memoir forming volume 13, being "A Catalogue of the Meteorites of North America," by Oliver C. Farrington, only awaits press-work and binding before it is issued.

The biographical memoirs of John Wesley Powell, Charles A. Schott, and Miers Fisher Longstreth have been published. The publication of the memoir of J. Peter Lesley, by William M. Davis, has been approved by the committee on publications; and the biography of Henry Morton, by Edward L. Nichols, has been printed and awaits the portrait.

Three members have died since the last annual meeting: Theodore Nicholas Gill, on September 25, 1914, elected in 1873; Charles Sedgwick Minot, on November 19, 1914, elected in 1897; and Henry Lord Wheeler, on October 30, 1914, elected in 1909.

Of our foreign associates, Edward Suess, elected in 1898, died on April 26, 1914; August Weismann, elected in 1913, on November 5, 1914; Hugo Kronecker, elected in 1901, on June 6, 1914; G. F. J. Arthur Auwers, elected in 1883, on January 24, 1915.

There are 134 active members on the membership list, 1 honorary member, and 43 foreign associates.

Respectfully submitted, ARTHUR L. DAY, Home Secretary.

REPORTS FROM COMMITTEES ON TRUST FUNDS

A report was received from the directors of the Bache Fund, stating that the vacancy caused by the death of Charles S. Minot had been filled by the election by the two remaining members of the Board of Ross G. Harrison, and that he has accepted the appointment. The Board elected Mr. Ira Remsen as its Chairman. The report contained also an announcement of the research grants made from the Bache Fund during the year ending April 19, 1915.

A report was received from the Trustees of the Watson Fund, signed by E. C. Pickering (chairman), W. L. Elkin, and E. B. Frost. The report stated that the wish expressed in the will of the late James Craig Watson that tables be prepared of the motions of all the planets discovered by him has now been carried out in a most satisfactory manner by Mr. A. O. Leuschner, so that the income which has been used for this purpose during the last fourteen years is now available for the promotion of Astronomical Science in other directions. In the report the trustees recommended that the sum of five hundred dollars from the income of the Watson Fund be appropriated to John A. Miller, Director of the Sproul Observatory, for measuring plates already taken for the determination of stellar parallaxes; and that the sum of three hundred dollars be appropriated from the income of the Watson Fund to John E. Mellish, to enable him to undertake observations at the Yerkes Observatory. It was recommended that the Watson Medal and the sum of one hundred dollars be awared to Armin Otto Leuschner of the University of California for the skill and ability which he has shown in supervising the preparation of tables of the Watson asteroids, involving original methods, and leading to results of much value to celestial mechanics. These recommendations were adopted by the Academy.

A report was received from the Committee on the Henry Draper Fund, signed by George E. Hale (chairman). The Committee recommended the approval of a grant from the Draper Fund of $500 to W. W. Campbell, Director of the Lick Observatory, for the purchase and construction of spectrographic and other apparatus for use with the Crossley Reflector. The Committee also recommended the approval of a grant of two hundred and fifty dollars to S. A. Mitchell, Director of the Leander McCormick Observatory, for the purchase of a machine for measuring astronomical photographs. These recommendations were adopted by the Academy.

A report was received from the Committee on the J. Lawrence Smith Fund, signed by Edward W. Morley (chairman). The report described the progress made on certain researches which have received aid from the Fund. It was announced that, in accordance with the vote passed by the Academy in 1912, the fourth $250 installment of the grant of one thousand dollars has been paid to C. C. Trowbridge, of Columbia University, in aid of his study of the luminous trains of meteors. The Committee recommended a grant of five hundred dollars to S. A. Mitchell of the University of Virginia in aid of computations of orbits of meteors. The Committee also recommended that the meteorites remaining from the purchases made by George P. Merrill, Curator in the Department of Geology in the United States National Museum, under the grants awarded to him in aid of the study of certain elements suspected to be present in small quantities in meteorites, be deposited by the Academy in the United States National Museum. These recommendations were adopted by the Academy.

A report was received from the Directors of the Wolcott Gibbs Fund, signed by C. L. Jackson (chairman), describing three grants made from the fund during the preceding year and outlining the progress made on the researches aided by those grants.

A summarized statement of the grants made from the various trust-funds of the Academy during the year ending April 15, 1915 will be found at the end of this number of the PROCEEDINGS.

REPORTS FROM STANDING COMMITTEES

A report was received from the Committee on Solar Research, signed by George E. Hale (chairman) calling the attention of the Academy to the publication of volume 4 of the *Transactions of the International Union for Cooperation in Solar Research*, which contains the complete proceedings of the last meeting in Bonn, reports of the various committees, resolutions adopted by the Union, and several scientific papers on solar and stellar phenomena. The report states that the four volumes of *Transactions* already published by the Solar Union may be obtained from Longmans, Green & Company, New York City, at $2.50 per volume.

A report was received from the Editorial Board of the PROCEEDINGS, signed by Arthur A. Noyes (chairman), as follows:

The Editorial Board of the PROCEEDINGS reports to the Academy that four numbers of the PROCEEDINGS have now been issued containing sixty-nine original papers in addition to the report of the Autumn Meeting, notices of scientific memoirs, and announcements. These numbers have consisted of 258 pages, an average of 64 pages per number and of about four pages per article. The papers are distributed among different sciences as follows: mathematics, 11; astronomy, 11; physics, none; chemistry, 13; geology, 2; paleontology, 1; botany, 4; zoology, 5; genetics, 5; bacteriology, 2; physiology, 8; pathology, none; anthropology, 5; psychology, 2. It will be noticed that the subjects of physics, of geology and paleontology, and of pathology, have been very inadequately represented; and the Editorial Board urges members of the Academy in these fields to endeavor to remedy this situation.

An edition of 3000 copies of these four numbers has been printed. Of this edition about 900 are to be sent abroad to the libraries of universities and other active research institutions upon a mailing list prepared with great care by the Foreign Secretary aided by members of the Editorial Board. Of this edition 1200 copies have also been distributed in this country by the Home Secretary to important libraries and to the thousand persons whose names are starred in Cattell's *American Men of Science.*

A report was received from the Committee on the Collection of Historical Portraits, Manuscripts, and Instruments, signed by Charles D. Walcott (chairman). The committee announced in this report that the collection of portraits of members of the Academy has been brought together and arranged alphabetically; that the foreign secretary has turned over to the Committee the medal from the Groningen Academy celebrating its four hundredth anniversary; and that many photographs and certain pieces of apparatus have been presented by Mrs. Henry Draper and have been deposited in the United States National Museum.

GENERAL BUSINESS

Upon recommendation of the Council it was voted to accept the bequest of Mrs. Mary Anna Palmer Draper provided for by the following paragraph of her will: "I give and bequeath to the National Academy of Sciences, Smithsonian Institution, Washington, D. C., the sum of twenty-five thousand dollars ($25,000)."

The President announced that an invitation had been received from the members of the Academy living in New York City to hold the Autumn Meeting there, and that this invitation had been accepted.

REVISION OF THE CONSTITUTION AND RULES

The draft of the Constitution as amended and adopted in Committee of the Whole at the Chicago Meeting was presented for final action and adopted by the Academy on April 21, 1915.

This Constitution is printed on pages 30–37 of the *Report of the Academy for the Year 1914*, in such a way as to show clearly the new amendments. The most important of these provides that 15 (instead of 10) new members may be elected annually until the total membership of the Academy shall reach 250 (instead of 150). Another amendment provides that "the Academy shall be divided by the Council into sections" (heretofore called standing committees) "representing the principal branches of scientific research;" and that "each section shall elect its own chairman" (heretofore appointed by the President), "who shall serve for three years." Another amendment provides that the annual meeting shall begin on the third Monday (instead of the third Tuesday) in April.

An amended code of rules was presented by the Committee on the Constitution and Rules; and this was adopted by the Academy on April 21, 1915. Minor and formal changes are thereby made in many of the former rules of the Academy. The new rules and those which have been substantially modified are as follows:

III. 5. A standing committee of three members called the Program Committee, a local committee of five members appointed for each meeting, and the Home Secretary, shall together constitute the Committee of Arrangements, of which the Home Secretary shall be chairman.

The term of service of the Program Committee shall be three years, one member retiring annually, the chairman to be designated by the President.

It shall be the duty of the Program Committee to prepare the scientific program for the Annual Meeting and for this purpose it shall be empowered to solicit papers from members or others. It shall also be empowered to ascertain the length of time required for reading papers to be presented at the scientific sessions of the Academy, and, when it appears advisable, to limit the time to be occupied in their presentation or discussion.

The local committee shall meet not less than two months previous to each meeting. It shall prepare the detailed program of each day, and in general shall have charge of all business and scientific arrangements for the meeting for which it is appointed except those specifically assigned to the Program Committee in the case of the Annual Meeting.

IV. 1. Each section of the Academy shall elect a chairman to serve for a term of three years. If the chairmanship of a section becomes vacant in an interval between stated meetings of the Academy the President shall appoint a chairman to serve until the next stated meeting, at which time the section shall elect a chairman for a term of three years.

IV. 2. The chairman of each section of the Academy shall submit to the members of his section, not later than November 1 of each year, a ballot containing the names of all those persons who received not less than two votes in the nominating ballot of the preceding year and of any other persons who were newly proposed for consideration at that time. Each member of the section shall be expected to return this ballot to the chairman within two weeks with his signature and with crosses placed against the names of those persons whom he is prepared to endorse for nomination. Each member may also write upon the ballot in a place provided for the purpose any new names which he desires to have included in the ballot to be submitted to the section in the following year. The vote resulting from this ballot shall be regarded as informal.

The chairman shall then submit to the members of his section a new ballot showing the results of the informal vote; and each member shall be expected to return this ballot to the chairman with his signature and with crosses placed against the names of those whom he will endorse for nomination.

The chairman shall then certify to the Home Secretary, prior to January 1, the names of those persons who have been voted for on this second ballot by a majority of the members of the section, and shall furnish him a list of the publications of these nominees, as required by the Constitution.

IV. 4. Preference ballots for election of members shall be sealed in a blank envelope which shall be inclosed in another bearing the name of the sender and which shall be addressed to the Home Secretary. Such envelopes shall be opened only by the tellers. If in any case the tellers are unable to determine who cast a ballot, or if the latter contains more names than are to be voted for, the ballot shall be rejected; but minor defects in a ballot shall be disregarded when the intent of the voter is obvious.

V. 1. The publication of the PROCEEDINGS shall be under the general charge of the Council, which shall have final jurisdiction upon all questions of policy relating thereto. The preparation of the PROCEEDINGS for publication shall be entrusted to an Editorial Board. This board shall consist of a chairman and a managing editor, both of whom shall be appointed by the Council for definite terms of service; of the Home Secretary and Foreign Secretary, ex officiis; and of a body of associate editors representing the various branches of science which are to be included in the scope of the PROCEEDINGS. The managing editor, who may be a non-member of the Academy, shall receive a salary which shall be fixed by the Council. It shall be the duty of the managing editor to prepare the PROCEEDINGS in detail for publication; but the chairman of the Editorial Board shall decide, in consultation with the managing editor, in regard to the acceptance, rejection, or substantial modification of papers offered for publication. The associate editors shall be appointed by the Council, upon recommendation of the chairman of the Editorial Board, for a period of three years, one-third of them retiring annually. The Home Secretary and the Foreign Secretary shall be responsible for the distribution of the PROCEEDINGS in their respective fields.

VI. 5. The standing committee on finance shall consist of the Treasurer ex officio as chairman and two members to be appointed annually by the President.

ELECTION OF COUNCILLORS AND OF NEW MEMBERS

J. M. COULTER and W. H. HOWELL were elected members of the Council for a period of three years, to succeed W. T. Councilman and R. S. Woodward, whose terms of service had expired.

The following persons were elected as new members of the Academy:

HENRY SEELY WHITE, Mathematician, Vassar College, Poughkeepsie, N. Y.
CHARLES GREELEY ABBOT, Astrophysicist, Astrophysical Observatory, Smithsonian Institution, Washington, D. C.
ROBERT ANDREWS MILLIKAN, Physicist, University of Chicago, Chicago, Ill.
ALEXANDER SMITH, Chemist, Columbia University, New York City.
SAMUEL WENDELL WILLISTON, Paleontologist, University of Chicago, Chicago, Ill.
WILLIAM ERNEST CASTLE, Zoologist, Harvard University, Cambridge, Mass.
FRANK RATTRAY LILLIE, Zoologist, University of Chicago, Chicago, Ill.
GRAHAM LUSK, Physiologist, Cornell University Medical College, New York City.
VICTOR CLARENCE VAUGHAN, Pathologist, University of Michigan, Ann Arbor, Mich.
GRANVILLE STANLEY HALL, Psychologist, Clark University, Worcester, Mass.

SCIENTIFIC SESSIONS

Two public lectures on the WILLIAM ELLERY HALE FOUNDATION were given on April 19 and 21 by THOMAS CHROWDER CHAMBERLIN, of the University of Chicago, on the Evolution of the Earth.

A public illustrated lecture was given by GEORGE H. PARKER, Official Representative of the Academy upon the Special Commission appointed by the President of the United States to study and report upon the Alaskan Fur Seals during the summer of 1914, on The Fur Seal Herd of the Pribilof Islands.

Three public scientific sessions were held on April 19 and 20 at which the following papers were presented:

Thomas H. Morgan: Localization of the hereditary material in germ cells.

Jacques Loeb: Stimulation of growth.

Lafayette B. Mendel: Specific chemical aspects of growth.

Eugene F. Du Bois, Medical Director, Russell Sage Institute of Pathology (by invitation of the Program Committee): Basal metabolism during the period of growth.

I. S. Kleiner and **S. J. Meltzer**: Retention in the circulation of injected dextrose in depancreatized animals and the effect of an intravenous injection of an emulsion of pancreas upon this retention.

Joel Stebbins, Draper Medallist: The electrical photometry of stars.

George E. Hale: A vortex hypothesis of sun spots.

Edwin B. Frost: The spectroscopic binary, Mu Orionis.

Robert W. Wood: One-dimensional gases and the experimental determination of the law of reflection for gas molecules.

Robert W. Wood: The relations between resonance and absorption spectra.

Edward L. Nichols and **H. L. Howes**: On the polarized fluorescence of ammonio-uranyl chloride.

Robert A. Millikan (by invitation of the Program Committee): Atomism in modern physics.

William Morris Davis: Problems associated with the origin of coral reefs, suggested by a Shaler Memorial study of the reefs of Fiji, New Caledonia, Loyalty Islands, New Hebrides, Queensland, and the Society Islands, in 1914.

F. W. Clarke: Inorganic constituents of marine invertebrates.

Henry Fairfield Osborn and **J. Howard McGregor**: Human races of the Old Stone Age of Europe, the geologic time of their appearance, their racial and anatomical characters.

Charles A. Davis, Geologist, Bureau of Mines (by invitation of the Program Committee): On the fossil algae of the petroleum-yielding shales of the Green River Formation.

Nathaniel L. Britton: The forests of Porto Rico.

J. Walter Fewkes: Pictures on prehistoric pottery from the Mimbres Valley in New Mexico and their relation to those of Casas Grandes.

Charles B. Davenport: Inheritance of temperament.

Charles B. Davenport: Inheritance of Huntington's chorea.

AWARD OF THE HENRY DRAPER MEDAL

At the Annual Dinner of the Academy held at the Hotel Raleigh on April 20, 1915, the Henry Draper Medal was awarded to Joel Stebbins, of the University of Illinois, in recognition of his work on the Application of the Selenium Cell to Stellar Photometry.

RESEARCH GRANTS FROM THE TRUST FUNDS OF THE ACADEMY

During the twelve months preceding the Annual Meeting of the Academy the following grants for the promotion of research were made from trust funds of the Academy.

GRANTS FROM THE BACHE FUND

No. 182, W. C. Kendall, $600. April 30, 1914. For illustrations in color and incidental expenses in connection with part II (Salmonidae) of Fishes of New England to be published by the Boston Society of Natural History.

No. 183, C. G. Abbot, Smithsonian Institution, $250. June 29, 1914. To complete and test on Mt. Wilson in California an apparatus consisting of a concave cylindrical mirror of about 100 sq. ft. surface adapted to heat oil to circulate through a reservoir containing ovens and water pipes, and thereby to utilize solar radiation for cooking and for heating water for domestic purposes.

No. 184, P. W. Bridgman, Harvard University, $500. September 14, 1914. To continue the work on high pressures, especially to investigate the phase changes brought about in various substances by very high pressure.

No. 185, Robert W. Hegner, $160. December 26, 1914. To determine the visible changes that take place during the differentiation of the germ cells in the embryos of hermaphroditic animals, and to discover, if possible, the cause of these changes.

No. 186, J. Voûte, $800. February 9, 1915. For the determination of parallaxes of Southern Stars by transits, to be conducted at the Royal Observatory, Cape of Good Hope, wholly at the expense of Mr. Voûte, except for these grants. (A grant of $1000 was previously made for the same purpose.)

No. 187, H. H. Lane, $500. April 14, 1915. For a comparative study of the embryos and young of various mammals, in order to determine, by physiological experimentation and morphological observations, the correlation between structure and function in the development of the special senses.

GRANT FROM THE J. LAWRENCE SMITH FUND

No. 4, C. C. Trowbridge, Columbia University, $250. In aid of his study of the luminous trains of meteors.

GRANTS FROM THE WOLCOTT GIBBS FUND

No. 3, W. J. Hale, University of Michigan, $100. May 15, 1914. For assistance in a research on derivatives of 2.3-diacetyl pentadiene.

No. 4, W. D. Harkins, University of Chicago, $200. November 25, 1914. For a special potentiometer and galvanometer to study cobaltamines and ternary systems of fused salts.

No. 5, Mary E. Holmes, Mt. Holyoke College, $100 (second grant), March 18, 1915. For her research on the electrolytic determination of cadmium.

GRANT FROM THE WATSON FUND

No. 9, A. O. Leuschner, University of California, $960. April 21, 1914. For the preparation of tables of the Watson asteroids.

GRANT FROM THE DRAPER FUND

C. G. Abbot, Astrophysical Observatory of the Smithsonian Institution, $500. April 21, 1914. For the services of a computer for the reduction of observations of the solar-radiation and the absorption of the sun's atmosphere.

The following additional grants were authorized by the Academy at the business session of April 19, 1915.

GRANTS FROM THE WATSON FUND

No. 10, John A. Miller, Sproul Observatory, Swarthmore College, $500. For the measurement of plates already taken for the determination of stellar parallaxes.

No. 11, John E. Mellish. To enable him to undertake observations at the Yerkes Observatory.

GRANTS FROM THE DRAPER FUND

W. W. Campbell, Lick Observatory, University of California, $500. For the purchase and construction of photographic and other apparatus for use with the Crossley Reflector.

S. A. Mitchell, Leander McCormick Observatory, University of Virginia, $250. For the purchase of a machine for measuring astronomical photographs.

GRANT FROM THE J. LAWRENCE SMITH FUND

No. 5, S. A. Mitchell, University of Virginia, $500. For computation of the orbits of meteors.

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OF THE
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PROCEEDINGS
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Volume 1 JUNE 15, 1915 Number 6

CONFIRMATORY EXPERIMENTS ON THE VALUE OF THE SOLAR CONSTANT OF RADIATION

By C. G. Abbot, F. E. Fowle, and L. B. Aldrich
SMITHSONIAN INSTITUTION, WASHINGTON, D. C.

Presented to the Academy, April 27, 1915

We have made hitherto nearly 1000 determinations of the intensity of solar radiation outside the atmosphere at mean solar distance, termed the solar constant of radiation. The mean value found is 1.93 calories per square centimeter per minute. Langley's spectro-bolometric method was employed. This consists in determining the distribution of the energy in the solar spectrum at different solar zenith distances, and thereby computing coefficients of atmospheric transmission suitable to determine the energy curve outside the atmosphere. The bolometric measurements are reduced in terms of standard 15° calories per square centimeter per minute by the aid of comparisons made each day of observation with standardized pyrheliometers. Observations have been made at Washington (sea level); Bassour, Algeria (1160 meters); Mount Wilson, California (1730 meters); and Mount Whitney, California (4420) meters. They have continued during all the years 1903 to 1914. Great changes from day to day and from place to place in temperature, in barometric pressure, in humidity, in haziness, while of course greatly affecting measurements of intensity at the stations, and of atmospheric transparency computed, nevertheless have not produced differences of the solar constant values. This seems to us to be strong evidence of the soundness of the method.

In the second place it has been shown by Fowle that the atmospheric transmission coefficients obtained at Mount Wilson fit well with Lord Rayleigh's theory of atmospheric scattering, except for those regions of spectrum where numerous atmospheric lines and bands of true absorption are known to occur. Fowle has computed from the transmission coefficients that the number of molecules per cubic centimeter of air at standard temperature and pressure is $(2.70 \pm 0.02) \times 10^{19}$. This value is very close to Millikan's determination by absolutely independent observations and methods, namely $(2.705 \pm 0.005) \times 10^{19}$.

In the third place simultaneous solar-constant observations at Mount Wilson and Bassour, separated by one-third the earth's circumference, unite in showing a substantial irregular variability of the sun from day to day. This solar variability has been of late independently confirmed by us by examination of the distribution of brightness along the diameter of the sun's disk. The latter observations show variations of distribution from day to day, and these accompany pretty closely the variations of the total solar radiation. It seems to us that, as the fact of solar variability is thus independently confirmed as a real phenomenon, it speaks favorably for the substantial accuracy of our solar constant measurements that it was through them that the irregular variations of from 1 to 5 or, very rarely, 10% were first discovered.

Notwithstanding these evidences of the soundness of our solar constant work, various attacks upon it have been made, tending to show that the solar constant is much higher than 1.93 calories, perhaps even 3.5 to 4.0 calories. A principal argument is that the atmospheric transparency continually diminishes as the sun rises within 75° zenith distances, so that our values of atmospheric transmission are much too great, and have no relation to the transmission of an atmosphere of constant transparency. Secondly it is said that measurements of solar radiation exceeding 1.93 calories have been made on mountain tops, and from free balloons. Various other objections are raised, which we discuss in our paper now being published by the Smithsonian Institution.

On two days, September 20 and 21, 1914, we continued solar constant observations at Mount Wilson from the instant of sunrise until about 10 o'clock. We have reduced the work by the aid of Bemporad's air-mass formulae and tables. As these postulated uniform optical quality of the atmosphere from bottom to top, it was necessary to apply certain corrections to them varying with the wave-length, depending upon the extinction by water vapor residing in the lowest atmospheric strata. We were enabled to determine these corrections by Fowle's studies of the effects of water vapor. We find on both days that the atmospheric transparency remained sensibly unaltered from sunrise to 10 o'clock. Closely identical values of the solar constant are obtained, whatever the range of air masses used to determine the atmospheric transmission. We made three independent estimates for each day, for air-mass ranges 1.3 to 4; 4 to 12; and 1.3 to 20 air-masses, respectively. All six solar constant values thus found fall between 1.90 and 1.95 calories. The smallest air masses, as it happens, yield slightly the highest values. We conclude that our previous results have not been made too small by neglecting to observe during the time when the sun is within 15° of the horizon.

On July 11, 1914, in coöperation with the United States Weather Bureau, a recording pyrheliometer attached to sounding balloons was sent up to the altitude of about 24 km., where the barometric pressure was 3 cm. of mercury, which is only one twenty-fifth of that which prevails at sea level.

Good records of solar radiation were obtained over a period of more than an hour, and including the period when the instrument reached highest elevation. The mean value of the best three records made at highest altitudes, as reduced to mean solar distance, comes out 1.84 calories per cm.^{2} per minute. We believe about 2% should be added to represent radiation scattered and absorbed in the atmosphere above the level reached, making for the probable value of the solar constant, from this day's work, 1.88 calories. This value falls decidedly within the range of solar constant values we have observed. We state in connection with it the following results which are the highest reliable direct observations of solar radiation at the various altitudes, as reduced to mean solar distance and vertical sun:

Station	*Washington*	*Mount Wilson*	*Mount Whitney*	*Manned Balloon*	*Free Balloon*
Altitude..........	127 m.	1730 m.	4420 m.	7500 m.	24000 m.
Barometer.........	75 cm.	62 cm.	45 cm.	30 cm.	3 cm.
Radiation	1.58	1.64	1.72	1.755	1.84 cal.
Observer	Kimball	Abbot	Abbot	A. Peppler	(Smithsonian)

VARIATION OF FLOWER SIZE IN NICOTIANA

By T. H. Goodspeed and R. E. Clausen

DEPARTMENT OF BOTANY, UNIVERSITY OF CALIFORNIA

Presented to the Academy, April 28, 1915

During the past five years considerable attention has been given to the study of the inheritance of flower-size in *Nicotiana* at the University of California Botanical Gardens. In the course of these investigations it has been found that flower size varies markedly under different conditions attending development. This report is concerned with an analysis of some of these conditions, and the bearing of such variations on the study of flower-size inheritance in *Nicotiana*.

The conclusions drawn in this paper are based on some 25,000 measurements taken on the following pure lines and hybrids of *Nicotiana:*

1. *N. Tabacum* var. *macrophylla*[1] in the growing season 1913. This *N. Tabacum* variety has been grown at the University of California Botanical Gardens for six years in the pure line, under the garden number 22/07.

2. The F_1 hybrid, *N. Tabacum* var. *macrophylla* × *N. sylvestris*,[2] which will be referred to as F_1H_{38}. Flowers of this hybrid were measured on plants from the first sowing of the hybrid seed in 1912, on the same plants cut back and coming up from the roots in 1913, and on a second sowing of the same hybrid seed in 1913.

3. F_1H_{137}, the reciprocal of the above cross, flowers of which were measured in 1913.

4. The F_1 hybrid produced by crossing the F_1 of the hybrid *N. Tabacum* var. Maryland × *N. Tabacum* var. Cavala with *N. sylvestris*, the garden number of which is H_{44}. As with H_{38}, flowers of this hybrid were first measured in 1912, and again on the same plants cut back and coming up from the roots in 1913. In addition cuttings of these plants were made and measurements taken on them.

From the data on these and other populations it has been found possible to determine a number of factors which influence flower size in *Nicotiana*. For convenience these are treated under various rather arbitrarily selected headings. Under the 'age of plant' heading are included not only a consideration of the difference in the size of flowers borne early in the season as compared with those borne late in the season on the same plants, but also a consideration of the difference in the size of flowers the first blooming season of the plant compared with the size of the flowers produced the second year on the same plants cut back and coming up from the roots. Under the 'age of flower' heading are included first, a consideration of the difference in the size of flowers borne on the terminal inflorescences just going out of flower and those borne at the same time on laterals and second, the influence of age on the individual flower; i.e., comparisons of measurements of flower fully opened, before and after shedding pollen. Other factors such as the influence of the removal of flowers and developing seed capsules, the behavior of cuttings under various conditions, and the influence of soil fertility have likewise furnished data upon this variability of flower size under conditions attending development.

When plants first come into flower the spread and length of corolla are greater than the spread and length of flowers produced on the same plants later in the growing season. Thus the mean spread of corolla of 75 plants of *N. Tabacum* var. *macrophylla* at the beginning of the blooming period was 28.49 ± 0.09 mm. and the length 45.60 ± 0.07 mm. Measurements of flowers taken on an average one month later gave a mean corolla spread for the same plants of 24.19 ± 0.22 mm. and for length 42.25 ± 0.15 mm. For the same plants means derived by grouping together the measurements of 25 of these plants early in the season, 25 in the middle of the period, and the remaining 25 toward the end of the period of measurement gave for spread 26.69 ± 0.20 mm. and for length 43.96 ± 0.18 mm., which figures lie very close to the means for all the plants for the entire season which are 26.83 ± 0.09 mm. and 44.27 ± 0.07 mm. respectively.

The latter arrangement brings up a practical matter of considerable importance in connection with genetic investigations of flower size in *Nicotiana*. In this case although the means were practically identical for the two arrangements, the coefficient of variability where each third of the plants was measured at a different time was 9.77 ± 0.54% whereas that based on the distribution of means for all the plants for the entire season was less than half as great, 4.45 ± 0.24%. For length the corresponding coefficients of variability, 5.18 ± 0.29% and 2.13 ± 0.12% respectively, show a closely proportionate correspondence in this respect. This higher variability is exactly what would be obtained if a large F_2 population were measured progressively, that is beginning at one end of the field and working through until the last plant had been measured. Such a method of measurement used in comparing large F_2 populations with relatively small parent and F_1 populations might very easily lead to erroneous conclusions on the critical point of the experiment, namely the extent of increase of variability in F_2.

In the case of the semi-sterile hybrids of *N. Tabacum* varieties with *N. sylvestris* another factor enters in, namely the lack of seed production and the consequent prolonged blooming period. There is, therefore, not so sudden a decrease in corolla size as in the case of *N. Tabacum* var. *macrophylla;* but nevertheless a marked decrease occurs rather later in the blooming season. In the case of spread of flowers of F_1H_{137} there was a decrease during the period from August 9 to September 19 from 38.33 ± 0.11 mm. to 31.67 ± 0.26 mm. For length the decrease was less striking, from 56.22 ± 0.09 mm. to 53.44 ± 0.28 mm. Similarly in 13 (11) F_1H_{38} there was a decrease in corolla spread from August 4 to November 7 from 39.10 ± 0.31 mm. to 33.10 ± 0.22 mm., and in length from 57.10 ± 0.22 mm. to 52.60 ± 0.29 mm. Flowers of $13F_1H_{38}$, genetically the same but a different population blooming the second year on their own roots, were measured at the beginning of the flowering season during the period from May 21 to June 20. During this time the flower size did not decrease as is shown by comparing the mean spread on the first dates of 38.81 ± 0.31 mm. with 40.05 ± 0.35 mm., the mean spread for the last dates, and in the case of length of corolla, 55.19 ± 0.15 mm. and 53.81 ± 0.24 mm., a slight decrease. On Plant 6, however, flowers were measured again on July 31. It was found that a mean spread of 43.41 mm. based on measurements of all the flowers on June 16 had decreased to 37.38 mm. based on measurements of all the flowers on July 31. H_{44}, which was measured during this same period, similarly shows only a slight decrease in flower size, but in every case where flowers were measured on the later date a striking decrease in size was obtained. We have taken the results of similar determinations and condensed them in the following table in which not only the means and their probable errors are given, but also the differences between measurements of different dates and the probable errors of these differences. These differences are to be regarded as significant in case they exceed two and a half or three times the corresponding probable error. In each case the constant for spread is given on the first line, that for length on the second.

Influence of Age of Plant on Flower Size. Measures in Millimeters

GARDEN NUMBER	MEANS FOR DATES			DIFFERENCES BETWEEN DATES		
	First	Mid	Last	First and Mid	Mid and Last	First and Last
$13F_1H_{44}$	38.53 ± 0.38	40.16 ± 0.44	39.32 ± 0.64	−1.63 ± 0.58	0.84 ± 0.77	−0.79 ± 0.74
	54.05 ± 0.26	54.53 ± 0.28	52.84 ± 0.49	−0.48 ± 0.39	1.69 ± 0.57	1.21 ± 0.56
$13F_1H\ _{38}$	38.81 ± 0.31	39.05 ± 0.40	40.05 ± 0.35	−0.24 ± 0.51	−1.00 ± 0.53	−1.24 ± 0.47
	55.19 ± 0.15	55.43 ± 0.15	53.81 ± 0.24	−0.24 ± 0.21	1.62 ± 0.28	1.38 ± 0.28
$13(11)F_1H_{38}$	39.10 ± 0.31	36.10 ± 0.43	33.10 ± 0.22	3.00 ± 0.53	3.00 ± 0.49	6.00 ± 0.38
	57.10 ± 0.22	54.10 ± 0.29	52.60 ± 0.29	3.00 ± 0.37	1.50 ± 0.41	4.50 ± 0.37
$13F_1H_{137}$	38.83 ± 0.11	36.67 ± 0.44	31.67 ± 0.26	1.66 ± 0.45	5.00 ± 0.51	6.66 ± 0.28
	56.22 ± 0.09	55.00 ± 0.18	53.44 ± 0.28	1.22 ± 0.21	1.56 ± 0.34	2.78 ± 0.30
13 22/07	28.48 ± 0.09		24.19 ± 0.22			4.29 ± 0.24
	45.60 ± 0.07		42.25 ± 0.15			3.35 ± 0.17

With respect to the influence of removal of flowers on corolla size, the measurements show a striking correspondence between the size of corolla and the presence of developing seed capsules on the plant. By removing all flowers from the plant as fast as they go by it is possible to keep up the flower size to nearly that of the first flowers produced, and in some cases to double the life of the plant. A case in point is that of two plants of the *N. Tabacum* var. *macrophylla* series. On Plant 14 only the flowers measured were removed, while on Plant 15 all flowers too old to be measured and all developing seed capsules were removed twice a week. In a month the spread of corolla for Plant 14 decreased 5.78 mm. and for Plant 15 only 0.75 mm. At the end of two months the decrease amounted to 5.40 mm. for Plant 14 and 2.29 mm. for Plant 15. The length behaved similarly in this case, in Plant 14 there was a decrease of 4.55 mm. and in Plant 15 of 0.25 mm.; and at the end of two months, a decrease of 4.00 mm. in Plant 14 and 0.79 mm. in Plant 15.

During the period which elapses from the time a flower is fully opened to the time at which pollen is shed, there is a considerable increase in corolla spread and associated with it little or no increase in corolla length. On ten plants of F_1H_{38}, the increase in spread amounted on an average to 4.53 ± 0.23 mm. and for length there was an increase of 1.62 ± 0.22 mm. Similarly the spread of corolla for F_1H_{137} averaged 3.18 ± 0.17 mm. greater for flowers after shedding pollen, but in this case the length was 0.47 ± 0.11 mm. smaller, a slight discrepancy undoubtedly due to the indirect method of comparison made, which still further emphasizes the increase in spread of corolla during this period. This increase in spread has, also, been repeatedly confirmed on individual marked flowers.

That there is a differential distribution of flowers on tobacco plants according to size at any given time is shown by the comparative measurements of flowers borne among developing seed capsules on the terminal inflorescence of a plant and those borne on laterals of the same plants. In the case of plants of *N. Tabacum* var. *macrophylla* the flowers borne on laterals were found to have an average spread greater by 2.56 ± 0.16 mm. than those of the terminal inflorescence, and in the case of length, 1.24 ± 0.11 mm. greater. In the hybrids studied the difference is rather more marked, so much so that there is a distinctly noticeable difference in size between these two classes of flowers.

Similarly we have found that numerous other factors have a like influence on flower size, some relatively great and others less marked. For instance cuttings of F_1H_{44} growing in the greenhouse produced flowers 3.95 mm. smaller in spread and 1.42 mm. greater in length than those on the field plants from which the cuttings were taken. Pure line populations of *N. sylvestris* grown in a shaded situation on rich moist garden soil produced distinctly larger flowers than plants growing on higher unfertilized soil and not shaded, and pot experiments likewise showed that flower size could be distinctly influenced by applications of sodium nitrate, and in a direction parallel to that of the influence on vegetative characters.

The conclusion seems irresistible that flower-size in *Nicotiana* is not so constant as it has been assumed to be, but that it is affected by a number of conditions, and that at least some of these may not affect length and spread in the same manner. Attention has been called to these facts because they have not been given adequate consideration in genetic research on the behavior of flower size in *Nicotiana* and other genera.

The experiments reported upon above have, in part, been made possible by an allotment from that portion of the Adams Fund of the United States Department of Agriculture granted to the Agricultural Experiment Station of the University of California.

[1] Setchell, Studies in *Nicotiana*, I, Univ. of Cal. Pub. Bot., **5**, 8, 1912.

[2] Ibid., p. 29.

RETENTION IN THE CIRCULATION OF DEXTROSE IN NORMAL AND DEPANCREATIZED ANIMALS, AND THE EFFECT OF AN INTRAVENOUS INJECTION OF AN EMULSION OF PANCREAS UPON THIS RETENTION

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The content of dextrose in the circulating blood of normal animals is almost constant, it amounts to about 0.1 percent. The carbohydrates of the foodstuffs form the main source of dextrose in the body. On their way from the digestive tract the carbohydrates are transformed into various forms of saccharides; but all are finally converted largely into glycogen, which is stored up mostly in the liver. The blood obtains its supply of dextrose from the glycogen of the liver, and distributes it among the tissues of the body according to their demand for it. In the normal animal none of the dextrose escapes through the kidneys. Accordingly the constancy of the amount of dextrose in the blood is regulated by a mechanism which controls either of the two factors; the *supply* of or the *demand* for it.

In diabetes the dextrose content of the blood is higher than normal, is variable in amount, and, when it is sufficiently high, dextrose escapes through the kidneys. The cause of the increase of the blood dextrose, or hyperglycaemia, may be found either in a decrease in the demand of the tissues for dextrose, that is, the tissues burn dextrose less readily than in normal conditions; or in an increase in the supply, that is, the liver supplies the blood with more dextrose than in normal conditions.

Both possibilities have groups of followers who have brought forward numerous experiments in support of either of the respective theories.

It was experimentally established years ago that the complete removal of the pancreas brings on a severe hyperglycaemia, leading to considerable glycosuria and to the death of the animal. Here we are confronted again with the question, whether the presence of the pancreas in the normal animal favors the burning of the dextrose by the tissues, or whether it restricts the supply of dextrose to the blood by the liver. But there also arose another question, namely, whether the effect of the removal of that gland was due to a removal of a controlling substance, or, whether it was due, in consequence of the operation, to an interference with the nervous mechanism which controls the supply of the dextrose. In case of the first alternative, namely, that the resulting hyperglycaemia is due to a removal of some substance, normally secreted by the pancreas, we have to assume that that substance belongs to that group of substances which are provided by glandular and other tissues for the proper maintenance of functions of the animal body, i.e., *internal secretions*, since a complete suppression of the external secretions of the pancreas never leads to hyperglycaemia or diabetes.

We wish to report briefly results of several series of experiments which are likely to shed some light upon these questions. Most of our experimental series have one feature in common, namely, that the blood was provided with dextrose directly and not by way of the liver. In one series of experiments large quantities of dextrose (20 cc. of a 20% solution of dextrose per kilo body weight) were injected intravenously into normal dogs. It was found by us, as it was found by some investigators before us, that the injected large surplus of dextrose disappeared quite rapidly from the circulation. *Ninety minutes after the infusion, the dextrose content of the blood reached nearly the same level which it had before the injection.* About one-half of the injected quantity of dextrose left the circulation through the kidneys; the other half went into the tissues. In nephrectomized animals also the dextrose returned to the previous level in the same length of time. *Here we have a striking example of the ability and readiness of the circulation of a normal animal to rid itself of a large surplus of dextrose.*

In another series of experiments similar injections of dextrose were made into animals, the pancreas of which was removed by an operation a day or two before the dextrose infusion. The sample of blood taken from the animal before the intravenous injection of dextrose was strongly hyperglycaemic. It was then found that in these experiments *the dextrose content of the sample of blood, which was taken ninety minutes after* *the end of the dextrose injection, was at least twice as high as that of the sample taken before the injection.* Apparently the removal of the pancreas interfered with the power of the circulation to rid itself readily of a surplus of dextrose. The possible ability of the liver to provide the circulation with a greater supply of dextrose in the absence of the pancreas can certainly be no important factor in these results. Our experiments show, then, *that in the presence of the pancreas the circulation rids itself easily of intravenously introduced dextrose; but that it is unable to do it satisfactorily in the absence of the pancreas.* On the basis of these facts is it not plausible to assume further that the pancreas exerts exactly the same influence, when the dextrose in the circulation is being supplied in some other manner than by a burette into a vein, for instance, by the liver through its connections with the circulation? Or, in other words, do not our experiments make the assumption plausible that pancreatic hyperglycaemia is due to the fact that *in the absence of the pancreas the circulation is unable to dispose properly of the dextrose which it receives in normal amounts from the liver?*

In a third series, the experiments were again made by injecting dextrose intravenously in depancreatized dogs. But in these cases a strained pancreatic emulsion was added to the dextrose solution, and in some instances the infusion of the pancreatic emulsion was continued for some time after the infusion of the dextrose was finished. We present here merely a preliminary communication; we have not yet mastered all the details of this part of the investigation. But there seems to be no doubt about the nature and validity of the main result, which is this: In such experiments, *ninety minutes after the end of the dextrose infusion, the dextrose content of the blood is again at about the same level as it was before the infusion of the dextrose.* Apparently the presence of a pancreatic emulsion within the blood helps the circulation to get rid of the surplus of dextrose injected intravenously. In these experiments it is evident that the *effect of the pancreas can be only of a chemical and not of a nervous nature*, or, in other words, it is the internal secretion of the pancreas which helps the circulation to get rid of the surplus of dextrose.

To the above results we wish now to add the following preliminary statement derived from three experiments. *In depancreatized animals the blood of which has shown a marked hyperglycaemia, an intravenous infusion of a pancreas emulsion brought the dextrose content of the blood to 0.09% and even 0.08%.* The hyperglycaemia returned next day and could be reduced again to the normal level by an infusion of pancreas emulsion.

Conclusions.—In normal animals the circulation possesses the ability to get rid readily of a surplus of dextrose injected intravenously. In the absence of the pancreas this ability of the circulation is impaired. This ability can be temporarily restored by an intravenous injection of a pancreas emulsion.

Furthermore, an intravenous injection of a pancreas emulsion is capable of reducing the hyperglycaemia due only to depancreatization to a normal level of the dextrose content of the blood.

As to the nature of the factors which may constitute the ability or inability of the circulation to get rid of a certain degree of surplus of the dextrose content of the blood, we are not willing to discuss it at this stage of our investigation. We are rather bent upon seeing how many more facts we shall be enabled to bring to light on the basis of the hypothesis which set us to work on these problems.

PARTHENOCARPY AND PARTHENOGENESIS IN NICOTIANA

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The occurrence of parthenogenesis in the genus *Nicotiana* has, in general, been considered to be a negligible factor so far as the interpretation of breeding experiments with tobacco is concerned. The only outstanding instance in which castrated or mutilated tobacco flowers have yielded viable seed is to be found in a report describing the experiments of Mrs. R. H. Thomas.[1] Her experiments, apparently conducted with due regard to the various sources of error, indicate that for her cultures parthenogenesis in various species and hybrids of *Nicotiana* is of frequent occurrence. Conflicting evidence is furnished by the experiments of others. Thus, Howard,[2] following experiments which involved the emasculation of over 5000 flowers on many strains of Indian Tobacco, found but five capsules containing seed and in only two cases was it shown that this seed was viable. Hartley[3] obtained two capsules of seed 'by treating fully receptive stigmas with magnesium sulphate,' employing in his experiments flowers of 'Cuban Tobacco (*Nicotiana Tabacum*).' The seeds thus produced proved, however, to be nothing more than empty shells. Further, East[4] and Wellington[5] claim that hybrid seed, produced by crosses between certain species of *Nicotiana*, has given plants 'like the mother species and also true hybrids,' plants 'like the mother species and no true hybrids,' and that this seed gave 'no true hybrids on one occasion but did produce true hybrids on other occasions.' Certain species-crosses in *Nicotiana* made by Gärtner, also, gave seed that produced the mother species as well as true hybrids. These latter observations have been explained by the assumption that cross pollinations, 'by the extraordinary irritation of foreign pollen' (East, l.c.), may induce an apogamic or parthenogenetic development of a portion of or all of the ovules affected. Wellington, by a variety of stimulations and irritations of immature tobacco flowers was able to produce 'some abortive seed without embryos' while from a considerable number of simple castrations of buds, viable seed was produced in only one doubtful case. Our own results, based upon over 1500 castrations of tobacco flowers, borne on many species and varieties of *Nicotiana* in the University of California Botanical Garden, are entirely negative. No viable seed without pollination has been produced and in the appearance of numerous hybrid progenies we have, in no case, had reason to assume the production of apogamic or parthenogenetic seed as a result of cross pollination.

On the assumption, then, that errors of technique will not entirely account for the results which Mrs. Thomas obtained, it appeared possible that the production of viable seed without pollination was actually peculiar to certain of the particular strains of tobacco which she possessed. Faulty technique has been the only explanation offered to account for the unusual results which Mrs. Thomas reported while, so far as I know, no one has attempted to repeat her experiments with the strains of tobacco she used. Seed of the *Nicotiana Tabacum* variety, described in her paper as a frequent producer of seed without pollination, was kindly sent me by Mrs. Thomas. On plants from this same seed Bateson[6] confirmed Mrs. Thomas' results. The seed was received under the name '*Nic. tabaccum* Cuba' and represented a portion of the original seed 'gathered in the Garden of Casa Loring at Malaga in 1908.' Ninety-five plants were grown from this seed during 1914. The stand was entirely uniform and the plants were taller than and as vigorous as any of the *N. Tabacum* varieties in our cultures (cf. Setchell[7]). As described by Mrs. Thomas (loc. cit.) they are white-flowered plants, considerable numbers of four-parted flowers being produced especially at the opening of the flowering season. In vegetative characters they resemble the commercial types collectively known as 'Cuban Tobacco.'

Eight hundred buds on these ninety-five plants of '*Nic. tabaccum* Cuba' were treated according to one of the following methods—(1) simple emasculation of the flower by picking off the anthers near the tops of the filaments, (2) castration of the bud plus the pinching off, with the forceps, of the stigma at the very top of the style and (3) pinching off the stigma in the bud without the removal of the anthers. The crushed tip of the style in such cases dries down considerably before the flower matures or pollen is shed. The last two types of treatment, (2) and (3), will be referred to as 'mutilations.' In addition a few decapitation experiments were attempted (cf. Thomas, l.c.). The number of simple emasculations far exceeded the other two types of treatment.

The length of every bud treated was noted by a measurement from the point of union of calyx and pedicel to the tips of the folded corolla lobes. The maximum length of unopened buds was 49 mm. and, with the exception of a few late flowers, the anthers do not open until the corolla lobes begin to open back. The number of the flower parts was, also, determined for each bud on the supposition that four-parted and five-parted flowers might give different results following emasculation or mutilation. Finally, the records were so kept that the position on the plant of a given treated bud can be determined from the field notes—i.e., whether it occurred on the terminal inflorescence, on one of the upper 'bald suckers', or on one of the lower leafy laterals. More than one bud was often treated and bagged on a single inflorescence but in most cases such a group of buds were all of approximately the same length.

To obviate the possibility that grosser errors of technique could affect the results of the various treatments, the corollas of treated flowers were carried along with the seed bags and are, in most cases, still available for examination. If, in castration, an anther is overlooked the fact is at once noted when the withered corollas are examined both at the time of cleaning and at the time of sowing the seed produced. In only one case among the 800 treatments were anthers overlooked and a considerable amount of viable seed resulted. This check upon the results of castration and mutilation experiments is, to my mind, imperative. Willingness to discard all treatments concerning which there is the slightest doubt as to purity, is the only security obtainable in the finer details of performing the emasculations or mutilations. Despite the sterilization of instruments a bud was discarded if the forceps touched the stigma during any operation. Similarly, though only the middle one-third of the corolla tube was opened to admit the forceps, the too-close proximity of open flowers on neighboring inflorescences at the moment of treatment caused the discarding of the bud being operated upon. In addition to the 800 castrations and mutilations considered unimpeachable, 150 doubtful treatments were thrown away. The results, in the case of a few doubtful treatments which were saved, showed the necessity of such rigid discrimination.

Following the castration or mutilation of these 800 buds, there were 112 bags, involving nearly 200 flowers, in which one or more fruits developed to normal size. These fruits contained matured seed all of which was normal in appearance, though the majority was small in size, and a small proportion of which was normal in every way including the presence of endosperm and embryos. That there is a ready and frequent parthenocarpy, taking this term simply to indicate the production of normal fruits without pollination, in '*Nic. tabaccum* Cuba' there can be no doubt. This is the more remarkable since other Nicotiana species and varieties uniformly exhibit an early dropping of those flowers in which emasculation is not closely followed by pollination. It may be noted that, of the castrations and mutilations which were followed by the production of normal fruits, 75% were on the terminal inflorescence and the four or five 'bald suckers,' normal to this variety of *N. Tabacum*, and 25% only were on the lower leafy laterals. Similarly, buds treated within two weeks after the opening of the first flower on the plant matured fruits in 65 instances as compared with 57 instances in the case of treatments performed after that period. Certain plants were not treated until they had passed their first and heaviest flowering period. Such plants gave no significant increase in the proportion of fruits matured after treatment. The number of fruits matured from treated buds under 35 mm. in length was almost identical with the number produced from treated buds more than 35 mm. long. Almost equal numbers of four-parted and five-parted flowers produced fruits following treatment. However, seed containing endosperm and embryos or endosperm only seems to have resulted, in nearly all cases, from the treatment of the more normally formed, five-parted flowers. Similarly, the use of a single bud on an inflorescence for treatment seems, as was perhaps to be expected, to have been more efficient for the production of normal seed than the castration of more than one. Of the total 800 treatments less than one-third involved the pinching off of the stigma in addition to emasculation. Approximately one-tenth of the total number involved the pinching off of the stigma only. Nine treated flowers produced a little viable seed and, of these nine flowers, three represent treatments in which the stigma was removed.

The great majority of the seed produced in the parthenocarpic fruits was normal in appearance and well filled out but these seeds were smaller than the self-pollinated seeds and consisted of empty seed coats only. Samples of seed from each seed packet were bleached in strong Eau de Javelle and examined under magnification. Controls were employed in which self-fertilized seed of '*Nic. tabaccum* Cuba' was similarly treated. Three types of seed were isolated, according to this method, from the parthenocarpic fruits. First, seed which consisted of nothing but empty seed-coats normal in structure and completely filled out. Second, seed of the first type which contained traces of endosperm but no embryos.[9] Such endosperm tissue occurred as a sheath of cells lining the seed-coat or as a mass of cells at one end of the seed, the contents of which was rich in starch. These first two types were small seeds. The third type of seed isolated was identical with the self-pollinated seed both in size and in the possession of normally developed endosperm and full sized embryos. Seeds of type one were overwhelmingly in the majority in every case and the total number of seeds shown to be in every way normally matured, either by examination under the microscope or by the fact that they produced normal seedlings, was approximately thirty-five. They were produced, along with empty seeds and seed containing endosperm only, in approximately equal numbers in the nine parthenocarpic fruits. Four seedlings are maturing normally and are of fair size from nine seeds which were germinated out of a total of eighteen seeds available for germination. The eighteen seeds represent the proportion of the fifty undoubtedly viable seeds not used in the bleaching test.

We may conclude, then, that parthenocarpy is of frequent occurrence in '*Nic. tabaccum* Cuba' and that parthenogenesis, employing the term to mean the production of viable seed without pollination (cf. Winkler[8]), is also peculiar to this variety of *N. Tabacum.* With these unusual phenomena manifesting themselves during this first year of cultivation in our cultures I feel that there is a possibility, at least, that after further cultivation here parthenogenesis and parthenocarpy may become more nearly equal in the frequency of their occurrence. The experimental results above described should furnish a partial confirmation of those reported by Mrs. Thomas for her experiments on '*Nic. tabaccum* Cuba.' Nothing that has been said, however, must suggest that I desire to confirm her general results on the basis of which she concludes that parthenogenesis is peculiar to *Nicotiana* species in general. It must, on the contrary, be emphasized that our experiments, with all other species and varieties of tobacco, and those of a number of other investigators, point to exactly the reverse conditions and that we have no reason to suppose that parthenogenesis has ever before occurred in our cultures. I have no suggestion to offer, at the present time, as to the possible origin of this *Nicotiana tabacum* variety which exhibits such marked divergence from the restricted method of fruit and seed production peculiar to other varieties of this species and to all other species of tobacco as far as known.

The experiments reported upon above have, in part, been made possible by an allotment from that portion of the Adams Fund of the United States Department of Agriculture granted to the Agricultural Experiment Station of the University of California. A more detailed report will appear in the University of California Publications in Botany, Volume 5.

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[6] Bateson, *4th Conf. Inter. Gen.*

[7] Setchell, *Univ. Cal. Pub. Bot.*, vol. 5, No. 1.

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EXOGAMY AND THE CLASSIFICATORY SYSTEM OF RELATIONSHIP

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Lewis H. Morgan, in his *Systems of Consanguinity and Affinity* (Washington, 1871), established the fact that in a large part of North America, in India, in Africa, and in Oceania the natives use terms of relationship that designate not individuals but groups of individuals, and accordingly he labeled these systems as 'classificatory.' Later E. B. Tylor and others advanced the view that the classificatory system and exogamy—the rule that a person must marry outside of his own social group (clan or gens)—were merely two aspects of a single institution: that, in other words, primitive man classed together individuals belonging to the same exogamous division and separated individuals of different divisions. Quite recently this view has been advocated by W. H. R. Rivers. In his *Kinship and Social Organisation* (London, 1914) he correlates the classificatory system with exogamy, our own system with the family in the narrower sense of the term, and the descriptive system of, say, the Nilotic Negroes (in which a few primary terms designate the basic relationships and serve by their combination to describe all other relatives) with the patriarchal or extended family. The correlation of the classificatory nomenclature with exogamy now requires empirical verification in the several areas of the globe, and the following is an attempt to make this test for North America.

Before undertaking this inquiry, however, the concept 'classificatory' must be supplanted by one that more adequately represents the phenomenon under discussion. For any particular 'classificatory' system is not moulded by a single factor but by a series of factors, and these are developed in varying degree in different systems. Hence the test must be applied to that common element which, consciously or unconsciously, differentiated the primitive terminologies in question from those current among ourselves in the minds of investigators. An examination of Morgan's earliest expressions on the subject indicates that it was the merging of lineal and collateral relatives—the use of a single term, e.g., for mother and mother's sister, for father and father's brother—that impressed this pioneer investigator, and this is the feature that actually characterizes the classificatory systems of all the regions of the globe. Our query is thus reduced to this form: Is the confusion of collateral with lineal relatives a function of exogamy?

The first question is, of course, how the exogamous tribes compare with those 'loosely organized,' i.e., those lacking exogamous divisions. One of the principal exogamous areas of North America is found in the United States east of the Mississippi. Practically throughout this immense territory the custom of exogamy is associated with a terminology that fails to distinguish collateral and lineal relatives. Among the Northwest Coast tribes of Canada the same association holds, and this applies likewise to those of the Plains tribes that possess a clan or gentile organization. The one doubtful exogamous region is the Southwest, for which we have practically no data except from the Tewa, where the correlation does *not* hold. An inquiry into the as yet unknown systems of the Keresan, Hopi, and Zũni pueblos is of the highest theoretical importance. When we turn to the loosely organized tribes we meet again with one exceptional region, that of the Mackenzie River, and several sporadic cases outside, where non-exogamous tribes are reported to possess a kinship terminology that is *ex hypothesi* to be expected together with exogamy. On the whole, however, the agreement with the Tylor-Rivers theory is highly satisfactory. The Eskimo, the Plateau Indians, the Californian tribes are loosely organized; and all of them tend to emphasize the distinction of such relatives as father and paternal uncle, mother and maternal aunt. It is important to note that these terminologies by no means resemble those of European languages. Among the Shoshonean tribes and the Kootenai, for example, relatives distinguished in English are classed together through the extensive use of reciprocal terms, members of a related pair addressing each other by a common term. But these systems are 'classificatory' only in an etymological sense of the word, the basis of the classification being wholly distinct from that which moulds the collateral-lineal terminology of the systems customarily designated as 'classificatory.'

Summing up, we may say that there is a very high degree of correlation between the practice of exogamy and the ranging in a single category of collateral and lineal kin. The aberrant cases are relatively few and some of them are readily intelligible as the result of cultural influences from neighboring tribes. It is, of course, possible that the correlation may ultimately turn out to have an unexpected meaning, for example, that the emphasis on exogamy is misplaced and should be on definite organizations of any kind, whether exogamous or not. Such an interpretation might perhaps eliminate some of the at present anomalous instances of non-exogamous tribes sharing the nomenclature of exogamous peoples. At all events, the North American data furnish strong evidence in support of the Tylor-Rivers theory.

That the classification of kin by certain tribes is a function of the exogamous grouping, may be demonstrated in more rigorous fashion. Within the Siouan family there are tribes with kinship systems that not only fail to distinguish between collateral and lineal relatives but also class together members of distinct generations, which is contrary to the usual form of 'classificatory' nomenclature. Rivers has suggested exceptional forms of marriage to account for the exceptional mode of classification; as a matter of fact it may be shown that the apparent exceptions are merely the result of an unusually consistent application of the exogamous principle of grouping.

The following are the facts. The Crow and Hidatsa, Siouan tribes tracing descent through the mother, class the father's sister's son with the father; the father's sister's daughter, father's sister's daughter's daughter and all her female descendants through females with the father's sister. It is to be proved that these classifications are connected with the exogamous social grouping.

The facts in the case may be summed up by saying that a single term is applied to male members of the father's clan regardless of generation, and a single term to female members of the father's clan who belong to his own and all descending generations. If this objective statement also represents the psychological basis of the grouping, the terminology should be modified as soon as we pass outside the clan. We pass outside the clan when we take not the father's sister's daughter's daughter, but the father's sister's *son's* daughter, since with exogamy and maternal descent she cannot belong to the father's clan. And actually we find that among the Hidatsa and Crow this relative is no longer classed with the father's sister but with the sister, this latter relationship following from the fact that *her* father is classed with *my* father. But between the status of a father's sister's daughter's daughter and a father's sister's son's daughter there is no ascertainable difference except that of clan affiliation. Therefore the terminological classification is a function of the exogamous group, which was to be proved.

The same conclusion may be established by eliminating the hypothetical factor in another way. The Siouan family embraces a number of tribes with patrilineal descent, of which the Omaha are the best known. In such tribes the father's sister's descendants are no longer, as among the Crow or Hidatsa, members of the father's exogamous group; and we find, as a matter of fact, that her son and daughter are classed not with her but with the sister's son and daughter. With paternal descent my father's sister is my group sister, and while the Omaha have a distinct term for the father's sister it seems that in some ways she is still regarded as a sister—both as regards her children and as regards her husband, who is classed with the brother-in-law. On the other hand, the mother's brother's son, mother's brother's son's son, and so forth, are all members of the same exogamous group if there is paternal descent, and the Omaha actually designate them by a single term. And again, as soon as we pass out of the exogamous group, the terminology varies: my mother's brother's *daughter's* son is my brother, not my mother's brother, since he no longer belongs to my mother's gens but is related to me solely through his mother, who is my 'mother' because *she* does belong to my mother's gens.

In short, passing from tribes with matrilineal to tribes with patrilineal descent within the Siouan stock, we find precisely those differences that logically follow from the assumption that the exogamous group lies at the basis of kinship classification; and passing within a particular tribe from relatives within the same group to relatives of otherwise similar status outside the group we at once find a difference in nomenclature. Hence the exogamous factor must have been a real cause in moulding the kinship terminology of at least some so-called classificatory systems.

SOLUTION OF AN INFINITE SYSTEM OF DIFFERENTIAL EQUATIONS OF THE ANALYTIC TYPE

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Presented to the Academy, April 20, 1915

Consider the infinite system of differential equations*

$$\frac{dx_i}{dt} = f_i\,(t;\, x_1, x_2, \cdots) = a_i + f_i^{(1)} + f_i^{(2)} + \cdots \qquad (i = 1, 2, \cdots), \tag{1}$$

where a_i is a constant and $f_i^{(j)}$ is the totality of terms of f_i which are homogeneous in t, x_1, x_2, ... of degree j. That is, $f_i^{(1)}$ is a linear function of the infinitely many variables t, x_1, x_2, ...; $f_i^{(2)}$ is a quadratic function of the same variables; and so on. From the analogy with analytic functions of a finite number of variables, f_i will be said to be of the analytic type.

It is assumed that the following hypotheses are satisfied:

(H_1). $x_i = 0$ $(i = 1, 2, \ldots)$ *at* $t = 0$.

(H_2). *Finite real positive constants* $c_0, c_1, c_2, \ldots; r_0, r_1, r_2, \ldots, A$ *and* a *exist such that*

$$s = c_0 t + c_1 x_1 + c_2 x_2 + \ldots \tag{2}$$

converges if

$$|t| \leqq r_0, \qquad |x_i| \leqq r_i \qquad (i = 1, 2, \ldots) \tag{3}$$

and such that $A r_i s^j$ *dominates* $f_i^{(j)}$ *and* $|a_i| \leqq A r_i a$.

Since the series (2) converges if the relations (3) are satisfied, a finite constant M exists such that

$$S = c_0\frac{r_0}{M} + c_1\frac{r_1}{M} + c_2\frac{r_2}{M} + \cdots < 1. \tag{4}$$

Consequently if $M|t| \leqq r_0$ and $M|x_i| \leqq r_i$ are satisfied, then

$$|f_i| \leqq A\, r_i\{a + S + S^2 + \cdots\} = A\, r_i\left\{a + \frac{S}{1-S}\right\}. \tag{5}$$

---

* The problem of an infinite system of differential equations was first treated by E. H. Moore in his paper read at the Fourth International Congress of Mathematicians, held at Rome in 1908. His treatment was in the sense of General Analysis (cf. *Introduction to a Form of General Analysis*, New Haven Mathematical Colloquium, 1906), in which a general variable p is used in place of the index i of this paper having the range of positive integers, and his functions were not limited to those of the analytic type. The solution was made to depend upon the solution of an integral equation, in general non-linear, of the Volterra type. Simplifications and extensions of this theory were presented before the National Academy of Sciences, at Chicago, December 9, 1914.

Therefore under these limitations not only do the terms of each degree in the right member of equations (1) converge, but the whole right members converge.

The particular form of (H_2) was chosen so as to secure by one analysis as wide a range as possible of permissible values of $x_1, x_2, \dots$. For example, if infinitely many of the c_i are bounded from zero the x_i must tend to zero for $i = \infty$. On the other hand, if Σc_i converges all the conditions so far imposed can be satisfied by x_i which are bounded from zero. In the latter case Σf_i does not converge.

If an analytic solution of (1) satisfying the initial conditions (H_1) exists, it will have the form

$$x_i = A_i^{(1)}t + A_i^{(2)}t^2 + \dots \qquad (i = 1, 2, \dots). \tag{6}$$

On substituting these series in (1) and equating coefficients of corresponding powers of t, it is found that

$$\begin{aligned} a_i^{(1)} &= a_i \qquad\qquad (i = 1, 2, \cdots), \\ 2a_i^{(2)} &= \frac{\partial f_i^{(1)}}{\partial t} + \sum_{j=1}^{\infty} \frac{\partial f_i^{(1)}}{\partial x_j} a_j^{(1)}, \\ & \dots\dots\dots\dots\dots , \\ n a_i^{(n)} &= P_i^{(n)}(a_j^{(1)}, \cdots, a_j^{(n-1)}) \qquad (j = 1, 2, \cdots), \end{aligned} \tag{7}$$

where $P_i^{(n)}$ is a polynomial in $a_j^{(1)}, \dots, a_j^{(n-1)}$ whose coefficients are linear functions of the coefficients of f_i with positive numerical multipliers. Hence the formal analytic solution of (1) is unique.

In order to prove the convergence of the series (6) for values of t whose moduli are sufficiently small, consider the solution of

$$\frac{d\xi_i}{dt} = A\, r_i \left\{ a + \frac{\sigma}{1 - \sigma} \right\} \qquad (i = 1, 2, \cdots), \tag{8}$$

where

$$\sigma = c_0 t + c_1 \xi_1 + c_2 \xi_2 + \cdots . \tag{9}$$

The right members of (8) dominate the respective right members of (1).

The formal analytic solution of (8) is

$$\xi_i = \alpha_i^{(1)} t + \alpha_i^{(2)} t^2 + \cdots \qquad (i = 1, 2, \cdots). \tag{10}$$

The coefficients of these series can be obtained by equations analogous to (7). They are therefore real and positive, and it follows from the fact that the right members of (8) dominate the right members of (1) that

$$\alpha_i^{(j)} \geqq | a_i^{(j)} | \qquad (i, j = 1, 2, \cdots). \tag{11}$$

Therefore if (10) converge for $|t| \leqq \rho$, then (6) also converge for at least the same values of t.

It follows from (8) that

$$\frac{1}{r_1}\frac{d\xi_1}{dt} = \frac{1}{r_2}\frac{d\xi_2}{dt} = \cdot\ \cdot\ \cdot = \frac{d\xi}{dt}.$$

The initial values of ξ_1, ξ_2, ... are zero; hence on taking $\xi(0) = 0$, it follows that

$$\xi_i = r_i\,\xi \qquad (i = 1, 2, \cdot\ \cdot\ \cdot). \tag{12}$$

Therefore each of equations (8) reduces to

$$\frac{d\xi}{dt} = A\left\{a + \frac{c_0\,t + C\xi}{1 - c_0\,t - C\xi}\right\}, \tag{13}$$

where

$$C = c_1 r_1 + c_2 r_2 + \cdot\ \cdot\ \cdot\ , \tag{14}$$

which is a finite constant by (H_2).

It follows from the ordinary theory for a finite number of differential equations that (13) has an analytic solution which converges if $|t|$ is sufficiently small. Therefore equations (10) and (6) converge for at least the same values of t.

In general the limitations placed on t in order that the solution of (13) shall be known to converge are so restrictive that the corresponding x_i do not attain the boundary of the region for which the right members of (1) converge. The question arises whether the solution can be continued beyond its original domain.

Suppose equations (6) converge for $t = t_0$ and let the corresponding value of x_i be $x_i^{(0)}$. Suppose

$$c_0|t_0| + c_1|x_1^{(0)}| + c_2|x_2^{(0)}| + \ldots = S_0 < S_1 < 1.$$

Then let

$$x_i = x_i^{(0)} + y_i, \qquad t = t_o + \tau. \tag{15}$$

The differential equations (1) become in the new variables

$$\frac{dy_i}{d\tau} = b_i + g_i^{(1)} + g_i^{(2)} + \cdot\ \cdot\ \cdot \qquad (i = 1, 2, \cdot\ \cdot\ \cdot), \tag{16}$$

where $g_i^{(j)}$ is the totality of terms in the ith equation which are homogeneous in τ, y_1, y_2, .. of degree j.

It follows from (1) that the explicit expressions for b_i, $g_i^{(1)}$, $g_i^{(2)}$, . . . are

$$
\begin{aligned}
b_i &= f_i\,(t_0;\ x_1^{(0)},\ x_2^{(0)},\ \cdot\ \cdot\ \cdot\ ),\\
g_i^{(1)} &= \frac{\partial f_i}{\partial t}\,\tau + \frac{\partial f_i}{\partial x_1}\,y_1 + \frac{\partial f_i}{\partial x_2}\,y_2 + \ \cdot\ \cdot\ \cdot\ ,\\
&\cdots\cdots\cdots\cdots\cdots\cdots\\
g_i^{(n)} &= \frac{1}{n!}\left[\frac{\partial f_i}{\partial t}\,\tau + \frac{\partial f_i}{\partial x_1}\,y_1 + \frac{\partial f_i}{\partial x_2}\,y_2 + \ \cdot\ \cdot\ \cdot\ \right]^n,\\
&\cdots\cdots\cdots\cdots\cdots\cdots ,
\end{aligned}
\tag{17}
$$

where the power indicated in the last equation is symbolic such that

$$
\left(\frac{\partial f_i}{\partial t}\,\tau\right)^{n_0}\left(\frac{\partial f_i}{\partial x_1}\,y_1\right)^{n_1}\left(\frac{\partial f_i}{\partial x_2}\,y_2\right)^{n_2}\cdots\cdot = \frac{\partial^n f_i}{\partial t^{n_0}\partial x_1^{n_1}\partial x_2^{n_2}\cdots}\,\tau^{n_0}y_1^{n_1}y_2^{n_2}\cdots
$$

$$
(n_0 + n_1 + \cdot\ \cdot\ \cdot\ \cdot = n). \tag{18}
$$

After the partial derivatives have been formed, t, x_1, x_2, $\cdot\ \cdot\ \cdot$ are replaced by t_0, $x_1^{(0)}$, $x_2^{(0)}$, $\cdot\ \cdot\ \cdot$ respectively.

Since the transformation (15) is linear, $g_i^{(n)}$ depends only on $f_i^{(n)}, f_i^{(n+1)}, f_i^{(n+2)}, \cdot\ \cdot\ \cdot\ \cdot$. It follows from the fact that $f_i^{(n)}, f_i^{(n+1)}$, $\cdot\ \cdot\ \cdot$ are dominated by Ar_iS^n, Ar_iS^{n+1}, $\cdot\ \cdot\ \cdot$ respectively that

$$
\left|\frac{\partial^n f_i}{\partial t^{n_0}\,\partial x_1^{n_1}\,\partial x_2^{n_2}}\right|_{\substack{t=|t_0|\\ x_j=|x_j^{(0)}|}} \leqq A r_i \left[\frac{\partial^n\left(\dfrac{S^n}{1-S}\right)}{\partial t^{n_0}\,\partial x_1^{n_1}\,\partial x_2^{n_2}\cdots}\right]_{\substack{t=|t_0|\\ x_j=|x_j^{(0)}|}} .
$$

It is easily found that

$$
\left[\frac{\partial^n\left(\dfrac{S^n}{1-S}\right)}{\partial t^{n_0}\,\partial x_1^{n_1}\,\partial x_2^{n_2}\cdots}\right]_{\substack{t=|t_0|\\ x_j=|x_j^{(0)}|}} = \frac{n!\ c_0^{n_0}\ c_1^{n_1}\ c_2^{n_2}\cdots}{(1-S_0)^{n+1}} . \tag{19}
$$

Now let

$$
c_0^{(1)} = \frac{c_0}{1-S_0}, \qquad c_1^{(1)} = \frac{c_1}{1-S_0}, \qquad c_2^{(1)} = \frac{c_2}{1-S_0} \tag{20}
$$

The series

$$
T = c_0^{(1)}\,\tau + c_1^{(1)}\,y_1 + c_2^{(1)}\,y_2 + \ \cdot\ \cdot\ \cdot
$$

converges if

$$
|\tau| \leqq r_0\,(1-S_0), \qquad |y_1| \leqq r_1\,(1-S_0), \qquad |y_2| \leqq r_2\,(1-S_0),\ \cdot\ \cdot\ \cdot\ ,
$$

and $|T| < 1$ if

$$
|\tau| \leqq \frac{r_0\,(1-S_0)}{M}, \qquad |y_i| \leqq \frac{r_i\,(1-S_0)}{M}.
$$

Moreover $g_i^{(n)}$ is dominated by $\frac{Ar_i}{1-S_0}T^n$. Therefore equations (16) have the essential properties assumed to hold for (1), and their solution converges for $|\tau|$ sufficiently small.

It is easy to imagine a physical problem which satisfies the conditions of this theory. For example, suppose the number of mutually gravitating bodies in the universe whose masses are bounded from zero is infinite. If beyond a finite number of them (which may be arbitrarily great) their initial distances from one another increase, as the number of bodies increases, with sufficient rapidity, it is easy to show that all the hypotheses are satisfied. In this case there is a rigorous, though limited, solution of the problem of infinitely many bodies moving subject to their mutual attractions.

SEX RATIO IN PIGEONS, TOGETHER WITH OBSERVATIONS ON THE LAYING, INCUBATION AND HATCHING OF THE EGGS

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Presented to the Academy, April 30, 1915

The conclusions[1] here presented are the results of a study of the records which have accumulated from 1907 to 1914 in connection with investigations of inheritance in pigeons begun at the Rhode Island Agricultural Experiment Station and later continued at the Experiment Station at Madison, Wisconsin. It is impossible in a brief space to present the data upon which the conclusions are based; for these the reader is referred to the complete report. Furthermore, although the conclusions are here presented somewhat dogmatically and as if of general application, and while we believe that they will probably be found in the main to apply generally to domestic pigeons, they are nevertheless based almost entirely on the data of the experiments mentioned and there is, therefore, no positive assurance that the results would be the same with other stock or under different conditions. The number of data obtained were, however, very considerable for pigeons, and it is felt they accordingly furnish a good foundation for the conclusions drawn.

It is commonly believed by pigeon breeders and others that from the two eggs of a clutch a pair of offspring, that is a male and a female, are produced either invariably, or at least in a great majority of instances. Furthermore, it is maintained that of this pair the male hatches from the egg which is laid first, while the egg laid later produces the female. The breeder has also found it difficult to explain the apparent excess of males which he commonly finds in the birds saved for breeding stock. The results here presented throw conclusive light on these and on other similar questions.

Sex ratio.—The normal ratio of the sexes of pigeons hatched is 105 males to 100 females.

Death rate.—The death rate of squabs is especially high for the first two or three days after hatching, and at about 10 to 15 days of age.

Differential mortality.—(1) When the two squabs from the same clutch of eggs are of distinctly different size before the banding age (10 to 15 days) the larger squab is more often a male than a female. (2) The death rate for the two sexes in bisexual broods is essentially equal. (3) There is no marked tendency for one sex to be weaker than the other in bisexual broods, and there is only a slight indication that more males than females from such broods survive to adult life—placed at 6 months. (4) A consideration of the ratio of males to females in each of the age groups defined[2] does not indicate a high relative mortality of females in the ages preceding the adult stage. (5) There is a high mortality of both sexes during the first two or three years of their adult life, and this is especially high in the females between the ages of one and two years. (6) The higher mortality of females at early adult ages, together with the higher proportion of males hatched (105: 100), may be in large part responsible for the prevailing notion of a considerable excess of male pigeons in adult populations and seems to furnish real substantiation for this notion. The fact that males are generally more easily recognized than females probably adds to this impression.

Ratio of bisexual to unisexual broods.—(1) The number of unisexual broods, in which the squabs are either both male or both female, somewhat exceeds in our records the bisexual broods (one squab of each sex), but the odds against the numbers obtained representing a potential equality are very slight. These facts are directly contradictory to the commonly accepted statement that the two eggs usually produce a male and a female squab. (2) Considering only the unisexual broods, the number of 'both males' to 'both females' is practically equal.

Sex with respect to order of laying.—A comparison of the numbers of each sex hatched from first eggs and from second eggs respectively, shows no tendency for the former to produce exclusively males and the latter females but, as a matter of fact, more males than females are hatched from both.

Time of laying.—(1) The mean time of laying of the first egg is about 5 p.m., and of the second egg about one o'clock of the afternoon of the second day following. (2) The mean interval between the laying of the two eggs is practically 44 hours. (3) The mean time between the laying of the first and second eggs decreases progressively in the months from February to July inclusive. (4) There is a very sensible positive correlation (0.38 ± 0.057) between the time of laying of the first and of the second egg. The equation of the regression straight line is $y = -0.48 + 0.378x$, where x is the time p.m. of laying of the first egg, and y that of the second. This enables one to calculate the most probable time of laying of the second egg when the time the first was laid is known.

Time of hatching.—(1) The mean time of hatching of the first egg is 16.5 days after the laying of the second. (2) The mean time of hatching of the second egg is 17 days after it is laid. (3) On the average, therefore, the time from laying to hatching of the first egg is nearly a day and a half longer than it is for the second egg. This is probably to be accounted for by the fact that the first egg receives very little incubation until the second is laid. (4) There is a high correlation between the times of hatching of the two eggs of a clutch.

Control of sex.—So far as the data obtained go, they appear to indicate that sex in pigeons is determined according to the laws of chance.

Time of sitting.—(1) In case the eggs do not hatch they are seldom abandoned at the end of the normal period of incubation, but the birds continue to sit on them for a time longer. (2) The length of time they will continue to incubate the eggs varies, but averages practically six days beyond the normal period, making the mean total time of incubation, when the eggs do not hatch, 23 days after the laying of the second egg. (3) This continuance of incubation beyond the normal time under such circumstances constitutes a 'factor of safety' in the incubating instinct. (4) A pair of Mourning Doves continued to sit on substituted eggs for four days after their own had hatched in an incubator, thus disproving Raspial's assertion that wild birds have an 'exact notion of the time required for the eggs to hatch.'

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[1] Summary of the results of a report bearing the same title and published as *Bulletin No. 162 of the Rhode Island Agricultural Experiment Station.*

[2] GROUP A. *Hatching period.* Embryos which were fully developed but which failed to hatch ('dead in shell') and birds which died within five days after hatching.

GROUP B. *Nestling period.* Squabs dying at from 6 to 28 days, inclusive.

GROUP C. *Immature or juvenile period.* Squabs living over 28 days but dying before six months of age.

GROUP D. *Adult period.* All birds living to the age of six months or more.

VIVIDIFFUSION EXPERIMENTS ON THE AMMONIA OF THE CIRCULATING BLOOD

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Presented to the Academy, May 5, 1915

The fact that the ammonia content of shed blood under aseptic conditions increases is well known and is taken into account as much as possible by a rapid procedure in methods for the determination of preformed ammonia. Suggestions as to the source of this slowly liberated ammonia have as yet led to no positive findings. My experiments were undertaken to determine whether with aseptic measures the formation of ammonia occurs in diffusible constituents of the blood after their separation from the non-diffusible constituents according to the vividiffusion method of John J. Abel[1] and his collaborators. The dialysate, obtained when the vividiffusion apparatus was attached to the femoral artery and femoral vein of a dog for periods varying from three to seven hours, was studied for the production of ammonia in excess of that present at the time of dialysis. The results were compared with those from shed blood under similar conditions. It was found that in a dialysate obtained from circulating blood there is no liberation of ammonia comparable to that which takes place under aseptic conditions in shed blood.

Blood	*Milligrams of* NH_3-N *per 100 cc.*
Sample taken at close of dialysis	0.30
Sample taken at close of dialysis and allowed to stand 36 hours	0.63
Dialysate	
Sample after 5½ hours dialysis	0.27
Sample after 7 hours dialysis	0.29
Sample removed after 7 hours dialysis and allowed to stand 36 hours	0.28

The ammonia content of the blood is doubled in twenty-four hours while that of the dialysate shows no increase. An equilibrium between the ammonia content of the dialysate and of a sample of the circulating blood was reached.

The slowly evolved ammonia has its source therefore in non-diffusible constituents of the blood.

Details of the methods with additional tables appear in the June (1915) number of the *Journal of Biological Chemistry.*

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[1] On the removal of diffusible substances from the circulating blood of living animals. *J. Pharmacol. Exp. Therap.*, 5, 275 and 611 (1914).

126 PARABOLIC ORBITS OF METEOR STREAMS

By Charles P. Olivier

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Presented to the Academy, May 10, 1915

The study of meteors as astronomical bodies began in the early part of the last century. The occurrence of the great showers of 1833 and 1866 showed the importance of the meteoric astronomy. When it was proved, soon after the latter appearance, that this meteor stream and Tempel's Comet of 1866 had the same orbit about the sun, the interest in meteors increased enormously. Though millions of meteors are swept up by the earth every twenty-four hours, still these little bodies also obey the law of gravitation, and, in obedience to this law, meteors pursue their orbits about the sun.

Astronomy is interested in proving the connection between comets and meteors, in investigating the distribution of meteoric matter in space with special regard to that near the earth's orbit, in finding the cause of the zodiacal light, and the action of meteoric matter as a resisting medium and as an absorber of light in space. Meteorology is interested in the study of meteor trains and meteor heights, because the former give us our only information about the velocity and directions of wind currents in the upper strata of our atmosphere, the latter certain information as to how high this atmosphere must extend.

To arouse interest in this subject the American Meteor Society was organized in 1911, and the results mentioned in this paper were worked out from the 2800 observations contributed by its members. These results appear in full in Volume 2, Part 4, of the Publications of the Leander McCormick Observatory of the University of Virginia. While the most important contribution to science was the 126 parabolic orbits of meteor streams, the most interesting result, undoubtedly, was the final proof of the connection of Halley's Comet and the η Aquarid meteors. These bodies were shown to follow orbits similar to Halley's Comet and present one of the best instances of the slow disintegration of such a body. The question whether radiants are stationary was studied in considerable detail, and a negative conclusion was reached, principally because motion was detected in the radiant point of the Orionid meteors, which are held to be the typical case by certain authorities. The tabular results of the paper are of interest to anyone who has even touched on the subject, showing as they do at a glance just what one can expect from a few hours' observation at favorable times of the year. The National Academy of Sciences at its annual meeting has granted a small sum from the J. Lawrence Smith fund for the purpose of extending the work.

For observations of meteors a technical knowledge of astronomy is not necessary. Anyone interested as an amateur is invited to write to the Leander McCormick Observatory. Maps and directions for observing will gladly be forwarded. It is earnestly hoped that a large number of amateurs will assist us in the extension of this work.

THE BASAL SILURIAN FORMATIONS OF EASTERN NORTH AMERICA

By Charles Schuchert

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Presented to the Academy, April 30, 1915

At the base of the Silurian system of rocks in the state of New York and in the Appalachian Mountains occurs the Medina sandstone. In tracing this formation from the gorge of the Niagara River northwestward into Ontario, the sandy phase of the Medina is seen to change gradually into a muddy one and finally into a limestone, as is the case on the Manitoulin Islands of Lake Huron. These early Silurian rocks of Ontario, known as the Cataract formation, have long been regarded as equivalent to the Clinton formation of New York, but are now known to hold the time of the Medina formation. In another direction, in Ohio, Indiana, and Kentucky, the basal Silurian strata, the Brassfield formation, are also seen to be the equivalent of the Cataract formation of the north.

The faunas of these three marine formations are quite different. The Medina sandstone has the smallest assemblage, with 22 species, while the Cataract limestones have at least 76 forms, and the Brassfield limestones 140 kinds of invertebrates. There are but 7 species in common between the Medina and Cataract, while the latter has 24 forms repeated in the Brassfield.

Each one of the three formations represents a different marine basin. The Medina is of the Appalachian province, is a sandstone formation, and finally invades to a slight extent the area of the Cataract. The Brassfield province lies in the main west of the Cincinnati axis, is of southern origin, with limestone-making seas, spreads also up the southern portion of the Appalachian province, and finally likewise invades slightly the area of the Cataract sea. On the other hand, the Cataract province spreads westward through the Saint Lawrence embayment, and finally, in eastern Ontario and northwestern Ohio, unites with the other two provinces. The normal marine junction of the Cataract and Brassfield seas is prevented by the Medina delta of sands. For these reasons, Medina, Cataract, and Brassfield are to be retained as names for independent marine faunas and formations. The details leading to these conclusions are set forth in a contribution entitled 'Medina and Cataract formations of the Siluric of New York and Ontario,' *Bulletin of the Geological Society of America.*

A METHOD OF OBTAINING COMPLETE GERMINATION OF SEEDS IN *OENOTHERA* AND OF RECORDING THE RESIDUE OF STERILE SEED-LIKE STRUCTURES

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Presented to the Academy, May 4, 1915

All genetical workers with *Oenothera* shortly become aware that generally only a small proportion of the seed-like structures sown in their seed pans produce seedlings before the pans are emptied to give place in the hot house for the developing culture. My own practice has been to keep seed pans from eight to ten weeks only, as it is uncertain, if sowings are made in January, whether seedlings appearing later can be brought to maturity during the summer. For the past three seasons I have counted the seeds sown and thus have obtained some information on the relative degrees of fertility in my cultures which in some cases have been surprisingly low. But this procedure does not give accurate data on the proportion of fertile seeds to sterile seed-like structures for the reason that germination of *Oenothera* seeds in earth is very irregular and may be delayed far beyond the time that it is convenient or possible to keep the seed pans. Seeds sown in earth are obviously lost for further enquiry as to the fact of their viability, a proportion of seedlings appears but, as for the residue, that cannot be examined.

The technical problem of obtaining from sowings of seeds cultures that will accurately represent the genetical possibilities of the sowing is under these conditions difficult. In *Oenothera* work this problem is vital for exact studies since through the delayed germination may be lost not only peculiar individuals but possibly, in the case of hybrids, entire classes of segregates. It is probably safe to say that no culture of *Oenothera* has as yet been described in which we may feel confident that all of the viable seeds have germinated. Consequently we cannot be certain that any of the reported percentages of 'mutants' or ratios of segregates from hybrids are correct. Furthermore it is important to determine for oenotheras the ratio of fertile seeds to the structures that in outward appearance cannot be distinguished from seeds but which contain no living tissue or only a small amount of endosperm, with an embryo, if present at all, so small or abnormal that it cannot produce a seedling. We are indebted to O. Renner[1] for the first detailed investigation of the sterile seeds of an *Oenothera* and for the formulation of a most interesting hypothesis as the result of his studies on *O. Lamarckiana.*

During the past winter I have tested the seed fertility of fifty species, races, or hybrids of *Oenothera* and as a result of these experiments I have become convinced that genetical research in this group must in the future adopt methods that will ensure a rapid and complete germination of the viable seeds and at the same time conserve the sterile seeds or undeveloped seed-like structures for examination. Such methods will require that seeds be germinated in some convenient receptacle and subsequently be set in earth leaving as a residue within the receptacle all sterile structures.

The method employed this winter is briefly as follows: In a Petri dish 3¾ inches in diameter was placed a pad of circular filter papers 3 inches across and about ¼ inch thick. The dish, cover, and paper pad were then sterilized by heat after which boiled tap-water was added in such quantity that the thoroughly soaked paper pad lay in the center of the dish surrounded by water. The seed-like structures were then spread over the surface of the pad and the dish covered. The seeds, therefore, lay on a very wet surface and in a moist chamber; more water was added when evaporation materially lowered the level in the dish. The tap-water was boiled to avoid the introduction of algae; fungi appeared in some of the cultures but apparently found the seed coats an unfavorable substratum for they grew very little; bacteria were likewise not troublesome.

The Petri dishes with seeds were placed in shaded parts of the same hot house with cultures sown in earth and were thus under similar conditions as to temperature. The possible germination within the dishes was always very much more prompt than for a similar sowing of seeds in the earth and proved to be complete when sufficient time was allowed. The advantages of Petri dishes as receptacles are those of convenience in the removal of germinated seeds and in the collection and examination of the residue. Petri dishes are also readily stacked and upon labels over the covers may be recorded the data of the experiment.

The rate of germination in the Petri dishes under the conditions of the hot house was irregular. Complete germination in some species required only three weeks, in other forms as much as from six to ten weeks, and in some cultures I found a few ungerminated seeds after three and one half months. In warm, sunny weather the house during part of the day was at 90°F., and these high temperatures were frequently followed by a burst of germinations. It is thus clear that the best results in experimental germination will be obtained when cultures are placed in chambers or incubators the temperature of which can be regulated with accuracy.

Hugo De Vries[2] has recently published a method of stimulating seed germination in *Oenothera* that seems likely to eliminate largely the irregularity and slowness of germination described above. He reports that well soaked seeds after being subjected to a pressure of 6–8 atmospheres for 1–3 days at room temperature will germinate in large percentages when removed to an oven at 30°C. His hypothesis for this interesting behavior assumes that the hard inner coats of the *Oenothera* seeds have narrow microscopic slits filled with air and that ordinarily water enters the seed with difficulty but under pressure can be forced to points where it may readily be absorbed. De Vries thus contributes the practical suggestion of an automobile pump and a strong chamber such as an autoclave into which receptacles containing the well soaked seeds may be packed. If complete germination is not obtained after one treatment to compressed air successive treatments alternating with normal atmospheric pressure and perhaps variation in temperature conditions may bring forward stubborn cultures.

The residue that is left after germination is believed to be complete has in my experience generally ranged from structures fully as large as the larger of the viable seeds to structures smaller than seeds, which in turn grade into the remains of numerous unfertilized, or abortive ovules, represented in the seed capsules by a light brown powder. It is an easy matter to determine whether or not the structures in the residue, seed-like in size and appearance, contain an embryo. The structures may be opened with the point of a needle or scalpel, or they may be squeezed flat between the tips of a strong pair of forceps. The presence of an embryo is at once made evident and the germ may be readily examined. Generally the seed-like structures will be found to be entirely empty or to contain only a trace of soft tissue. Structures smaller in size than normal seeds can be examined microscopically. The residue may be conveniently preserved as a record by arranging the structures on paper and covering them with a solution of shellac in alcohol, or with a solution of glue.

To illustrate the advantages of germinating *Oenothera* seeds under experimental conditions over the old practice of sowing upon earth I will give the results for a problem as attacked under the old and new method. In 1914 I germinated on earth seeds from an F_1 hybrid plant, 13.35ac, of the cross *O. franciscana* × *O. biennis*. A sowing of 819 seed-like structures produced a culture of 402 seedlings, a germination of about 50%. The culture was grown partly to test the inheritance of a character (red coloration of the papillae on the stems and ovaries) present in *franciscana* and absent in *biennis*. This character was fully dominant in the F_1 of this cross and in its reciprocal. It seemed reasonable to expect that a proportion of the plants in the F_2 generation would present the clear green stems and ovaries (recessive) of *biennis*, but I found no plants of this type in the culture of 1914. This winter (1915) I germinated in Petri dishes seeds from the same F_1 hybrid plant, 13.35ac, and obtained 761 seedlings from about 921 seed-like structures,—a germination close to 82%. The new method had raised the proportion of germination more than 30%. Of the 761 seedlings I was able to bring 748 to rosettes, the 13 which died probably belonging to a small group of etiolated dwarfs difficult to grow. It becomes a matter of interest to see whether or not in this culture of 1915 a group of green-stemmed plants will appear.

[1] Renner, O., Befruchtung und Embryobildung bei *Oenothera Lamarckiana* und einigen verwandten Arten. *Flora, Jena*, **107**, 115 (1914).

[2] De Vries Hugo, The Coefficient of Mutation in *Oenothera biennis* L. *Bot. Gaz., Chicago*, **59**, 169 (1915).

THE OSMOTIC PRESSURE OF THE IONS AND OF THE UNDISSOCIATED MOLECULES OF SALTS IN AQUEOUS SOLUTION

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The concentration of the ions (C_i) and that of the undissociated molecules (C_u) are involved in the equation, $C_i^2 / C_u = K$, expressing the application of the law of mass-action to solutions of electrolytes. Hence the large divergence from this law which strong electrolytes exhibit may be due to the behavior of the ions, to that of the undissociated molecules, or to that of both. This so called anomalous behavior may be simply expressed by saying that for one or for both of these molecular species van't Hoff's law, $\Pi = CRT$, where Π is the osmotic pressure, C is the concentration in mols per unit-volume, R is the gas-constant, and T is the absolute temperature, does not hold; for, if all of the molecular species entering into an equilibrium obey this law, the law of mass-action is a thermodynamic necessity.[1]

In this paper is outlined and applied a method for determining the extent to which the ions and the undissociated molecules deviate from van't Hoff's law, that is, for determining the relation between the osmotic pressure of the ions or of the undissociated molecules and their concentration. The significance of the results, particularly as applied to the calculation of the degree of ionization and to the validity of the law of mass-action, is discussed.

Besides the laws of thermodynamics, the assumptions involved in these calculations are that (1) in a dilute solution of a di-ionic electrolyte the osmotic pressure due to each of the two ions is the same, and that (2) the degree of ionization (γ) is given by the conductance-viscosity ratio ($\lambda\eta/\lambda_o\eta_o$), the concentration of each ion then being $C\gamma$ and that of the undissociated molecules $C(1-\gamma)$. These assumptions are generally accepted; they are, however, briefly discussed below.

The total osmotic pressure (Π) of a solution is due in part to that of each of the two ions and in part to that of the undissociated molecules. Since by the first assumption above stated, the osmotic pressure of one ion is equal to that of the other, the relation $\Pi = 2\Pi_i + \Pi_u$ follows. The principles of thermodynamics, together with the same assumption, lead for equilibrium in a solution of di-ionic electrolyte, to the relation[1]

$$2d\Pi_i/C_i - d\Pi_u/C_u = 0 \tag{1}$$

If now C_i and C_u are calculated from conductance data, and if the total osmotic pressure of the solution be known (from direct measurements or from freezing-point determinations, etc.), there remain in these two equations but two unknown quantities Π_i and Π_u.

The solution of these two equations gives for the rate of change of the osmotic pressure of the ions with their concentration the relation:

$$\frac{d\Pi_i}{dC_i} = \tfrac{1}{2}\left[\frac{\Pi}{C} + \frac{\gamma}{2\cdot 303}\,\frac{d(\Pi/C_i)}{d\log C_i}\right] \tag{2}$$

The first term Π/C may be calculated directly from the data. The second term, which is small in comparison with the other, may be conveniently evaluated by the graphic method of plotting values of Π/C_i against those of $\log C_i$ and determining the tangents. Having calculated $d\Pi_i/dC_i$ for a series of concentrations, values of Π_i/C_i may be determined by integration, either graphic or algebraic.

In order to determine the similar ratios for the undissociated molecules, the equation corresponding to (2) might be employed. In practice, however, it is more convenient to use a different method. By eliminating K, C_i and C_u from Storch's equation, $[C_i^n / C_u = K]$, from the differential equation obtained from this and from equation (1), the relation

$$d\Pi_u/dC_u = \frac{2}{n} d\Pi_i/dC_i$$

is obtained. That is, having calculated values of $d\Pi_i/dC_i$ and of n at any given concentration, by the methods which have been previously developed,[2] over a concentration-range, values of $d\Pi_u/dC_u$ may be determined, and hence by integration the osmotic pressure of the undissociated molecules may be calculated.

The results of such calculations for a few representative di-ionic electrolytes for whose solutions reliable freezing-point data[3] are available are given in the table. They are given as ratios of the form $\frac{d\Pi}{dC}/RT$. The deviations of these from unity show the degree of deviation from van't Hoff's law at the different concentrations. For most purposes the ratio $d\Pi/dC$ here considered is more useful than the integral ratio Π/C; for it is the former ratio which is introduced into thermodynamic relations in order to derive the mass-action law, the electromotive-force equation, etc. Further, the differential ratio gives information of a more definite character regarding the properties of the solution at any given concentration than does the integral ratio. The value of the former depends only upon the properties of the solution of the concentration in question, while that of the latter is influenced by, and to a great extent depends upon, the properties of the solutions of all concentrations from the infinitely dilute solution up to that in question.

Values of the Ratio $\frac{d\Pi_i}{dC_i}\Big/$ *RT for the Ions*

Equivalents per liter	LiCl	NaCl	KCl	KNO_3	$CuSO_4$
0.001	.....	0.996	0.987	.....	0.927
0.002	.....	0.993	0.983	.....	0.919
0.005	.....	0.990	0.983	0.990	0.899
0.01	0.985	0.986	0.979	0.983	0.867
0.02	0.993	0.983	0.977	0.972	0.824
0.05	0.986	0.980	0.970	0.946	0.756
0.1	1.000	0.973	0.967	0.926	0.734
0.2	1.015	0.963	0.960	0.888	.....
0.3	1.046	0.965	0.958	0.850	.....
0.5	1.110	0.967	0.956	.....	.....

Values o the Ratio $\dfrac{d\Pi_u}{dC_u} \Big/ RT$ *for the Undissociated Molecules*

Equivalents per liter	LiCl	NaCl	KCl	KNO_3	$CuSO_4$
0.001	.....	1.263	1.249	.....	1.090
0.002	.....	1.280	1.262	.....	1.091
0.005	.....	1.305	1.289	1.279	1.085
0.01	1.334	1.323	1.305	1.281	1.067
0.02	1.366	1.343	1.325	1.275	1.046
0.05	1.390	1.375	1.350	1.246	1.025
0.1	1.439	1.394	1.375	1.217	1.055
0.2	1.500	1.413	1.396	1.159	.....
0.3	1.577	1.446	1.412	1.100	.....
0.5	1.728	1.488	1.432	.....	.....

The results in the table show that, even in solutions as dilute as 0.001 normal, neither the ions nor the undissociated molecules obey van't Hoff's law, though with decreasing concentration both become more nearly normal.

The osmotic pressure of the ions is less than that calculated from this law. This is true in every case except for solutions of lithium chloride at 0.1 normal and above; hydration is doubtless the cause of this exception.

It can be shown, moreover, that the behavior of the ions may be represented within the error of the freezing-point determinations by an equation of the form

$$\frac{d\Pi_i}{dC_i} = RT\,(1 + kC_i^m)$$

The results seem to justify the assumption that the osmotic pressures of the two ions in a solution of a di-ionic electrolyte are the same up to moderate concentrations. For example, the behavior of the chloride ion is within the experimental error the same whether calculated from the data for sodium chloride or from that for potassium chloride.

The undissociated molecules have an osmotic pressure considerably greater than that calculated from van't Hoff's law. At moderate concentrations the various salts do not behave similarly. In the case of the chlorides the deviations continue to increase with the concentration, but in that of the nitrates a maximum is reached about 0.02 normal. This somewhat surprising result, that the behavior of the least hydrated is the most complex, is connected with the fact that the exponent n is for these salts a minimum at about 0.02 normal.

For solutions more dilute than 0.01 normal and in some cases to a much higher concentration, an empirical equation of a form similar to that which holds for the ions expresses the relation between the osmotic pressure of the undissociated molecules and their concentration. In the case of potassium chloride these relations, one for the ions and one for the undissociated molecules, express the results for solutions between 0.001 normal and 0.5 normal, so closely that the average difference between the total osmotic pressure of the solution calculated from them and that experimentally determined is but 0.10 percent.

The behavior of the undissociated molecules of copper sulphate is remarkable in two particulars. First they show an almost constant deviation from van't Hoff's law from 0.001 normal to 0.02 normal. In the second place, while the bivalent ions are considerably more abnormal than are the univalent ions, the undissociated molecules of this salt obey van't Hoff's law much more closely than do those of the uni-univalent salts.

H. Jahn[4] and recently G. N. Lewis[5] have considered it to be less reasonable to assume that (1) the concentration of the ions is proportional to the electrical conductivity of the solution, than it is to assume that (2) the concentration of the ions is proportional to their osmotic pressure or activity, that is, that the ions are normal or that they obey van't Hoff's law. Parallel to this latter assumption is the one that (3) the undissociated molecules are normal. The degree of ionization calculated upon any of these three assumptions is different from that calculated upon the other two. That is, the acceptance of one compels the rejection of the others.

It may be shown in another way that the experimental data are such that it is not possible for both the ions and the undissociated molecules of strong electrolytes to be normal. By combining the mass-action law relation, $C_i^2/C_u = K$, and the differential equation derived from this with the thermodynamic equation (1) above, it can be shown that the necessary and sufficient condition that the law of mass-action hold over a certain concentration range is that at all concentrations $d\Pi_i/dC_i = d\Pi_u/dC_u$; that is, that the rates of change of the osmotic pressures of the ions and of the undissociated molecules with their respective concentrations be the same. Since the mass-action law is not obeyed by solutions of strong electrolytes this relation does not hold, and the relation $\Pi = CRT$ cannot be true for both molecular species. The computation of the mol-number i from freezing-point data, and the calculation of the degree of ionization on this basis, assumes that both the ions and the undissociated molecules are normal. Since the resulting values do not obey the mass-action law, the method is inconsistent and theoretically incorrect. It has long been recognized that actual gases are not 'perfect' gases. Unfortunately the idea that in general solutes may be treated as normal or 'perfect' solutes is much more persistent.

The magnitude of the deviation of the strong electrolyte from the normal behavior depends greatly upon the terms in which this deviation is expressed. The equilibrium expression C_i^2/C_u increases with the concentration. Thus for potassium chloride it increases from 0.026 at 0.001 normal to 0.55 at 0.1 normal, an increase of 21 fold. The results given in the table show that this behavior is accounted for by a deviation of 3.3% in the osmotic pressure of the ions and of 37% in that of the undissociated molecules. The percentage deviation when expressed as here in terms of osmotic pressure is smaller than when it is expressed, as Lewis[5] has done, in terms of activity; this follows from the logarithmic relation between the two.

A more thorough study of these topics is appearing in *The Journal of the American Chemical Society.*

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[1] E. W. Washburn, *J. Amer. Chem. Soc.*, 32, 484–5 (1910).

[2] S. J. Bates, *J. Amer. Chem. Soc.*, 35, 519 (1913).

[3] The data have been taken in great measure from the compilation of A. A. Noyes and K. G. Falk, *J. Amer. Chem. Soc.*, 32, 1011 (1910). This has been supplemented by the recent measurements of F. Flügel, *Zs. physik. Chem.*, 79, 577 (1912), of W. A. Roth, *Ibid.*, 79, 599 (1912), of E. W. Washburn and D. A. McInnes, *J. Amer. Chem. Soc.*, 33, 1686 (1911) and of L. H. Adams, *Ibid.*, 37, 481 (1915).

[4] *Zs. physik. Chem.*, 33, 545 (1900).

[5] *J. Amer. Chem. Soc.*, 34, 1631 (1912).

THE EXTENSION OF THE SPECTRUM BEYOND THE SCHUMANN REGION

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The violet limit of the spectrum determined by direct eye observation lies in the neighborhood of 4000 angstrom units; with a glass prism and lenses the spectrum may be followed photographically to wave length 3000 or thereabout; with a quartz system or with a reflecting grating, the limit may be pushed to wave length 1850. Victor Schumann showed that the absorption of the air and of the gelatine of the photographic plate were responsible for the abrupt termination of the spectrum. By employing a vacuum spectroscope and a special photographic plate whose emulsion was very poor in gelatine, he was able to push his observations to wave length 1230. At this point he was stopped by the opacity of the fluorite of which his lenses were made.

I have employed a vacuum spectroscope containing a concave diffraction grating[1] arranged in such a manner that the light path from the source to the photographic plate is wholly in gas. Briefly, the apparatus may be described as a brass tube about 11 cm. in diameter and rather over a meter long. This tube is closed at each end by brass plates ground to fit suitable flanges. In my most recent experiments the light is generated electrically in a discharge tube of quartz provided with tungsten electrodes. This discharge tube fits air tight on one of the two brass plates just mentioned; light from it having passed through a slit, traverses the length of the apparatus and falls upon the diffraction grating by which, having been analyzed into its component colors, it is brought to focus on a special photographic plate placed in close proximity to the slit.

As the discharge tube is in no way separated from the body of the spectroscope it is obviously necessary to choose for the experiment some gas which will not only yield radiations in the region under investigation but which will be transparent to these radiations. My earlier experiments were conducted with hydrogen, since it had shown the necessary characteristics in that part of the spectrum investigated by Schumann. With this gas at a pressure of 2 or 3 mm., and by employing a strong disruptive discharge, about eighteen months ago I was able to extend the spectrum to wave length 900.

A tedious investigation having convinced me that nothing more was to be expected from the use of hydrogen, at least in this neighborhood, I turned my attention to helium, being guided by some of my earlier experiments which had proved that this gas possesses the necessary transparency. At the same time, I made some improvements in my apparatus which, though they left its general form unchanged, resulted in making it far more air tight than ever before. The success of this improvement may be judged by the fact that I have recently been able to leave the spectroscope for over fourteen hours at a pressure of about 3 mm. without being able to detect any leak, either by a McLeod Guage reading to 0.007 mm. or by the appearance of impurities in the spectrum of the gas content. I also took particular pains to purify the helium which I employed.

I have been rewarded for my trouble by a very considerable extension of the spectrum for with helium free from nitrogen, at a pressure of 2 or 3 mm., by the use of the disruptive discharge, and with an exposure of about ten minutes, I have repeatedly observed a number of new lines the most refrangible of which has a wave length of 600. All this has been accomplished with a grating ruled on speculum and with photographic plates prepared exactly as recommended by Schumann. The wave length 600 represents an extension beyond Schumann's limit greater than that which Schumann himself achieved beyond λ 1850.

The relations of the spectra of hydrogen and helium have recently come into prominence through the theoretical researches of Bohr, Nicholson and others. It so happens that the region on the more refrangible side of wave length 1250 offers an important ground for the study of these relations. In order that the conclusions may be of value, however, it is necessary that the gases under observation should be free from impurities. In the best form of closed discharge tubes the difficulties arising from the contamination of the gas by the material of the tube or by the electrodes, are very considerable, especially when a disruptive discharge is used. In my apparatus, where the brass spectroscope and quartz discharge tube communicate directly, these difficulties are considerably increased. I trust in time to overcome them; but for the present I must confine myself to the following statements:

Prominent in the spectrum of hydrogen is the line at 1216 which forms the first member of a series predicted, on theoretical grounds, by Ritz. I have also found the two next members near wave length 1026 and 972. With hydrogen, they are best seen when a disruptive discharge is used, but with helium containing that trace of hydrogen so difficult to remove, they occur quite strongly with a simple alternating current from a 60 cycle transformer. This is a good illustration of the curious behavior of helium, for an atmosphere of this gas seems sometimes to facilitate the production of the spectra of other gaseous substances which occur in it as impurities. As far as I am able to observe with the uncondensed discharge, all the lines seen with helium are also found in hydrogen. It is only with the disruptive discharge that the new lines on the more refrangible side of λ 900 make their appearance; they occur in helium alone and number seven or eight; some are quite strong.

The wave length of the X-rays as determined by the Braggs, is of the order of one angstrom unit. There is, therefore, a gap of some six hundred units between the region of the ordinary Röntgen rays and the limit which I have reached. Several attempts have been made recently to produce less refrangible X-rays,[2] but there is no satisfactory way of estimating their wave length unless perhaps one relies on the Planck-Einstein formula $Ve = h\nu$. Taking V as the potential drop which gives the cathode particle the velocity necessary to produce a very soft X-ray, e as the charge on the particle and giving to h the usual value ascribed to Planck's constant, the experiments of Dember[3] were made to yield a wave length for the X-ray longer than that which I have determined. Such speculations, though they are very interesting, cannot be given any great weight. There is still a considerable region between wave length 600 and wave length *one* which must be experimentally explored.

My present limit is set by the adjustment and dimensions of my apparatus. Now that we know that a Schumann plate can be used and that a speculum grating is efficient, I see no insuperable difficulty to a still further extension by purely spectroscopic means.

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[1] *The Spectroscopy of the Extreme Ultra-Violet,* Longmans, Greene & Co.

[2] J. J. Thomson, *Phil. Mag., London,* 28, 620 (1914); E. R. Ladd, *Ann. Physik, Leipzig,* 46, 605 (1915).

[3] *Berlin, Verh. D. Physik. Ges.,* 15, 560 (1913).

UNSYMMETRICAL LINES IN TUBE-ARC AND SPARK SPECTRA AS AN EVIDENCE OF A DISPLACING ACTION IN THESE SOURCES

By Arthur S. King

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The most effective laboratory means of causing a displacement of spectrum lines is pressure around the light-source, which has been found to be generally effective in changing the wave-length of lines in the spectra of the arc, spark and electric furnace. These displacements vary in magnitude for different groups of lines at the same pressure. Recently other influences have been found effective in causing small displacements, these manifesting themselves usually in making the spectrum lines unsymmetrical, usually with the red side stronger. A measurement of the maximum of the line under these conditions gives a different wave-length from that obtained for the same line when symmetrical.

We have then the two-fold problem of finding what features in the light-source produce these dissymmetries and the degree to which various lines are subject to such disturbing influences.

About two years ago, the writer described the spectrum of the 'tube-arc,' and noted that many lines were made highly unsymmetrical in this source. The tube-arc is obtained when the tube used in the electric furnace is made thin at its middle so that when the voltage is applied the tube quickly burns apart, forming an arc which carries a very large current at low voltage. This ring of arc is viewed axially and the spectrum shows many peculiarities.

Further use of the tube-arc has now been made in studying the spectrum of iron from λ 3600 to λ 6600, special regard being paid to the dissymmetries produced and the manner in which certain types of lines are intensified.

The tube-arc shows dissymmetries in a very pronounced degree which are present in a more or less incipient state in the ordinary arc, especially in the region near the negative pole. Such dissymmetries are naturally most conspicuous in the case of reversed lines, and but few of these show a fully symmetrical structure in the tube-arc spectrum, the red side usually being stronger than the violet.

The unsymmetrically reversed iron lines fall into two main groups, and these groups show a close connection with the behavior of the lines at various temperatures of the electric furnace and with their pressure displacements. The first group is made up of lines whose dissymmetry in the tube-arc is small or moderate in amount. These appear in the electric furnace at low temperature. The arc under pressure gives them approximately symmetrical and with small or moderate pressure displacements. The lines of the second group are reversed very unsymmetrically in the tube-arc, the red side being often 6 to 8 times as strong as the violet side. They are relatively much stronger in the arc than in the furnace. Under pressure they are very unsymmetrical and show large pressure displacements. In the ordinary iron arc the lines of the second group show a decided dissymmetry near the pole as compared with the center of the arc.

A similar concordance of behavior is shown for a group of strong lines in the green-yellow. These widen toward the violet near the pole of the iron arc and are displaced toward the violet by pressure. In the tube-arc these are unreversed and nearly symmetrical, but show strong displacements toward the violet. Reversal with the violet side strongest has not been observed among the iron lines, but this structure is shown by the tube-arc for the *H* and *K* lines in the calcium spectrum.

The measurement of these tube-arc lines, difficult and uncertain with a micrometer microscope, is made much more accurate by the registering micro-photometer, the curves of which show the structure of the lines and permit the interval between neighboring lines to be measured.

A number of favorably situated lines have been thus measured, the distance being taken from a line approximately symmetrical to one showing strong dissymmetry. The latter line in each case was found to be displaced by amounts ranging from 0.02 to 0.04 A. An interesting feature is that when a line is made narrow by the presence of very little iron vapor in the tube-arc, the displacement becomes very small, although stronger and wider lines on the same plate retain their one-sided character.

It has long been known that a strongly condensed spark tends to produce unsymmetrical lines in its spectrum. The writer has made a study of this effect using a powerful and very disruptive transformer spark. The dissymmetries produced are strikingly similar to those of the tube-arc. Examination of different regions of the spark showed the dissymmetry to be such as would result from the interior vapor of the spark giving a line displaced toward the red, while the outer vapor gives a symmetrical line coinciding with the usual arc line. Tube-arc lines displaced toward the violet show strong shading on that side in the condensed spark.

For the cause of these dissymmetries, which amount in each case to a displacement of the maximum of the spectrum line, it appears that we must look to an agency other than pressure. Dissymmetries in the spark have sometimes been ascribed to this, but they correspond closely to those of the tube-arc, which is operated in a partial vacuum. Also, in both sources there is no tendency for the displacement to increase in magnitude with increasing wave-length, which is a feature of the pressure effect.

A high vapor density increases the dissymmetry of tube-arc lines, but apparently cannot by itself cause such dissymmetry. Evidence on this point is supplied by the electric furnace, the spectrum lines of which are notably symmetrical with high vapor density. In the arc also, lines may be very wide when much vapor is present, and yet remain symmetrical. When the poles of the arc are approached, however, the occurrence of enhanced lines indicates a condition resembling the spark, and in this region the lines more sensitive to displacement become one-sided, an effect which is magnified in the tube-arc and in the disruptive spark.

At present these effects seem to be harmonized by considering as a necessary condition the presence of electrified particles moving at high velocities, these being given in the arc and spark by the strong potential gradients and in the tube-arc by the enormous consumption of energy by the ruptured carbon tube. If such swiftly moving electrons are present, the effect of their velocities in causing a disturbance of vibration period should be increased by the crowding together due to high vapor density. On this account the relation of the effects here described to the effect of the crowding due to pressure may be a close one, as is borne out by the similarity of the observed displacements.

A combination of electron bombardment with high pressure should give larger displacements for lines which are strengthened by strong electrical conditions. The following evidence bears this out: (*a*) The lines which are strong in the arc relatively to the furnace have large pressure displacements. (*b*) Enhanced lines as a class are found to be displaced by pressure more than arc lines. (*c*) For a given pressure the enhanced lines are displaced more in the spark than in the arc.

The details of this investigation, with corroborative evidence from other spectra, will be published in the *Astrophysical Journal*.

ON THE FACTORIZATION OF VARIOUS TYPES OF EXPRESSIONS

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Not a few mathematicians have dealt with the problem of setting up criteria by means of which the irreducibility of certain expressions in certain domains may be seen at a glance from the character of the expressions. Gauss, Kronecker, Schoenemann, Eisenstein, Dedekind, Floquet, Koenigsberger, Netto, Perron, M. Bauer, and Dumas have written on the subject.[1] The work of these authors may be said to center around the Schoenemann-Eisenstein theorem, which, however, is an exceedingly special case of various theorems obtained (for example, of theorem IV when interpreted for situation 1). With the exception of Floquet and Koenigsberger, the authors mentioned deal exclusively with the case where the expressions are polynomials, and chiefly with the case of polynomials whose coefficients belong to the domain of rational numbers. Floquet and Koenigsberger have extended the investigation to the case of linear homogeneous differential expressions.

A variety of methods have been employed. Thus the theory of algebraic fields, the theory of algebraic functions, the character of the solutions of linear homogeneous differential equations, and even geometric representation (Dumas, loc. cit.) have been used. Elementary methods, requiring no such means, have succeeded in yielding only the less general results.

One of our objects is to show that elementary and comparatively short considerations may be made to yield results more general than any hitherto obtained. In fact, for a complete comprehension of the proofs, little more is required in the way of specific knowledge than an understanding of the definition of the various expressions considered

Moreover, our results have a most intimate connection with the work of every author cited, and, in most cases, the results bearing on the problem before us that are contained in the articles quoted subordinate themselves as special cases to the theorems obtained here.[2]

A sharp distinction has been made in the literature (as regards polynomials) between those investigations that are based on the *divisibility* properties of the coefficients and those that proceed from the consideration of the *magnitude* of the coefficients.[3] One of the interesting results is that the gap between these two types of investigation may, in a certain sense, be bridged. As a consequence, it is possible to show how the various theorems obtained flow from certain general considerations as a common source, and thus a surprising unification of the material is achieved.[4]

Our work deals with polynomials (for the various kinds of coefficients considered, see situations 1–5 incl.), linear homogeneous differential expressions, linear homogeneous difference expressions (for the first time as regards our problem, as far as the writer knows), and more general expressions. There is no better way of making what is essential in the proofs come to the front than by treating the subject in abstract, postulational fashion. Thereby also the interconnection between the results and the unification above referred to are made manifest. Thus, it is easy to see, as Koenigsberger has pointed out by an example, that the Schoenemann-Eisenstein theorem cannot be directly extended to the case of linear homogeneous differential expressions; but our abstract treatment furthermore lays bare the underlying reason—by no means evident otherwise—why it breaks down, and at once indicates what analogous theorem may replace it.[5] The abstract treatment is, moreover, especially fitting here because a small number of simple assumptions is sufficient for the foundations of the theory.

We start[6] with any *aggregate* $\mathfrak{S}$ whatsoever and shall deal with finite, ordered subaggregates $E = (e_0, e_1, e_2, \cdots, e_m)$ of $\mathfrak{S}$, e_ν $\{\nu = 0, 1, 2, \cdots, m\}$ being an element of $\mathfrak{S}$.

Such a finite, ordered subaggregate E we shall call a '*parenthesis*' of $\mathfrak{S}$; m will be called the '*order*' of E. We assume that the '*product*' $A \cdot B = (a_0, a_1, a_2, \cdots, a_r) \cdot (b_0, b_1, b_2, \cdots, b_s)$ of any two parentheses of $\mathfrak{S}$ is equal to a parenthesis $C = (c_0, c_1, c_2, \cdots, c_n)$ of $\mathfrak{S}$ for which $n = r + s$. We assume furthermore that with every element e of $\mathfrak{S}$ there is associated a single number η, called the '*rank* of e,' η being an integer or $-\infty$ (never $+\infty$), such that when $A \cdot B = C$ one of the following 3 sets of relations holds (see situations 1–10 that make either I or II or III valid):

$$\text{I}\quad a)\quad \gamma_\nu \leqq \max_{\lambda+\mu=\nu}(\alpha_\lambda+\beta_\mu), \quad \{\nu=0, 1, 2, \cdots, n\},$$

$$b)\quad \gamma_\nu = \max_{\lambda+\mu=\nu}(\alpha_\lambda+\beta_\mu), \quad \{\nu=0, 1, 2, \cdots, n\},$$

if $\alpha_\lambda+\beta_\mu$ attains its maximum, λ and μ varying so as to satisfy the relation $\lambda+\mu=\nu$, for a single pair of values of λ and μ. Here α_λ, β_μ, γ_ν = rank[7] of a_λ, b_μ, c_ν.

$$\text{II}\quad a)\quad \gamma_\nu \leqq \max_{\lambda+\mu\leqq\nu}(\alpha_\lambda+\beta_\mu), \quad \{\nu=0, 1, 2, \cdots, n\},$$

$$b)\quad \gamma_\nu = \max_{\lambda+\mu=\nu}(\alpha_\lambda+\beta_\mu), \quad \{\nu=0, 1, 2, \cdots, n\},$$

if $\alpha_\lambda+\beta_\mu$ attains its maximum M_ν (λ and μ varying so as to satisfy $\lambda+\mu=\nu$) for a single pair of values of λ and μ, and if $M_\nu \geqq \alpha_\lambda+\beta_\mu$ whenever $\lambda+\mu<\nu$.

$$\text{III}\quad a)\quad \gamma_\nu \leqq \max_{\lambda+\mu+\sigma=\nu}(\alpha_\lambda+\beta_\mu+\sigma), \quad \{\nu=0, 1, 2, \cdots, n\},$$

σ being an integer such that $0 \leqq \sigma \leqq \nu$.

$$b)\quad \lambda_\nu = \max_{\lambda+\mu=\nu}(\alpha_\lambda+\beta_\mu), \quad \{\nu=0, 1, 2, \cdots, n\},$$

if $\alpha_\lambda+\beta_\mu$ attains its maximum M_ν (λ and μ varying so as to satisfy $\lambda+\mu=\nu$) for a single pair of values of λ and μ, and if $M_\nu > \alpha_\lambda+\beta_\mu+\sigma$ for $\lambda+\mu+\sigma=\nu$ and $\sigma>0$.

It is to be noted that I implies II and that II implies III. *Hence every theorem proved for all* $\mathfrak{S}^{\text{III}}$ (*i.e., for all* $\mathfrak{S}$ *having property* III) *is valid for every* $\mathfrak{S}^{\text{II}}$ *and* $\mathfrak{S}^{\text{I}}$; *and every theorem proved for all* $\mathfrak{S}^{\text{II}}$ *holds for every* $\mathfrak{S}^{\text{I}}$.

We shall now describe various important situations where I or II or III holds. For this purpose, we must define in each case the *aggregate* $\mathfrak{S}$, the *parentheses* of $\mathfrak{S}$, the *product* of two parentheses and the *rank* of every element of $\mathfrak{S}$.

I *holds in the following situations* (1–7 *incl.*):

1. $\mathfrak{S}$ consists of the set of rational numbers. We understand by the parenthesis $(e_0, e_1, e_2, \cdots, e_m)$ of $\mathfrak{S}$ the rational polynomial $e_0y^m+e_1y^{m-1}+\cdots+e_m$ in the letter y. The product of two parentheses $(a_0, a_1, \cdots, a_r)\cdot(b_0, b_1, \cdots, b_s)$ is defined, as usual, to be equal to the parenthesis $(c_0, c_1, \cdots, c_n)$, where $n=r+s$ and $c_0=a_0b_0$, $c_1=a_0b_1+a_1b_0$, $\cdots$, $c_n=a_rb_s$. The rank of an element $e=e'/e''$ (where e' and e'' are integers) is defined with reference to a fixed prime p. First let $e \neq 0$; let e' be divisible by $p^{d'}$ but not by $p^{d'+1}$; e'', by $p^{d''}$ but not by $p^{d''+1}$. We define the rank of e by the equation $\eta=d''-d'$. Moreover, we (naturally) define the rank of 0 to be $-\infty$.

2. $\mathfrak{S}$ consists of the class of the Hensel p-adic numbers. The parentheses of $\mathfrak{S}$ and the product of two parentheses are defined as in 1. The rank of the p-adic number e is the negative of Hensel's[8] 'order' of e with respect to p.

3. $\mathfrak{S}$ consists of the class of rational fractions $e=e'(x_1, \cdots, x_\kappa)/e''(x_1, \cdots, x_k)$ in k letters $x_1, x_2, \cdots, x_k$, e' and e'' being polynomials with

arbitrary complex numerical coefficients.[9] The parentheses and the product of two parentheses are defined as in 1. The rank η of an element e ($\neq 0$) is defined as $d'-d''$, where d' and d'' represent respectively the degrees (in the usual sense) of e' and e'' in the k letters $x_1, \cdots, x_k$. The rank of 0 is defined to be $-\infty$.

4. $\mathfrak{S}$ consists of the collectivity of elements $e = \sum_{\lambda=0}^{\infty} c_\lambda x^{\eta-\lambda}$, where the c's are arbitrary complex numbers, that is, e is a Laurent series having only a finite number of terms with positive exponents.[10] The parentheses and the products of parentheses are defined as in 1. The rank of e ($\neq 0$) is defined to be the exponent of x in the first non-zero term of the development of e. The rank of 0 is (naturally) defined to be $-\infty$.

5. $\mathfrak{S}$ consists of the collectivity of fractions $e = e'/e''$, where e' and e'' are power series in k letters $x_1, x_2, \cdots, x_k$ with arbitrary complex coefficients:

$$e' = \sum_{\lambda_1, \lambda_2, \cdots, \lambda_k = 0}^{\infty} c'_{\lambda_1, \lambda_2, \cdots, \lambda_k} x_1^{\lambda_1} x_2^{\lambda_2} \cdots x_k^{\lambda_k}, \qquad e'' = \sum_{\lambda_1, \lambda_2, \cdots, \lambda_k = 0}^{\infty} c''_{\lambda_1, \lambda_2, \cdots, \lambda_k} x_1^{\lambda_1} x_2^{\lambda_2} \cdots x_k^{\lambda_k}.$$

The rank of e ($\neq 0$) is defined as $d''-d'$, where d' and d'' represent the lowest degrees (in the usual sense) in the k letters $x_1, x_2, \cdots, x_k$ of a non-zero term of e' and e'' respectively. The rank of 0 is $-\infty$.

6. $\mathfrak{S}$ consists of all rational fractions $e(x) = e'(x)/e''(x)$ in x ($e'(x)$ and $e''(x)$ being polynomials) with arbitrary complex coefficients. The parenthesis $(e_0, e_1, \cdots, e_m)$ of $\mathfrak{S}$ is the linear homogeneous difference expression $e_0 y_{x+m} + e_1 y_{x+m-1} + \cdots + e_m y_x$. The product $A \cdot B = (a_0, a_1, \cdots, a_r) \cdot (b_0, b_1, \cdots, b_s)$ of two parentheses of $\mathfrak{S}$ is the ordinary *symbolic* product of the linear difference expressions A and B and is equal to $(c_0, c_1, \cdots, c_n)$ where $n = r + s$ and

$$c_\nu = \sum_{\lambda=0}^{\nu} a_\lambda(x)\, b_{\nu-\lambda}(x+r-\lambda), \qquad \{\nu = 0, 1, 2, \cdots, n\}.$$

The rank of $e(x)$ is what is ordinarily called the degree of $e(x)$, i.e., $d'-d''$, where $d' =$ degree of $e'(x)$ and $d'' =$ degree of $e''(x)$.

7. $\mathfrak{S}$ and rank are defined as in 4; parentheses and products of parentheses, as in 6.

II *holds in the following situations* (8–9 *incl.*):

8. $\mathfrak{S}$ and rank are defined as in 6. The parenthesis $(e_0, e_1, \cdots, e_m)$ of $\mathfrak{S}$ is the linear homogeneous differential expression

$$e_0(x) \frac{d^m y}{dx^m} + e_1(x) \frac{d^{m-1} y}{dx^{m-1}} + \cdots + e_m(x)\, y.$$

The product $A \cdot B = (a_0, a_1, \cdots, a_r) \cdot (b_0, b_1, \cdots, b_s)$ of two parentheses is the ordinary *symbolic* product of the linear differential expressions A and B and is equal to $(c_0, c_1, \cdots, c_n)$, where $n = r + s$ and

$$c_\nu = \sum_{\mu=0}^{\nu} \sum_{\lambda=0}^{\nu-\mu} \binom{r-\lambda}{\nu-\mu-\lambda} a_\lambda \frac{d^{\nu-\mu-\lambda} b_\mu}{dx^{\nu-\mu-\lambda}},$$

$$\left\{ \binom{r-\lambda}{\nu-\mu-\lambda} = \frac{(r-\lambda)!}{(\nu-\mu-\lambda)!\,(r-\nu+\mu)!} \right\}.$$

9. $\mathfrak{S}$ and rank are defined as in 4; parentheses and products of parentheses, as in 8.

III *holds in the following situation*:

10. $\mathfrak{S}$ consists of the collectivity of fractions $e = e'/e''$, where e' and e'' are power series in $x-h$ ($=x_1$, cf. 5) with arbitrary complex coefficients. The rank of e is defined as in 5. The parentheses of $\mathfrak{S}$ and products of parentheses are defined as in 8.

We shall now proceed to the statement of certain consequences of the assumptions made that relate to factorization properties of a given parenthesis $C = (c_0, c_1, \cdots, c_n)$ of $\mathfrak{S}$. We base all further considerations on the tentative assumption that C may be expressed as a product $A \cdot B = (a_0, a_1, \cdots, a_r)\,(b_0, b_1, \cdots, b_s)$ of two parentheses, where $r \geqq 1$ and $s \geqq 1$. When other assumptions to be made in the theorems contradict this assumption of 'reducibility,' it must be that C is under those later assumptions incapable of being expressed as a product of two parentheses whose orders are at least 1. C is then said to be 'irreducible.' (In general, the terminology employed for our abstract situation is parallel to that for the ordinary concrete situations.) *We assume furthermore throughout in what follows that* γ_0, *the rank of* c_0, *is finite* (i.e., $\neq -\infty$). We introduce the following notations (partly for the purpose of indicating our method of investigation and partly for the purpose of simplifying certain future statements):

$$\begin{cases} \Delta_0 = \alpha_0 - \alpha_0 = 0, \quad \Delta_1 = \alpha_1 - \alpha_0, \cdots, \quad \Delta_r = \alpha_r - \alpha_0. \\ \Delta'_0 = \beta_0 - \beta_0 = 0, \quad \Delta'_1 = \beta_1 - \beta_0, \cdots, \quad \Delta'_s = \beta_s - \beta_0. \end{cases}$$

$$\begin{cases} G = \max_{1 \leqq \nu \leqq r} (\Delta_\nu);\ \Delta_\tau = \text{last } \Delta_\nu\ \{\nu = 1, 2, \cdots, r\} \text{ equal to } G. \\ G' = \max_{1 \leqq \nu \leqq s} (\Delta'_\nu);\ \Delta'_{\tau'} = \text{last } \Delta'_\nu\ \{\nu = 1, 2, \cdots, s\} \text{ equal to } G'. \end{cases}$$

$$\begin{cases} M = \max_{1 \leqq \nu \leqq r} \left(\frac{\Delta_\nu}{\nu}\right);\ \frac{\Delta_t}{t} = \text{last } \frac{\Delta_\nu}{\nu}\ \{\nu = 1, 2, \cdots, r\} \text{ equal to } M. \\ M' = \max_{1 \leqq \nu \leqq s} \left(\frac{\Delta'_\nu}{\nu}\right);\ \frac{\Delta'_{t'}}{t'} = \text{last } \frac{\Delta'_\nu}{\nu}\ \{\nu = 1, 2, \cdots, s\} \text{ equal to } M'. \end{cases}$$

The following results (lemmas[11] I–VI *incl., theorems* I–X *incl.) are valid for every* $\mathfrak{S}^{\text{II}}$:

Lemma I. *If* $G \leqq 0$ *and* $G' \leqq 0$, $\gamma_\nu - \gamma_0 \leqq 0$ *for every* ν.

Lemma II. *If* $G \geqq 0$ *and* $G' \geqq 0$, $\gamma_{\tau+\tau'} - \gamma_0 = G + G' \geqq \gamma_\nu - \gamma_0$ *for every* ν.

Lemma III. *If* $M > M'$ *and* $M > 0$ $(M' > M$ *and* $M' > 0)$, $M = \dfrac{\gamma_t - \gamma_0}{t} \geqq \dfrac{\gamma_\nu - \gamma_0}{\nu}\left(M' = \dfrac{\gamma_{t'} - \gamma_0}{t'} \geqq \dfrac{\gamma_\nu - \gamma_0}{\nu}\right)$ *for every* $\nu \neq 0$, *the* $>$ *sign alone holding when* $\nu > t$ $(\nu > t')$.

Lemma IV. *If* $M > M' \geqq 0$ $(M' > M \geqq 0)$, *there is no fixed* k *such that* $\gamma_k > \gamma_\nu$ *and* $\dfrac{\gamma_k - \gamma_0}{k} \geqq \dfrac{\gamma_\nu - \gamma_0}{\nu}$ *for every* $\nu \neq 0, k$.

Lemma V. *If* $M > M' > 0$ $(M' > M > 0)$ *there is no fixed* k *such that* $\gamma_k \geqq \gamma_\nu$ *and* $\dfrac{\gamma_k - \gamma_0}{k} \geqq \dfrac{\gamma_\nu - \gamma_0}{\nu}$ *for every* $\nu > 0$.

Lemma VI. *If* $M = M' > 0$, $\dfrac{\gamma_{t+t'} - \gamma_0}{t+t'} = M = M' \left(= \dfrac{\Delta_t}{t} = \dfrac{\Delta'_{t'}}{t'}\right) \geqq \dfrac{\gamma_\nu - \gamma_0}{\nu}$ *for every* $\nu > 0$, *the* $>$ *sign alone holding when* $\nu > t + t'$.

Theorem I. *If, for a fixed* k, $\gamma_k - \gamma_0 > 0$ and $\dfrac{\gamma_k - \gamma_0}{k} \geqq \dfrac{\gamma_\nu - \gamma_0}{\nu}$ $\{\nu = 1, 2, \cdots, n\}$, *at least one of the following* $n - k + 1$ *congruences holds:*

$$\gamma_k - \gamma_0 \equiv 0 \left(\mathrm{mod}\, \frac{k}{(k, \sigma)}\right),$$

where σ *takes the successive values* $k - r, k - r + 1, \cdots, n - r$ *(or* $k - s, k - s + 1, \cdots, n - s$*) and* (k, σ) *represents, as usual, the G.C.D. of* k *and* σ.

Theorem II. *If, for a fixed* k, $\gamma_k - \gamma_0 > 0$, $\dfrac{\gamma_k - \gamma_0}{k} \geqq \dfrac{\gamma_\nu - \gamma_0}{\nu}$ $\{\nu = 1, 2, \cdots, n\}$ *and* $(\gamma_k - \gamma_0, k) = 1$, *the parenthesis C contains an irreducible factor of order* $\geqq k$.

Theorem III (a special case of I). *If* $\gamma_n - \gamma_0 > 0$ *and* $\dfrac{\gamma_n - \gamma_0}{n} \geqq \dfrac{\gamma_\nu - \gamma_0}{\nu}$ $\{\nu = 1, 2, \cdots, n - 1\}$, $\gamma_n - \gamma_0 \equiv 0 \left(\mathrm{mod}\, \dfrac{n}{(n, r)}\right)$.

Theorem IV (a special case of III). *If* $\gamma_n - \gamma_0 > 0$, $\dfrac{\gamma_n - \gamma_0}{n} \geqq \dfrac{\gamma_\nu - \gamma_0}{\nu}$ $\{\nu = 1, 2, \cdots, n - 1\}$ *and* $(\gamma_n - \gamma_0, n) = 1$, *C is irreducible.*

Theorems V–X inclusive materially extend and generalize Perron's theorem, *J. Math., Berlin,* **132,** 304 (1907), which Perron designates as 'ein sehr allgemeines Kriterium.'

Theorem V. *If* $G \geqq 0$, $G' \geqq 0$, $\gamma_k > \gamma_\nu$ *for every* $\nu \neq k$ *and* $\dfrac{\gamma_k - \gamma_0}{k} \geqq \dfrac{\gamma_\nu - \gamma_0}{\nu}$ *for every* $\nu \neq 0$, *the following relations hold:*

$$\begin{cases} 0 < M = M' = \dfrac{\gamma_k - \gamma_0}{k}\left(= \dfrac{\Delta_t}{t} = \dfrac{\Delta'_{t'}}{t'} \right); \\ k = t + t';\ t = \tau,\ t' = \tau';\ (\Delta_t = G;\ \Delta_{t'} = G'). \end{cases}$$

Theorem VI. *If* $G \geqq 0$, $G' \geqq 0$, $\gamma_k > \gamma_\nu$ *for every* $\nu \neq k$, $\dfrac{\gamma_k - \gamma_0}{k} > \dfrac{\gamma_\nu - \gamma_0}{\nu}$ *for every* $\nu \neq 0, k$, *and* $(\gamma_k - \gamma_0, k) = 1$, *C is irreducible.*

Theorem VII. *If* $G > 0$, $G' > 0$, $\gamma_k \geqq \gamma_\nu$ *for every* ν *and* $\dfrac{\gamma_k - \gamma_0}{k} \geqq \dfrac{\gamma_\nu - \gamma_0}{\nu}$ *for every* $\nu \neq 0$, *the same conclusions may be drawn as in theorem* V *with the possible exception of* $t = \tau$ *and* $t' = \tau'$.

Theorem VIII. *If* $G > 0$, $G' > 0$, $\gamma_k \geqq \gamma_\nu$ *for every* ν, $\dfrac{\gamma_k - \gamma_0}{k} > \dfrac{\gamma_\nu - \gamma_0}{\nu}$ *for every* $\nu \neq 0, k$, *and* $(\gamma_k - \gamma_0, k) = 1$, *C is irreducible.*

Theorem IX (generalizes VI). *If* $G \geqq 0$, $G' \geqq 0$, $\gamma_k > \gamma_\nu$ *for every* $\nu \neq k$, $\dfrac{\gamma_k - \gamma_0}{k} \geqq \dfrac{\gamma_\nu - \gamma_0}{\nu}$ *for every* $\nu \neq 0$, *and if furthermore every parenthesis* $(e_0, e_1, \cdots, e_m)$ *that may occur as a factor of C is such that* $\eta_\nu \geqq \eta_0$ *for at least one* $\nu > 0$, *every decomposition of C into a product of parentheses contains at most* $(\gamma_k - \gamma_0, k)$ *factors.*

Theorem X (generalizes VIII). *If* $G > 0$, $G' > 0$, $\gamma_k \geqq \gamma_\nu$ *for every* ν, $\dfrac{\gamma_k - \gamma_0}{k} \geqq \dfrac{\gamma_\nu - \gamma_0}{\nu}$ *for every* $\nu \neq 0$, *and if furthermore every parenthesis* $(e_0, e_1, \cdots, e_m)$ *that may occur as a factor of C is such that* $\eta_\nu > \eta_0$ *for at least one* ν, *every decomposition of C into a product of parentheses contains at most* $(\gamma_k - \gamma_0, k)$ *factors.*

The lemmas leading up to theorems XI and XII are omitted.

Theorem XI. *Theorem* I *holds for every* $\mathfrak{S}^{\mathrm{I}}$ *if the inequality* $\gamma_k - \gamma_0 > 0$ *is replaced by* $\gamma_k - \gamma_0 < 0$.

Theorem XII. *Theorem* I *holds for every* $\mathfrak{S}^{\mathrm{III}}$ *if the inequality* $\gamma_k - \gamma_0 > 0$ *is replaced by* $\dfrac{\gamma_k - \gamma_0}{k} > 1$.

The results here outlined will be offered for publication *in extenso* to the *Transactions of the American Mathematical Society.*

[1] Gauss, *Disquisitiones arithmeticae* (1801) Art. 341; Kronecker, *J. Math., Berlin,* **29**, 280 (1845), **100**, 79 (1887), *J. Math., Paris,* **19**, 177 (1854) and ser. 2, **1**, 399 (1856); Schoenemann, *J. Math., Berlin,* **32**, 100 (1846) and **40**, 188 (1850); Eisenstein, *J. Math., Berlin,* **39**, 160 (1850); Dedekind, *J. Math., Berlin,* **54**, 27 (1857); Floquet, *Ann. Sci. Ec. norm., Paris,* (1879), cited by Koenigsberger; Koenigsberger, *J. Math., Berlin,* **115**, 53 (1895), **121**, 320 (1900), *Math. Ann., Leipzig,* **53**, 49 (1900); Netto, *Math. Ann.,* **48**, 81 (1897); Perron, *Math. Ann.,* **60**, 448 (1905), *J. Math., Berlin,* 132, 288 (1907); M. Bauer, *J. Math., Berlin,* 128, 87 and 298, (1905), **132**, 21 (1907), **134**, 15 (1908); Dumas, *J. Math., Paris,* ser. 6, **2**, 191 (1906).

[2] Thus the work on the subject before the publication of the Schoenemann-Eisenstein theorem is summarized and generalized by that theorem, which, as previously indicated, is a very special case of theorem IV. In fact, theorem I alone, for example, includes as special cases a great bulk of the results heretofore published, and, in particular, the following: the theorem of Floquet—quoted by Koenigsberger—, all the results contained in the 1895 paper of Koenigsberger, nearly all contained in his 1900 *Math. Ann.* paper, almost all the results of Netto and almost all the results in the 1905 paper of Perron. Cf. also the paragraph preceding the statement of theorem V.

[3] Perron, *J. Math., Berlin,* **132**, 288, and Loewy, in Pascal's *Repertorium* (1910) Analysis I_1, 292 and 293.

[4] I may perhaps be permitted to remark, for the purpose of indicating that the results given are less artificial than one would at first suppose, that I had obtained my chief results with little knowledge of the literature. Their intimate connection with results already obtained points to a degree of 'naturalness' of these results that one would hardly attribute to them in the absence of such a connection.

[5] As a matter of fact, two distinct theorems obtained may be properly regarded as (highly) generalized Schoenemann-Eisenstein theorems for the case of linear homogeneous differential expressions: theorem I for situation 8 and theorem XII for situation 10. Curiously, Koenigsberger himself has obtained theorems—special cases of XII—for differential expressions that may be properly regarded as generalized Schoenemann-Eisenstein theorems, without his having noticed this relation.

[6] It is possible to build up just as general a theory as ours by dealing exclusively with 'parentheses' whose elements are 'ranks'—see below in the same paragraph—and hence always integers or $-\infty$. The reader who prefers a more concrete, tho necessarily less general, discussion may, for example, at once interpret $\mathfrak{S}$, '*parenthesis*,' '*product*' and '*rank*'—see below in the same paragraph—as in situation 8. In that case, II will hold.

[7] In general, we denote the rank of an element represented by a Latin letter by the corresponding Greek letter. By $\underset{\lambda+\mu=\nu}{max}(\alpha_\lambda+\beta_\mu)$, $\underset{\lambda+\mu\leqq\nu}{max}(\alpha_\lambda+\beta_\mu)$, $\underset{\lambda+\mu+\sigma=\nu}{max}(\alpha_\lambda+\beta_\mu+\sigma)$—see II and III below—we naturally understand the largest value attained by the numbers of the set $\alpha_\lambda+\beta_\mu$, $\alpha_\lambda+\beta_\mu$, $\alpha_\lambda+\beta_\mu+\sigma$, λ, μ and σ varying so as to satisfy the relations $\lambda+\mu=\nu$, $\lambda+\mu\leqq\nu$, $\lambda+\mu+\sigma=\nu$, and in addition, of course, $0\leqq\lambda\leqq r$ and $0\leqq\mu\leqq s$.

[8] Hensel, *J. Math., Berlin,* **127**, 51–84, §2 (1904) or Zahlentheorie (1913), chaps. 3 and 6.

[9] More generally, the numerical coefficients may belong to any abstract system $(K, +, \times)$, where K is a class, such that if a and b are elements of K both $a+b$ and $a\times b$ are elements of K. This remark applies just as well to the numerical coefficients in situations 4–10 incl.

[10] The question of convergence does not enter here because the formal character of the series is sufficient for our purpose. More generally, we may have such series in two or more variables, the required change in the definition of rank being evident.

[11] These lemmas lead up to the theorems I–X and are given partly for the purpose of indicating the nature of the proofs and partly because they are believed to be of interest in themselves.

THE DIRECTION OF ROTATION OF SUN-SPOT VORTICES

By George E. Hale

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Read before the Academy, December 8, 1914. Received, April 15, 1915

Whirling storms in the earth's atmosphere, from the extensive and moderate cyclone to the small and destructive tornado, follow a well-known law of rotation: right-handed or clockwise in the southern hemisphere and counter-clockwise in the northern. These directions are easily explained by the increasing eastward velocity of the air from pole to equator. As sun-spots are vortex phenomena, analogous in many respects to tornadoes, it is interesting to inquire as to the law of their rotation, since this may throw light on their nature and origin.

It is a well-known fact that when a normal Zeeman triplet is observed along the lines of force of a magnetic field, the central component is absent, and the two side components are circularly polarized in opposite directions. A quarter-wave plate and Nicol prism mounted over the slit of the spectroscope permit either component to be cut off at will. Furthermore, if the polarizing apparatus be adjusted so as to extinguish one component, reversal of the current through the coils of the magnet will cause this component to reappear, while the other will be extinguished. The method thus offers a simple means of determining the polarity of a magnetic field, which can still be applied when the angle between the line of sight and the lines of force is as great as 60° or 70°. In this case, however, the central line of the triplet is present, and the elliptically polarized light of the side components can be only partially extinguished.

Every sun-spot exhibits a magnetic field, whose polarity is determined by the direction of rotation of the electrons in the spot vortex. To learn the polarity, and hence the direction of rotation, it is therefore only necessary to observe whether the red or the violet component of a spot triplet is transmitted by the polarizing apparatus.

In my earlier work attention was concentrated on a few of the largest spots, in order to secure unquestionable evidence as to the existence of the Zeeman effect. The possibility of finding a law of rotation appeared to be remote, as spots of opposite polarity were observed in the same hemisphere. Subsequently, when the characteristics of bipolar spot groups had been discovered, the search for a rotation law was renewed, with the results given in this paper.

The typical sun-spot group consists of two spots of opposite magnetic polarity, lying on a line which usually makes only a small angle with the solar equator. One of the spots may be replaced by several smaller ones, or even be absent altogether, its place in the latter case being indicated by calcium and hydrogen flocculi. Small companion spots, of either polarity, may attend either member of the pair. The characteristic feature of the group is the presence of magnetic fields, of opposite polarity, in the regions including its eastern and western extremities.

Deferring until another occasion a discussion of the probable origin of bipolar groups, it will suffice for present purposes to emphasize the necessity of recognizing the existence and importance of such pairs. The following quotation from a valuable paper by Father Cortie, based upon a study of some 3500 drawings of spots made at Stonyhurst College Observatory, is of special significance in this connection:

> The chief type, however, of which the above mentioned are in most, possibly in all, cases but phases, is the double spot formation, with a train of smaller spots between the two principal spots of the group, the whole group generally drifting into more or less parallelism with the solar equator. In this form the principal spot, which eventually becomes a normal spot of regular outline, is generally the leading spot, but in many cases it is the following spot, while sometimes the preponderance in area alternates between the two, as the group traverses the disk. In yet rarer instances both the chief spots develop as regular spots. (*Astrophys. J.*, **13.** 261, 1901.)

As the preceding (western) spot of a bipolar group usually lasts longer than the following spot, and is likely to be the larger of the two, we may take it as the dominant member of the group, and classify the polarities of northern and southern spots accordingly. Single spots, followed by a train of flocculi, are classified in Table 1 as preceding spots. Under the headings V and R are given the number of spots for which the "marked strip" of the compound quarter-wave plate transmitted the violet or red component respectively. The spots of the old and new cycles (observed before and after the recent sun-spot minimum) are tabulated separately.

TABLE 1

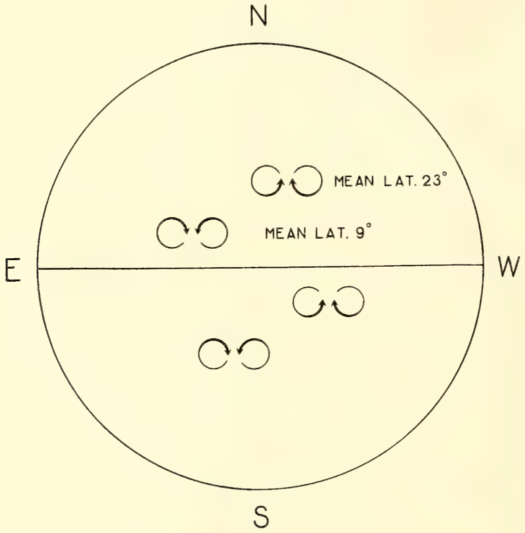
HEMISPHERE	OLD CYCLE				NEW CYCLE			
	Pr. Spots		Foll. Spots		Pr. Spots		Foll. Spots	
	V	R	V	R	V	R	V	R
N	5	0	0	1	0	13	10	0
S	2	16	8	0	9	1	1	7

TABLE 2

	OLD CYCLE		NEW CYCLE	
	N	S	N	S
Mean ϕ and No. of spots	10° (7)	9° (22)	22° (11)	26° (6)
Weighted Mean	9.0° (29)		23.2° (17)	

From this summary it appears that the five preceding spots observed in the northern hemisphere before the minimum were characterized by the violet component, while the single following spot gave the red component. In the southern hemisphere, with two exceptions, the polarities were of the opposite sign.

After the minimum, to my surprise, the polarities were found to be reversed in both hemispheres, as the tabulated results for the new cycle indicate. The explanation of this fact probably lies in the change of latitude which a new cycle introduces. While the last spots of the old cycle appear in low latitudes, the first spots of the new cycle occur much farther from the equator. Thus we apparently have on the sun two spot zones, in which the great majority of spot vortices rotate in opposite directions. The approximate mean latitudes of these zones are shown in Table 2.



From the best supplementary evidence at present available, the true direction of rotation of a preceding spot vortex in the low latitude zone is counter-clockwise, corresponding to that of a terrestrial tornado. The opposite direction obtains in the southern hemisphere, as on the earth. But in high latitudes the signs are reversed—a result which is likely to prove significant in future studies of the sun (see figure). The complete details of this investigation will be published in the *Astrophysical Journal.*

SOME VORTEX EXPERIMENTS BEARING ON THE NATURE OF SUN-SPOTS AND FLOCCULI

By George E. Hale and George P. Luckey

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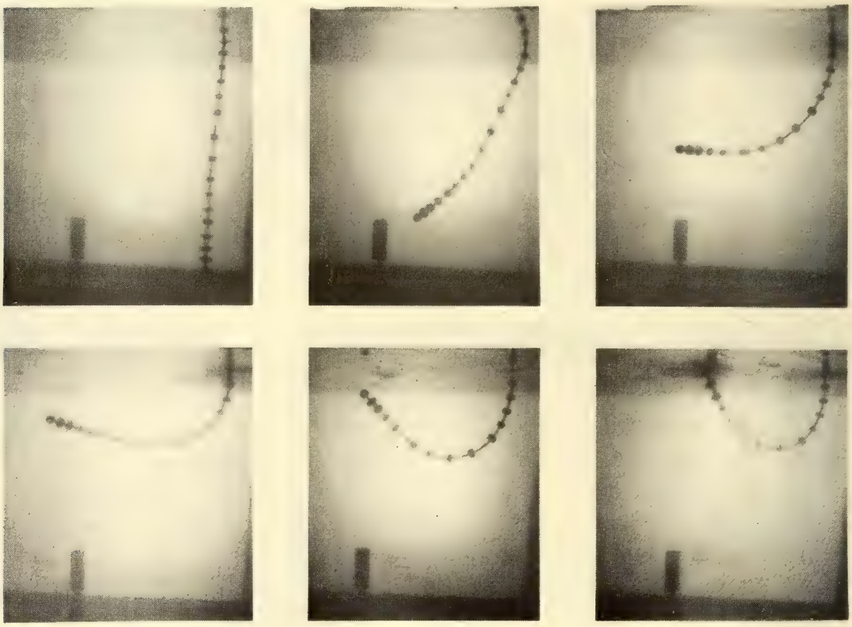
Read before the Academy, April 20, 1915. Received, May 22, 1915

The following working hypothesis of sun-spots was first proposed, in a somewhat different form, in the Annual Report of the Director of the Mount Wilson Solar Observatory for 1912:

> As the result of an eruption, or some other cause tending to produce rapid convection, a gaseous column moves upward from within the sun toward the surface of the photosphere. Vortex motion is initiated by differences in velocity of adjoining surfaces or by irregularities of structure and is maintained by convection. The circulation in the vortex is vertically upward and then outward along the photosphere, as in a terrestrial tornado. Expansion produces cooling at the center of the vortex, and a comparatively dark area (the umbra) results. As in Harker's electric furnace experiments, a rapid flow of negative ions sets in toward the cooler gases at the center from the hotter gases without. These ions, whirled in the vortex, produce a magnetic field, plainly indicated by the widened and resolved lines of the sun-spot spectrum.
>
> Hitherto we have assumed the spot to be single. The typical sun-spot group is double, though either or both of the principal members may be replaced by several smaller spots. The axis of the group makes a small angle with the solar equator, and the magnetic polarities of the preceding and following members are opposite in sign. Such a group may result from the tendency of any columnar vortex to turn up toward the photosphere, thus forming a semi-circular vortex ring.
>
> A secondary vortex, caused by the influence of the low-lying spot vortex on the gases of the solar atmosphere above it, partially determines the structure of the hydrogen flocculi, which are drawn inward at high levels, then downward and outward at the upper levels of the spot vortex. The hydrodynamic structure thus developed may be more or less modified by the influence of the spot magnetic field on the trajectories of the moving ions within it.

The presence of magnetic fields in all sun-spots and the spectroscopic observations of Evershed and St. John on the revolution of the spot vapors at various levels, seem to prove beyond doubt that sun-spots are vortices. The object of the present paper is to describe some recent experiments bearing on the suggested explanation of double or multiple spots, and the flocculi associated with them.

A closely wound helix of brass wire, with disks of wood threaded on it to increase friction, is hung vertically from the shaft of an electric motor, and spun at moderate velocity in water. A columnar vortex is thus set up with the helix as a core, which will remain nearly vertical as long as its angular velocity is low. But if the speed of the motor is increased, the lower end of the helix will gradually move toward the surface, giving rise to the successive forms shown[1] in figures 1 to 6. The semi-circular vortex ring finally produced, like an ordinary smoke ring, will have a motion of translation at right angles to its plane, in a direction corresponding to the direction of rotation of its inner edge. It is interesting to observe the depressions and whirls in the water when the end of the helix is approaching the surface. These represent in the hypothesis the incipient spots which often appear and disappear near one extremity of a bipolar group.



An instructive control of this experiment may be readily applied. The water in the tank is set into rotation (by an auxiliary paddle) about the axis of the helix before the motor is started. After the helix is set spinning it will turn up much more rapidly than in quiet water if the direction of the preliminary rotation is opposed to that of spin, i.e., if the water is whirling in the same direction as the second 'spot' of the group. If, however, the whole body of water is rotating in the same direction as the helix, the latter will turn up with difficulty, or remain in the vertical position without turning up at all.

Thus a sun-spot vortex, if produced within a region already rotating in the same direction, would tend to retain a columnar form, and the second spot of the group would not appear. Other conditions favorable to the production of single spots are low angular velocity of the vortex (similar in effect to the above case), a large ratio of diameter to length, and the presence within the sun of persistent eruptive centers, such as are suggested by the frequent recurrence of spots in the same region of the photosphere. It should be noted that convection may be a factor in prolonging the life of single spots.

If bipolar spots are semi-circular vortices, they should exhibit the proper motion characteristic of vortex rings. This motion, as already remarked, is at right angles to the plane of the ring, and in a direction corresponding with that of its inner edge. In a paper read before the Academy at its last autumn meeting,[2] it was shown that high and low latitude bipolar spot groups rotate in opposite directions. These directions are such that low latitude groups should move toward the equator, while high latitude groups should move toward the poles.

An investigation of the motion of spots in latitude was made many years ago by Carrington, who says:

> In the above table it will be remarked that there is more distinctly a trace of motion in latitude, the signs being on the whole + for latitudes higher N. or S. than 20°, though the daily polar motion between 20° and 40° of latitude on an average does not exceed 2′, a quantity which could only be deduced from the totality of a large number of single results. Between the parallels of 10° to 20° the motion in latitude is evidently very small; but the signs are generally negative and a feeble tendency towards the equator of about 1′ per diem is indicated. Within 10° of the Equator on either side no reliable motion in latitude appears to exist, the signs varying much and the mean results being of less weight.[3]

These results relate to all spots, both single and multiple, whose motions were observed by Carrington. His conclusion as to the reversed directions of motion of high and low latitude spots has recently been confirmed by Dyson and Maunder, who discussed the extensive observational material comprised in the Greenwich sun-spot measures from 1874 to 1912. They remark:

"A slight general tendency is suggested for high-latitude spots in either hemisphere to move away from the equator, but for those in lower latitudes to move toward it."[4]

This apparent confirmation of the hypothesis should not be taken too seriously, as it remains to be seen whether a closer comparison, excluding single spots and grouping the results in strict conformity with the observed polarities of bipolar pairs, would be equally favorable.

The need of caution is emphasized by the fact that the observed daily motions in latitude appear to be considerably smaller than theory would indicate. We have verified Kelvin's approximate expression for the motion of translation of vortex rings[5] by means of a semi-circular vortex ring in water, with paddles which can be driven at angular velocities varying through a wide range. But it does not follow that the same expression can be applied under the very different conditions existing in the sun, and we are seeking further information regarding this point before attempting to complete the quantitative test.

We may now inquire whether experimental methods will not throw some light on the difficult problem of the structure of the hydrogen flocculi. In 1908 we found that sun-spots are centers of vortex phenomena, plainly shown by the spectroheliograph when the sun is photographed with the light of the red line ($H\alpha$) of hydrogen. It was subsequenly proved by Evershed and St. John, however, that these are secondary vortices, extending down through the solar atmosphere from the hydrogen level, where their stream-lines are visible, toward an underlying vortex which constitutes the spot and produces its magnetic field. Are these secondary vortices of hydrodynamic origin, as St. John has maintained, or are they due to the effect of the magnetic field on the trajectories of the electrically charged particles in the solar atmosphere? The latter view, implied in Brester's aurora hypothesis of the flocculi and more clearly stated by Deslandres, has been fully developed mathematically by Störmer. Undoubtedly both hydrodynamic and electromagnetic phenomena must influence the structure of the solar atmosphere, but a satisfactory means of distinguishing between the two has been lacking.

The structure of the hydrogen flocculi shown on spectroheliograph plates can be more or less closely imitated with the aid of smoke in a closed glass box above vortices in water, and photographs of the stream-lines at different levels are easily obtained. The most interesting case we have yet observed is that produced when two paddles, rotating in opposite directions in water, set up a surface circulation similar to that in a bipolar spot group. The stream-lines in the smoke, seen when looking vertically downward toward the water, resemble those of a vortex ring of colored liquid rising through water from the orifice of a tube. When observed from a point in the plane of the ring, the stream-lines in the latter case appear to be closed on the upper or advancing side, while the surplus liquid on the following side moves along lines which are straight and axial opposite the center of the ring and become more and more convex on either side of this axis.[6] In the smoke similar stream-lines appear in horizontal planes, producing the effect of a hood or arch on one side of the line joining the two 'spots,' with the very different structure just described on the opposite or following side.

Such an appearance of asymmetry would seem to be faithfully reproduced on some of our photographs of the hydrogen flocculi surrounding bipolar spot groups. If this resemblance should prove to be other than superficial, it would apparently indicate that in these cases the hydrodynamic influence of the spot vortices outweighs their electromagnetic effect in determining the structure of the flocculi. It may be added that the inward (nearly horizontal) flow of the smoke at high levels, the downward flow at intermediate levels and the outward flow at low levels is in harmony with solar observations.[7] But no final conclusions can be drawn before the experiments have been multiplied, and the necessary magnetic influence introduced, if possible, by means of magnets within the liquid vortices, acting on the moving smoke particles, ionized by an X-ray discharge, and separated electrostatically. As other photographs of the hydrogen flocculi surrounding bipolar spots seem to show a more nearly symmetrical structure, resembling the lines of force about a bar magnet, it may be true that in such cases the electromagnetic influence predominates, since we have no evidence that one spot may be regarded as a source and the other as a sink. The possible effect of electric fields in and about sun-spots must also be considered.[8]

It has been shown in this paper that some of the phenomena of single and multiple sun-spots can be imitated by simple laboratory experiments, which may also assist in explaining certain characteristic structures and motions of the solar atmosphere. The full details of the investigation will be published in the *Astrophysical Journal.*

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[1] The vertical tube shown at the bottom of the tank, used for another experiment, has no effect on this result.

[2] See these PROCEEDINGS, **1**, 382 (1915).

[3] *Observations of Solar Spots*, p. 222.

[4] *London, Mon. Not. R. Ast. Soc.*, **73**, 687 (1913).

[5] Kelvin, *Mathematical and Physical Papers*, vol. 4, p. 67.

[6] See Mach, *Ann. Physik, Leipzig*, **68**, Plate III, fig. 13. A better resemblance is afforded by a photograph of a smoke ring by Wood, reproduced in *Nature*, **63**, 418, fig. 1.

[7] The stream-lines in different planes, either vertical or horizontal, are most clearly shown by a narrow beam of sunlight, admitted through the glass wall of the tank by means of a slit, which can be set in any desired position. In this way it is easy to imitate phenomena observed with the spectroheliograph in plan on the sun's disk and in elevation at the limb, where Slocum and others have photographed prominences while being drawn into sun-spots. Radial stream-lines in the horizontal plane, like those often indicated by the hydrogen flocculi about single spots, are well shown by the smoke at high levels.

[8] An error occurs in a paper by Hale and Babcock in a recent number of these PROCEEDINGS (March 1915, p. 125). Having in mind the lines of force between the separated charges in the vortex, the electric lines of force in the spot were described as tangential to the solar surface. This statement is more likely to apply only to the edges of the spot. At its center the lines of force are probably nearly radial.

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John Wesley Powell (1834–1902). By W. M. Davis. *Biographical Memoirs of the National Academy*, vol. **8**, pp. 1–83.

This Memoir discusses the life-work of its subject under the headings: Early Life; Service in the Civil War; Visit to the Rocky Mountains, 1867–1868; Exploration of the Colorado Canyon; Geological Survey of the Territories; Report on the Colorado Canyon; Antecedent Rivers; Geological Work; Physiographic Work; Baselevel of Erosion; Planation; Physiographic Essays; Lands of the Arid Region; the Geological Survey; Topographical Map; Reports and Folios; Irrigation Survey; Administration; Resignation from the Survey; Residence in Washington; Ethnological Work; Bureau of Ethnology; Indian Languages and Mythology; Savagery, Barbarism, and Civilization; Synthetic Essays; Mannerisms; Views on Evolution; Evolution of Music; Inductive Studies; Indian Linguistic Families; Philosophical Studies; Pentalogic Series; Truth and Error; Personal Estimates.

Charles Anthony Schott (1826–1901). By Cleveland Abbe. *Biographical Memoirs of the National Academy*, vol. **8**, pp. 87–133.

This Memoir is subdivided into sections entitled: Autobiography of the Early Life of Charles Anthony Schott; Bibliography; Hydrography and Office Work; the Computing Division; Longitude and Latitude; Chartography; Miscellaneous Observations; Terrestrial Magnetism; Climatology; Base-lines and Metric Standards; Geodesy; A Catalogue of the Works (Abridged Titles) of Charles Anthony Schott; Unpublished Works.

Miers Fisher Longstreth (1819–1891). By Rebecca C. Longstreth. *Biographical Memoirs of the National Academy*, vol. **8**, pp. 137–140.

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Volume 1 JULY 15, 1915 Number 7

NOVA GEMINORUM NO. 2 AS A WOLF-RAYET STAR

By Walter S. Adams and Francis G. Pease
MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

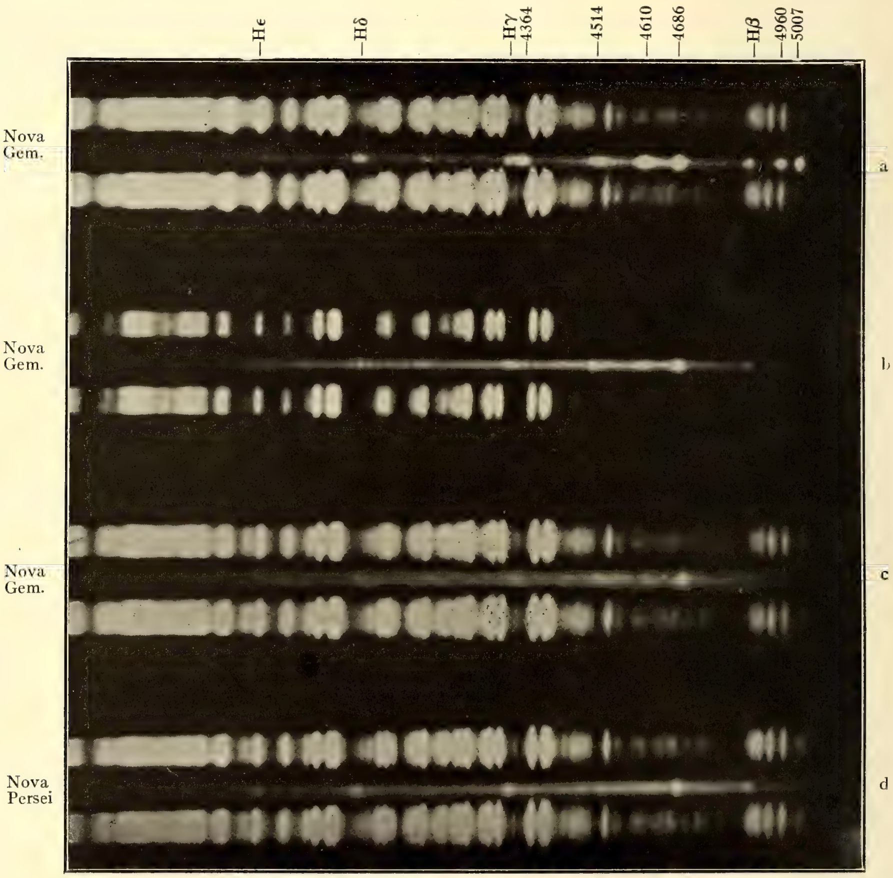
Presented to the Academy, June 7, 1915

It has been known for many years that the spectrum of temporary stars or Novae develops in a later stage into a spectrum nearly identical with that of planetary nebulae. First observed in the case of Nova Aurigae of 1891, the same result has been found to hold for all of the temporary stars observed since that time, including the most important Nova of recent years, Nova Persei of 1901. The question whether this nebular spectrum may be regarded as the final stage in the development of the spectra of temporary stars is of great importance because of its immediate bearing on the order of evolution of stars. In his book on Stellar Movements Eddington says in this connection:[1] "There is some justice in a remark of R. T. A. Innes: 'The fact that we have seen a star change into a nebula ought to outweigh every contrary speculation that stars originate from nebulae.' "

The faintness of the Novae in the later stages of their history has made observations of their spectra exceptionally difficult. In 1906, however, Hartmann[2] obtained an observation of Nova Persei, then of about the eleventh magnitude, which showed that two of the principal nebular lines had disappeared, and he called attention to the strong similarity of its spectrum to that of one of the Wolf-Rayet stars. Since that time the work has been taken up at Mount Wilson, with the 60-inch reflector and extended to include observations of other Novae.

Photographs of the spectrum of Nova Geminorum No. 2 (1912) taken during the past year have shown some peculiarly interesting developments in this most recent of the temporary stars. In February 1914, the spectrum consisted of prominent bright bands joined by a continuous spectrum of rather feeble intensity. These bands are those so conspicuous in the spectra of nebulae, the more important in order of brightness being λ 4686, 5007, 4610, 4364, 4960 and the hydrogen lines, $H\beta$, $H\gamma$, $H\delta$ and $H\epsilon$.

During the summer the star could not be observed on account of its position, but in November another photograph was secured. This showed some extraordinary changes. In addition to a great increase in the intensity of the continuous spectrum, the plate showed the disappearance of the chief nebular line at λ 5007 and the line so commonly associated with it at λ 4364, and a marked reduction in the intensity of λ 4610. The absence of λ 5007 is especially remarkable in view of the presence, although in reduced intensity, of the nebular line at λ 4960. These two lines, known as N_1 and N_2 of the nebular spectrum, without exception so far as we know, have always been observed together, and vary in intensity together, λ 5007 being about twice as strong as λ 4960.



Although no common origin has been established for these lines this peculiarity in behavior might be compared to the disappearance of D_2 in the sodium spectrum, while D_1 persisted in reduced intensity.

Another photograph of the spectrum of the star was obtained in April 1915. This showed that all of the characteristic nebular lines had disappeared, the line λ 4686 which is common to the spectra both of nebulae and Wolf-Rayet stars, and the hydrogen lines alone remaining. There are, however, traces of some of the Wolf-Rayet bands and the continuous spectrum is very strong.

In a communication published by us in 1914[3] we compared the spectrum as then observed of Nova Geminorum No. 2 with that of three of the temporary stars discovered in previous years. Two of these, Nova Aurigae (1891) and Nova Persei (1901), were found to have strong continuous spectra on which the hydrogen lines appeared as bright bands of moderate intensity. No characteristic nebular lines were present in their spectra, but λ 4686 was very prominent in Nova Persei, and present, though much less bright, in Nova Aurigae. The close agreement of these spectra and especially that of Nova Persei with the spectrum of some of the Wolf-Rayet stars was referred to in the course of the comparison. The interesting result now appears that the spectrum of Nova Geminorum No. 2 has also developed into that of a Wolf-Rayet star, being essentially identical with that of Nova Persei, although the hydrogen lines are somewhat weaker. Measurements of the negatives of Nova Persei and Nova Geminorum show the presence of the following Wolf-Rayet bands in addition to λ 4686 and the hydrogen lines:

Nova Persei	4021	4200	448	4514	454	461	4634
Nova Geminorum	402	4199	444	4512		461	469

In Plate I are shown direct enlargements of three spectra of Nova Geminorum No. 2 and one spectrum of Nova Persei. The enlargement is about 17 fold. The dates of the photographs are as follows:

	Date	*Exposure time*
Nova Geminorum (a)	1914, Feb. 22	2 hours
Nova Geminorum (b)	1914, Nov. 16	2 “
Nova Geminorum (c)	1915, Apr. 10–12	8 “
Nova Persei (d)	1913, Nov. 27	8 “

The photograph of April 10–12 was taken under very poor observing conditions.

The especial feature of interest attaching to these results is the fact that a continuous series of observations in the case of Nova Geminorum No. 2 has shown the development of the spectrum of this star through the successive stages characteristic of Novae into one very strongly resembling that of planetary nebulae; and then, by the gradual elimination of the nebular lines and their replacement by Wolf-Rayet bands, into a spectrum identical with this characteristic type of stellar spectra. It appears very probable from the observations of Nova Aurigae and Nova Persei that this stage of development is to be considered as in the nature of a permanent one. It is of interest to note, however, that the period required to reach this condition may be quite variable for different stars, since the nebular lines λ 5007 and λ 4960 were still by far the most prominent feature of the spectrum of Nova Lacertae (1910) as recently as the autumn of 1913. It seems entirely possible that the nebular lines found in the case of Novae are extraneous to the stars themselves and due to material in which they are involved. Of especial importance in this connection is the recent discovery by Wright[4] that the central star in certain planetary nebulae has the Wolf-Rayet type of spectrum.

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[1] *Stellar Movements and the Structure of the Universe,* p. 156.

[2] *Astr. Nachr. Kiel,* **177**, 113 (1908).

[3] *Contrib. Mt. Wilson Solar Obs.*, No. **87**; *Astrophys. J.*, **40**, 294 (1914).

[4] *Astrophys. J.*, **40**, 466 (1914).

A SINGULAR DARK MARKING ON THE SKY

By E. E. Barnard

YERKES OBSERVATORY, UNIVERSITY OF CHICAGO

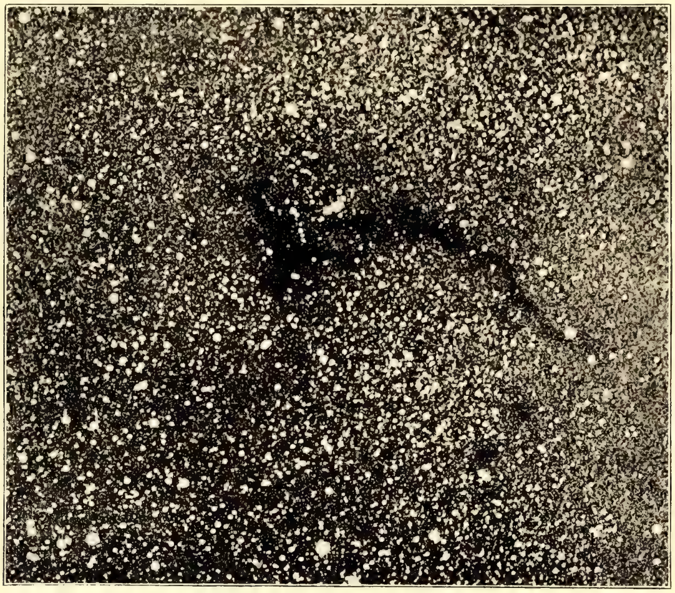
Read before the Academy, April 20, 1915. Received, June 4, 1915

In the search for dark or vacant regions in the sky I have photographed a number of very remarkable places, such as those near *ρ Ophiuchi* and *θ Ophiuchi*, and elsewhere.

These regions are generally connected with the presence of diffused nebulosity, but I have found several cases in which a well defined dark object seems superposed on the sky itself without any visible presence of ordinary nebulosity. One of these, and perhaps the most remarkable on account of its well defined character and singular form, is in the position: (1855.0)$\alpha = 20$ h. 48 m.; $\delta = +59°.6$, in Cepheus (see Plate). It resembles an irregular, roughly torn and curved opening through which a darker region is visible. It is one degree long and narrows down to something like 3′ across at its western end. It seems to be a part of the background of the sky independent of the starry stratum (there are no indications whatever of nebulosity immediately about it), and is apparently of a similar nature to the remarkable dark 'lanes' shown in my photographs of a region in *Taurus*. (See *Astrophys. J.*, **25**, 218, 1907). There are two very small dark spots near and south of the west end of the dark marking.

The accompanying plate is from a photograph which was taken by me on October 1, 1910, with the 10-inch Bruce telescope of the Yerkes Observatory, with an exposure of 6 h. 2 m. I had previously taken a photograph of this part of the sky on September 30, 1910, with an exposure of 5 h. 5 m. which verifies the above in all particulars.

NORTH



DARK OBJECT IN *CEPHEUS* ($\alpha=20^h\ 48^m$, $\delta=+60°$)
Scale: 1°=41mm.

There are two possible explanations of this object:

(1) That it is an opening in a widely diffused nebulous stratum.

(2) That it is an opaque, non-luminous object projected against space, which is itself luminous.

From this object and those in *Taurus*, I get the impression that the interstellar spaces (or possibly the regions beyond the stars), perhaps covering the entire heavens, are suffused with a feeble nebulosity that, with very prolonged exposures, affects the photographic plate; and that such phenomena (as the present one) are due to the projection upon this background of nearer, dark, opaque objects. If not this, then they are rifts or openings in a luminous background which look out into the blackness of space beyond. From investigations elsewhere in the sky, I lean towards the idea that these are relatively non-luminous, opaque bodies, seen against a luminous background.

THE RULING AND PERFORMANCE OF A TEN-INCH DIFFRACTION GRATING

By A. A. Michelson

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Presented to the Academy, May 13, 1915

The principal element in the efficiency of any spectroscopic appliance is its resolving power—that is, the power to separate spectral lines. The limit of resolution is the ratio of the smallest difference of wave-length just discernable to the mean wave-length of the pair or group. If a prism can just separate or resolve the double yellow line of sodium its limit of resolution will be (5896–5890)/5893 or approximately one one-thousandth, and the resolving power is called one thousand.

Until Fraunhofer (1821) showed that light could be analysed into its constituent colors by diffraction gratings these analyses were effected by prisms the resolving power of which has been gradually increased to about thirty thousand. This limit was equaled if not surpassed by the excellent gratings of Rutherford of New York, ruled by a diamond point on speculum metal, with something like 20,000 lines, with spacing of 500 to 1000 lines to the millimeter. These were superseded by the superb gratings of Rowland with something over one hundred thousand lines, and with a resolving power of 150,000.

The theoretical resolving power of a grating is given as was first shown by Lord Rayleigh by the formula $R = mn$, in which n is the total number of lines, and m the order of the spectrum. An equivalent expression is furnished by $R = \frac{l}{\lambda}(\sin i + \sin \theta)$, where l is the total length of the ruled surface, λ the wave-length of the light, i the angle of incidence, and θ the angle of diffraction; and the maximum resolving power which a grating can have is that corresponding to i and θ each equal to 90° which gives $R = 2\,l/\lambda$, that is twice the number of light waves in the entire length of the ruled surface.

This shows that neither the closeness of the rulings nor their total number determine this theoretical limit, and emphasizes the importance of a large ruled space.

This theoretical limit can be reached, however, only on the condition of an extraordinary degree of accuracy in the spacing of the lines. Several methods for securing this degree of accuracy have been attempted but none has proved as effective as the screw. This must be of uniform pitch throughout and the periodic errors must be extremely small.

For a short screw for example, one sufficient for a grating two inches in length, the problem is not very difficult but as the length of the screw increases the difficulty increases in much more rapid proportion. It was solved by Rowland in something over two years.

Since this time many problems have arisen which demand a higher resolving power than even these gratings could furnish. Among these is the resolution of doubles and groups of lines whose complexity was unsuspected until revealed by the interferometer and amply verified by subsequent observations by the echelon and other methods.

Others that may be mentioned in this connection are the study of the distribution of intensities within the spectral 'lines'; their broadening and displacement with temperature and pressure; the effect of magnetic and electric fields, and the measurement of motions in the line of sight, as revealed by corresponding displacement of the spectral lines in consequence of the Doppler effect.

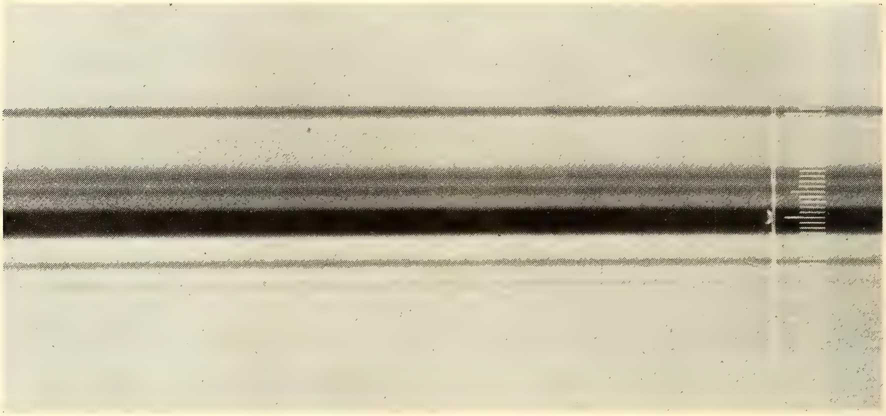
All of these have been attacked with considerable success by observations with the echelon, the interferometer and the plane-parallel plate. These methods have a very high resolving power, but labor under the serious disadvantage that adjacent succeeding spectra overlap making it difficult to interpret the results with certainty.

Some twelve years ago the construction of a ruling engine was undertaken with the hope of ruling gratings of fourteen inches—for which a screw of something over twenty inches is necessary. This screw was cut in a specially corrected lathe so that the original errors were not very large, and these were reduced by long attrition with very fine material until it was judged that the residual errors were sufficiently small to be automatically corrected during the process of ruling.

The principal claim to novelty of treatment of the problem lies in the application of interference method to the measurement and correction of these residual errors.

For this purpose one of the interferometer mirrors is fixed to the grating carriage, while a standard, consisting of two mirrors at a fixed distance apart, is attached to an auxiliary carriage. When the adjustment is correct for the front surface of the standard, interference fringes appear. The grating carriage is now moved through the length of the standard (one-tenth of a millimeter if the periodic error is to be investigated; ten or more millimeters if the error of run is to be determined) when the interference fringes appear on the rear surface. This operation is repeated, the difference from exact coincidence of the central (achromatic) fringe with a fiducial mark being measured at each step in tenths of a fringe (twentieths of a light-wave). As a whole fringe corresponds to one hundred-thousandth of an inch, the measurement is correct to within a millionth of an inch.

The corresponding correction for periodic errors is transferred to the worm wheel which turns the screw; and for errors of run to the nut which moves the carriage. In this way the final errors have been almost completely eliminated and the resulting gratings have very nearly realized their theoretical efficiency.



Enlargement of photograph of the green mercury 'line' λ5461 taken by H. E. Lemon with 10-inch diffraction grating in sixth order. Scale: 1 division = 0.01A.U. Ruled surface, $9\frac{3}{8}$ x $2\frac{7}{8}$ inches, 11700 lines per inch. Mounted in Littrow form with 8-inch lens by Brashear. Focal length, 20 feet.

A number of minor points may be mentioned which have contributed to the success of the undertaking.

(*a*) The ways which guide the grating carriage as well as those which control the motion of the ruling diamond must be very true; and these were straightened by application of an auto-collimating device which made the deviation from a straight line less than a second of arc.

(*b*) The friction of the grating carriage on the ways was diminished to about one-tenth of that due to the weight (which may amount to twenty to forty pounds) by floating on mercury.

(*c*) The longitudinal motion of the screw was prevented by allowing its spherically rounded end to rest against an optically plane surface of diamond which could be adjusted normal to the axis of the screw.

(*d*) The screw was turned by a worm wheel (instead of pawl and ratchet) which permits a simple and effective correction of the periodic errors of the screw throughout its whole length.

(*e*) A correcting device which eliminates periodic errors of higher orders.

(*f*) It may be added that the nut which actuates the carriage had bearing surfaces of soft metal (tin) instead of wood, as in preceding machines. It was not found necessary to unclamp the nut in bringing it back to the starting point.

Finally it may be noted that instead of attempting to eliminate the errors of the screw—by long continued grinding which inevitably leads to a rounding of the threads—it has been the main object to make these errors conveniently small; but especially to make them constant—for on this constancy depends the possibility of automatic correction.

The accompanying photograph made with a 10-inch grating sixth order, (actual ruled surface 9.4 inches by 28 inches), used in the Littrow form with an excellent 8-inch lens by Brashear, is given in evidence of its performance. The resolving power as shown by the accompanying scale of angstrom units is about 450,000. The original negative shows a resolving power of about 600,000. The theoretical value is about 660,000.

Doubtless the possibility of ruling a perfect grating by means of the light-waves of a homogeneous source has occurred to many—and indeed this was one of the methods first attempted.

It may still prove entirely feasible—and is still held in reserve if serious difficulty is encountered in an attempt now in progress to produce gratings of twenty inches or more. Such a method may be made partly or perhaps completely automatic, and would be independent of screws or other instrumental appliances.

It may be pointed out that an even simpler and more direct application of light-waves from a homogeneous source is theoretically possible and perhaps experimentally realizable.

If a point source of such radiations sends its light-waves to a collimating lens and the resulting plane waves are reflected at normal incidence from a plane surface, stationary waves will be set up as in the Lippman plates; these will impress an inclined photographic plate with parallel lines as in the experiment of Wiener; and the only limit to the resolving power of the resulting grating is that which depends on the degree of homogeneity of the light used. As some of the constituents of the radiations of mercury have been shown to be capable of interfering with difference of path of over a million waves, such a grating would have a resolving power exceeding a million.

This investigation has had assistance from the Bache Fund of the National Academy of Science, from the Carnegie Institution, and from the University of Chicago. In addition to the grateful acknowledgement to these institutions I would add my high appreciation of the faithful services rendered by Messrs. Julius Pearson and Fred Pearson.

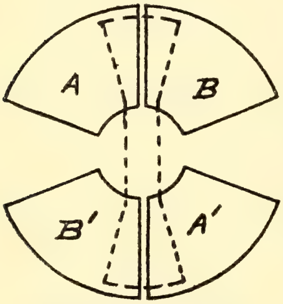
A HIGHLY SENSITIVE ELECTROMETER

By A. L. Parson

CHEMICAL LABORATORY, UNIVERSITY OF CALIFORNIA

Presented to the Academy, June 3, 1915

In this instrument the principle of working in a condition approaching instability is used to increase greatly the sensitiveness of the quadrant electrometer, which, even in its ordinary form, is more sensitive than other electrometers in which this condition of instability has been employed. The box-shaped quadrants are replaced by flat sectors subtending about 70° each at the centre and arranged as shown in the figure. Because of the two large gaps between A and B′, and between A′ and B, the needle hangs stably, as shown by the dotted lines, by virtue of electric forces quite apart from the torsion of the suspension. (Incidentally, this makes it possible to use a torsionless suspension or a float for the needle, which latter has been tried with some success.) If now a potential-difference is set up between AA′ and BB′ the needle is deflected, at first nearly proportionately to the potential difference, and then less and less as the large gaps are approached.



A HIGHLY SENSITIVE ELECTROMETER

The approach to instability and hence an increase in sensitiveness is brought about by means of a mechanical device, which on turning a micrometer screw can be made to widen the small gaps between A and B and between A′ and B′, thus making the needle less and less stable in its central symmetrical position. In this way the sensitivity which in the least sensitive condition is about 3 mm. per millivolt at a scale distance of 5 metres can be raised to as much as 60 mm. per millivolt easily, and with some care even to 150 mm. per millivolt.

A most important feature of the instrument is that, on account of the relative unimportance of the thickness of the quartz suspension, the sensitivity to potential-differences is practically independent of the size of the instrument. The last model made, with a needle 1.8 cm. long, has a capacity of 9 cm., but this could without difficulty be halved by reducing the size of the instrument; and it might be halved again by omitting one end of the needle and the pair of sectors below it.

Although the sensitivity of the electrometer (with the very efficient optical system used) is theoretically great enough to detect 10^{-6} volt, it has not as yet been made steady enough to detect an isolated potential-difference of less than about 3×10^{-5} volt.

The details of the construction of the instrument and the results of measurements made with it, together with a consideration of the cause of the residual unsteadiness will be discussed in a more extended article to be published shortly.

THE DISTRIBUTION AND FUNCTIONS OF TRIBAL SOCIETIES AMONG THE PLAINS INDIANS: A PRELIMINARY REPORT

By Clark Wissler

AMERICAN MUSEUM OF NATURAL HISTORY, NEW YORK

Presented to the Academy, June 2, 1915

For several years the American Museum of Natural History has been engaged in systematic anthropological field-work among the surviving Indians of the great Plains area in North America pertaining to one definite problem, viz., the distribution and functions of tribal societies. Early observers noted the existence of societies, chiefly for men, which within the tribe seemed to be correlated and in some cases organized into progressive series, or ranks. Some data more or less fragmentary were recorded by Lewis and Clark, Catlin, and Maximilian in early days, and later by Grinnell and J. O. Dorsey. The first serious investigation of such societies was undertaken by A. L. Kroeber among the Arapaho. This was followed by work among the Cheyenne by G. A. Dorsey, among the Assiniboin by R. H. Lowie, and among the Blackfoot by me. These pioneer studies revealed such striking similarities between these four tribal systems of societies as to suggest a case of culture diffusion.

The significance of the problem may appear from the following brief statement; two investigators had previously made special use of what data were then available on these societies in their respective efforts to explain such phenomena as manifestations of a yet to be discovered law of social evolution.[1] In each case the method was the same, one employed by many ethnologists and sociologists; viz., to collect examples of tribal societies from several parts of the world and to theoretically correlate them to form a consistent development series. The chief weakness of this method lies in the inadequacy of the data; for, so far as I know, the successive tribal organizations for an entire geographical area have never before been the object of detailed investigation. Hence, it is clear that such discussions as we have cited can at best be but hypothetical since the data upon which they are based do not in any way reveal the true characters of the phenomena involved.

Our investigation was planned to face the other way, or to proceed by observation and the collection of concrete data upon all the tribes of a geographical area as a preliminary condition to the interpretation of the phenomena.

The plan of work was developed in 1906 and was to include special investigations among the following tribes: Arikara, Blackfoot, Comanche, Crow, Dakota, Hidatsa, Iowa, Kansas, Kiowa-Apache, Mandan, Pawnee, Plains-Cree, Plains-Ojibway, Ponca, Sarsi, Shoshone, and Ute. The societies of the Arapaho, Gros Ventre, Omaha, Osage, Kiowa, and Cheyenne were excepted because they either had been or were then under investigation by others.

It is clear that an investigation of such magnitude could not be conducted by a single individual but must proceed by the coöperation of several field-workers. Accordingly, the problem was taken up as a departmental investigation conducted by the writer and his associates in the American Museum of Natural History, Dr. P. E. Goddard, Dr. R. H. Lowie, and Mr. Alanson Skinner. The field-work has been completed and the reports upon the several tribes published preparatory to an exhaustive correlation of the data, which will form the final paper to the volume.[2]

Some of the most important results of this final study may be summarized as follows:

1. It is now clear that the distinction of age found in some tribes, or where only persons of certain ages were eligible to membership, has no functional relation to the organization in which it is found for the very simple reason that in many cases what is obviously the same organization is found among other tribes independent of an age system. (Schurtz's conclusion was that all these societies were natural developments from simple age groups.)

2. The one feature that is almost universal is the exercising of police functions by some one or all of the tribal series of societies, but that the societies are expressions of a social tendency toward control is unlikely because they are often but the secondary or deputy police called upon for special service by permanent officials. In most cases the society is

called only for policing a buffalo hunt and rarely for ordinary services. Again, we have the policing of the buffalo hunt among some tribes by appointed individuals and so independent of a society.

3. Among some tribes these societies are predominately military and based upon military ideals while in others they have serious ceremonial and religious characters. This is well illustrated in the kit-fox society, one of the most widely distributed which among the Teton-Dakota is controlled by military ideals but among the Blackfoot is one of their most sacred organizations.

4. If a general résumé of the characteristics of societies for the different tribes is made, we find their ideals of organization or conceived functions exceedingly variable, much more so than the objective forms of organization, for there is marked uniformity in the number and duties of officers, kinds of regalia, and even in the names of individual societies. It is therefore difficult to consistently interpret these organizations as expressions of any special function in the tribal life, but rather as due to many and various causes.

While the data make it clear that by culture diffusion these societies have been spread from tribe to tribe, it has been diffusion of a desultory kind since in no case has a system of societies been carried along but only separate organizations. We have just noted how the same society appeared in different associations among different tribes. Further, a statistical study of the details of organization reveals a similar condition for it often happens that what is a distinctive feature of a society in one tribe will be found associated with quite a different organization in another. From this it appears that the phenomena can be most readily explained as due to mutual tribal borrowings. We have succeeded in tracing certain features of organization to particular tribes, so it is now clear that no one tribe can be the originator of the society system as a whole.

Whatever may be the ultimate interpretation of the data our work shows clearly that such ethnic phenomena can be made the object of scientific investigation and that very fundamental social problems can be successfully approached by proper inductive methods.

[1] H. Schurtz, *Altersklassen and Männerbunde, eine Darstellung der Grundformen der Gesellschaft*, Berlin, 1902; H. Webster, *Primitive Secret Societies*, New York, 1908.

[2] *Anthropological Papers*, American Museum of Natural History, volume 9. The descriptive papers are on the Teton-Dakota, Blackfoot, and Pawnee by Clark Wissler; the Eastern Dakota, Crow, Hidatsa-Mandan, Arikara, and Shoshonean tribes by Robert H. Lowie; the Plains-Cree, Plains-Ojibway, Iowa, Kansas, and Ponca by Alanson Skinner; the Sarsi and Kiowa-Apache by Pliny E. Goddard; and the final discussions will be contributed by Lowie and Wissler.

THE DETERMINATION OF SURFACE-TENSION

By T. W. Richards and L. B. Coombs
WOLCOTT GIBBS MEMORIAL LABORATORY, HARVARD UNIVERSITY

Received, May 20, 1915

This investigation is part of a series of investigations having for their object the study of the fundamental properties of liquids. It is hoped that, when a number of these properties have been determined with great accuracy, the essential relations between them may be discovered with greater certainty than is possible at present.

Among the significant properties of liquids surface-tension stands out as one of the most interesting. This somewhat unfortunately named property affords us a valuable clue concerning the cohesive forces which bind the substance together and cause it to become a liquid. Therefore, its exact determination is a matter of far-reaching importance to anyone who seeks to understand the fundamental nature of the liquid state.

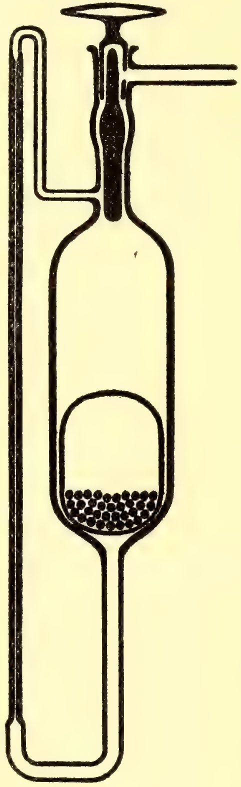
A glance at the published data concerning surface-tension leads one to conclude that much remains to be done. For example, the values obtained by experienced men for water at 20° vary all the way from 70.6 to 78, according to different methods. Even a single method (for example, that of the rise in capillary tubes) has yielded results in the last twenty-five years varying from 70.6 to 72.7, and no satisfactory evidence is forthcoming as to the reasons for the difference.

It seemed therefore worth while not only to study the surface-tension of a variety of new liquids, but also to discover the reason for the divergences between different methods, and to obtain results of absolute as well as of relative accuracy for liquids already studied. The present work, although only preliminary, seems to have been successful in locating several of the heretofore not adequately heeded sources of error.

The method chosen was the well known method of capillary rise in carefully measured tubes, because this method seems to be one of the most direct and least likely to lead to insoluble mathematical complications. The method has been used by many experimenters in the past.

Several features of the present work deserve emphasis. In the first place, the careful selection of the capillary tubes and the calibration by means of short columns of mercury received especial attention. Correction was made for the meniscuses of these columns in determining the exact diameter of the tube. Again, great care was taken to determine exactly the position of the meniscuses, both of the larger and of the smaller surface, by means of a finely adjustable black screen behind the tubes to be measured. It was found that the exact position was only to be observed when the meniscus appeared to be precisely tangent to the edge of the screen.

Both of these precautions have been more or less fully heeded by others; but another precaution, the determination of the diameter necessary for the larger tube in order to secure perfectly flat surface, has been often overlooked. We found that a tube over 35 mm. in diameter was required, and that even into this wide tube it was not permissible to insert a capillary; for such an insertion acted as another basis of support for the liquid and caused appreciable rise. By actual measurement we found that the capillary rise in a 20 mm. tube, counting only from the middle of the bottom of the meniscus, was over 0.5 mm., and the addition of a capillary tube in the middle of this raised it at least 0.3 mm. more. As apparatus of this sort has been used by most experimenters on surface-tension, most of the capillary rises which have been reported are in the neighborhod of 1 mm. too low—an error which accounts for a large part of the discrepancies between different methods.



APPARATUS IN DIAGRAMMATIC SECTION. (THE LOADED SINKER IS TO DIMINISH THE NECESSARY VOLUME OF LIQUID.)

Another error which does not seem to have received sufficient attention is that due to the weight of the liquid in the finer meniscus above its lowest point. The equation of Poisson, which is usually used for calculating this weight, gives an absurd result with tubes as wide as 1 cm., and therefore must be rejected. Another equation, that of Desains, gives a result for fine tubes which is not plausible; hence this also seemed unworthy of confidence. A careful measure of the height of the meniscus between its lowest point and its line of contact with the fully wetted walls showed that in very fine tubes this height is almost exactly equal to the radius, and that, therefore, the meniscus is here essentially hemispherical. As the tube widens, the hemisphere becomes somewhat flattened, and for moderate radii it appears entirely safe to apply, as a correction to be added on account of the meniscus, one-third of the meniscus height as actually measured. This method of correction was shown to give consistent results with tubes of different diameters.

Great difficulty and considerable liability for error were found in the inequalities of the glass of the tubes to be measured. Accordingly all measurements were made in reversible apparatus of the type shown in the diagram. This form of apparatus when exactly half filled with liquid is observed, first in an upright position in front and behind, and again in an inverted position in front and behind. Thus from the average, all the displacements due to refraction of irregular walls of the larger tube are entirely eliminated. The regularity of the walls of the smaller tube are tested by the calibration in the first place.

The preliminary results thus obtained are recorded in the table given below. All the measurements were made in the presence of air. The surface-tensions are calculated according to the well known equation $\gamma = \frac{1}{2} rhg\ (s_1 - s_2)$ in which the angle of contact of the meniscus in the tube is assumed to be zero.

CAPILLARY CONSTANTS AND SURFACE TENSIONS

Data obtained with Apparatus III (20.00°)

Radius of capillary = 1.0099 mm.

SUBSTANCE	AVERAGE HEIGHT IN MM.	CORRECTION FOR SMALL MENISCUS	CORRECTED HEIGHT IN MM.	DENSITY 20°/4°	CAPILLARY CONSTANT a^2	SURFACE TENSION DYNES PER MM^2
Water	14.394	0.321	14.715	0.99823	14.861	72.62
Benzene	6.351	0.311	6.662	0.8788	6.728	28.94
Toluene	6.369	0.311	6.680	0.8658	6.736	28.58

Data obtained with Apparatus IV (20.00°)

Radius of capillary = 0.1936 mm.

Benzene	34.620	0.061	34.681	0.8788	6.714	28.89
Methyl alcohol	30.063	0.061	30.124	0.7918	5.832	22.61
Ethyl alcohol	29.720	0.061	29.781	0.7892	5.766	22.27
Isobutyl alcohol	30.016	0.061	30.077	0.8019	5.823	22.86
Ethyl butyrate	29.403	0.061	29.464	0.8789	5.704	24.53

It will be noted that, in general, these results are higher than most of the earlier results, for the reasons already suggested. For example, Quincke found only 14.47 as the capillary constant of water, and Renard and Guye found 6.47 for that of benzene. The carefully obtained results of Walden and Swinne, although measured in a fairly satisfactory apparatus, are all subject to the same error, because the apparatus was calibrated by means of a value of the capillary constant of benzene which is too low. It is hoped that an exact evaluation of the amount of this error may make it possible to correct the comprehensive and valuable work of Ramsay and Shields and Aston, as well as that of Renard and Guye, to the standard herewith shown to be more trustworthy.

This paper is only a preliminary communication. A fuller report of the work will appear in the July number of the *Journal of the American Chemical Society*. Much more work upon the subject has already been finished, and yet more is in prospect. We hope that yet further accuracy may be attained in the future, bearing in mind the precautions to which attention has been called in this paper, and that the results may be capable of fruitful discussion.

In conclusion, we are glad to express our indebtedness to the Carnegie Institution of Washington for some of the apparatus we have employed.

Summary.—In the course of a series of determinations of capillary constants by measuring the capillary rise in fine tubes, the following precautions have been especially emphasized: (1) The detection and correction of inequalities in the glass tubes employed were effected by the use of a reversible apparatus. (2) Reference of the capillary rise was made to an unrestricted flat surface 38 mm. in diameter, the largest ever used. It was shown that much smaller surfaces are too small and that the insertion of a capillary in the middle of a larger tube causes appreciable error by increasing the capillary effect of the large tube. (3) Especial care was taken that the true bottom of the meniscus should be read. (4) The weight of the fine meniscus was in each case allowed for, and a new approximate formula was suggested for its calculation, depending upon the observed height of the meniscus.

Heeding these precautions, determinations of the capillary constants of several important liquids were determined at 20° as follows: water 14.861, benzene 6.721, toluene 6.736, methyl alcohol 5.832, ethyl alcohol 5.793, isobutyl alcohol 5.823, ethyl butyrate 5.704.

AN EXHIBIT IN PHYSICAL ANTHROPOLOGY

By Aleš Hrdlička

DIVISION OF PHYSICAL ANTHROPOLOGY, U. S. NATIONAL MUSEUM, WASHINGTON

Presented to the Academy, June 8, 1915

In the April number of the PROCEEINGS OF THE NATIONAL ACADEMY OF SCIENCES, I published a brief account of "Some recent anthropological explorations," which were carried out under my direction or by myself, for the Smithsonian Institution and the Panama-California Exposition at San Diego, in different parts of the world. I shall now point, in an equally brief way, to the material results of these expeditions in relation with the Exposition.

One of the main objects of the whole work was to bring together a comprehensive, instructive and harmonious exhibit relating to the natural history of man. Such exhibits have been attempted with more or less success at a number of former expositions, from that in Paris (1878) to those in Chicago (1892) and Dresden (1911); but all these efforts, while reaching creditable and useful results, suffered from the inclusion of archeological and other exhibits, as well as from defects due to heterogeny or lack of material, and especially to the impossibility of carrying out a uniform and necessarily costly plan.

In undertaking the preparation of the exhibits for San Diego, the whole subject of Anthropology or Man's Natural History, was divided into four sections. These were (1) Man's Evolution (Phylogeny); (2) Man's Development and Growth (Ontogeny); (3) Man's Variation; and (4) Man's Decline and Elimination. To these it seemed necessary to add a subdivision dealing with the modern means by which anthropology works and including a comprehensive library. In accordance with this plan, a stipulation was made for five connected moderate sized halls, to accommodate in logical progression the five sections. The exhibits were to be, furthermore, as far as possible original and of permanent museum value, so that they could serve if circumstances permitted as a nucleus or an important part of a future permanent anthropological museum on the Pacific Coast.

As mentioned in the former paper, only three years were available for the carrying out of the above large program. Illness among the staff, the European war, and other conditions interfered. Not a few of the links or specimens needed were not to be found or purchased; and other difficulties developed. In consequence, the results are not as complete and perfect as they might be; yet it is safe to say that in richness, instructiveness, and harmony, they surpass considerably anything attempted before in this line. These facts seem to justify the present remarks which call attention not so much to the existence of the collection at San Diego as to the material progress accomplished in a branch of science in which exhibits are particularly scarce and difficult.

The five moderate sized halls which harbor the exhibits occupy a special building marked 'Science of Man.'

The first hall is devoted entirely to Man's Evolution. It contains one of the largest existing series of accurate, first-hand casts of the most important authentic skeletal remains of early man. Added to this are numerous attempted reconstructions of the generally defective early crania. On the walls above the specimens are illustrations showing, in good sized photographic enlargements or colored charts, the localities of discovery of the various originals represented by the casts. Besides this, there are large charts relating to geology and stratiography so far as these relate to man; and charts showing the probable lines of man's ascent after the foremost authorities. Another series of illustrations, covering more than one entire wall, is devoted to the pictorial representations of early man, by the early man himself (cave drawings and sculptures), and by noted scientists and artists of the present day. The whole center of this hall is occupied by the most striking and interesting series of busts—reconstructions of early man, made by the talented Mascré and under the direction of R. Rutot, one of the foremost European students of early man. Finally, there is a large series of original specimens showing in a progressive way the crania of existing primates, or more exactly those from the lemur to man. The anthropoid apes are each represented in this series by skulls of a full grown male and female and by one of a young animal of the same species.

The second hall, or that devoted to *Ontogeny,* contains six series of true-to-nature busts, made at the National Museum by one of the best modelers in this country, and showing, by different age stages, from birth onward and in both sexes, the three principal races of this country, namely the 'thorough-bred' white American (at least three generations American on each parental side), the Indian, and the full-blood American negro. These series form a unique, costly exhibit, nothing like which has ever been attempted before in this or any other country. Each set consists of fifteen busts and proceeds from infants at or within a few days after birth, to the oldest persons that could be found. The oldest American negro represented is 114. After the new-born, the stages are 9 months, 3 years, 6, 10, 15, 20, 28, 35, 45, 55, 65, and 75 years, and then the oldest person obtainable. Special care was exercised in ascertaining the age of the subjects, particularly among the Negroes and the Indians. No choice was made of the subject beyond that due to the requirements of pedigree, age, and good health. The whites and negroes were obtained with a few exceptions in the District of Columbia and vicinity, but their places of birth range over a large part of the eastern, southern, and middle states; for the Indian I chose the Sioux, a large, characteristic, and in a very large measure still pure blood tribe, and one in which the determination of the ages in the subjects was quite feasible.

On the walls of this room are eight original charts relating to the physical decline of man or normal senility, and a series of other charts, mostly in colors, relating to the development of the child in different races. In a case along the walls are series of brains, skulls and other bones of the body, almost all in original specimens, showing the development of these parts from early fetal to adult life, or even towards old age.

The third hall is devoted to racial, sexual, and individual variations. The main exhibits consist of ten pairs of original busts, showing important groups of humanity; and over one hundred original bronzed masks, illustrating the individual variation in physiognomy among different races. The latter series includes the masks of 19 Bushmen, which are especially rare and valuable. The wall is covered with charts showing racial classification, distribution, and statistics. The middle of the hall is given to specially constructed steel fixtures, for over two hundred colored and plain portrait transparencies of racial types. These portraits include several special series, such as 'The Indian child in different parts of the continent,' 'Beauty among Indians' etc. In the wall case are numerous exhibits of original specimens relating to racial, sexual and individual variation in the skull and other parts of the skeleton.

The fourth hall is devoted to the illustration of the causes which, outside of strictly normal senility, contribute to the decline of the human organism, and in the vast majority of cases cause death. The geographical distribution of the principal diseases is represented in a series of small colored maps on the wall, and there are charts relating to causes and frequency of mortality. The main exhibit in this room, however, consists of a very large series of skeletal remains of the prehistoric American Indian, showing his entire, or almost entire pathology. This exhibit is unique and could scarcely be duplicated. It is supplemented by a large number of Indian crania which show wounds and by 60 original cases of pre-Columbian Indian trephining for such wounds.

The fifth hall is fitted out as a modern anthropological laboratory and is also used as a lecture room in which frequent demonstrations are given bearing on the collections in this section. There are, further, the library; two cases of modern anthropometric instruments; anthropometric charts on the walls, and a series of portraits of the most eminent deceased representatives of anthropology.

It will be seen from the above that the exhibits here briefly described, are quite different from the usual exhibits at an Exposition. They are of permanent value, have attracted from the first most encouraging attention, and are capable of forming the foundation of an anthropological center in a locality eminently fitted for such a purpose.

THE COMPRESSIBILITIES OF THE ELEMENTS AND THEIR RELATIONS TO OTHER PROPERTIES

By T. W. Richards

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Presented to the Academy, May 22, 1915

Eight years ago the compressibilities of thirty-five elements were determined at Harvard, in order to discover the relation of these compressibilities to one another and to other properties of these fundamental substances. All the values were referred to that of mercury, with the understanding that, although the compressibility of mercury was not at that time very well known, they could all be easily corrected to the true standard when more complete knowledge of this quantity had been attained.

This time now seems to have come. The recent research of Bridgman at high pressures and that of the author in conjunction with E. P. Bartlett at lower pressures have furnished such a consistent and convincing outcome that the compressibility of mercury is no longer in serious doubt. Accordingly, it is appropriate that the older results should be corrected to the true standard. Moreover, a better standard of pressure is now available, and the pressure gauges used in the early work have been very carefully compared with it, so that the older results can be corrected in this respect also.

The table below contains the corrected values of the compressibilities in question, together with new values for tungsten and tantalum, determined with the help of E. P. Bartlett, and of boron, determined with the help of J. H. Hodges.

The megabar (or megabarie) is chosen as the measure of pressure because this absolute unit bids fair to become the scientific standard of the future, being the pressure of one megadyne per square centimeter. It is almost 2% greater than the kilogram per square centimeter, and 1.3% less than the 'atmosphere.'

In addition to the compressibilities, the table records also several other properties, which will be shown to be more or less related. Many of these also have been determined at Harvard, but some are due to the work of others. Most of these latter are easily found in the literature; complete references would occupy far too much space for the present publication.

Compressibilities, Atomic Volumes, Densities, Melting Points, Coefficients of Expansion and Atomic Weights of a Majority of the Commonly Solid and Liquid Elements Between 100 and 500 Megabars (if the compressibility of mercury = 0.00000395).

	Average Compressibility at 20°(see above)	*Atomic Volume*	*Density at 20°*	*Melting Point (absolute temp.)*	*Cubic Coefficient of Expansion × 100,000*	*International Atomic Weight (1914)*
Lithium......	9.0	13.1	0.534	453	19.	6.94
Boron........	0.3	4.7	2.34	2800		11.0
Diamond.....	[0.7] (?)	3.4	3.51	Very high	0.3	12.00
Graphite......	3.	5.4	2.26	Very high	7.2	12.00
Sodium.......	15.6	23.7	0.971	371	22.	23.00
Magnesium...	2.9	13.3	1.74	927	7.8	24.32
Aluminium....	1.47	10.1	2.60	930	7.2	27.1
Silicon........	0.32	11.4	2.42	1733	2.3	28.3
Phosphorus, red	9.2	14.4	2.15	863		31.04
Phosphorus, white	20.5	16.6	1.82	317	36.	31.04
Sulphur......	12.9	15.5	2.07	384	18.	32.07
Chlorine......	[95] (?)	25.	1.42	171		35.46
Potassium....	31.7	45.5	0.862	335	25.	39.10
Calcium......	5.7	25.3	1.53	1073		40.07
Chromium....	0.9	7.7	7.06	1823		52.0
Manganese...	0.84	7.7	7.37	1533		54.93
Iron.........	0.60	7.1	7.85	1790	3.6	55.84
Nickel.......	0.43	6.7	8.7	1725	4.2	58.68
Copper.......	0.75	7.1	8.92	1356	5.0	63.57
Zinc.........	1.7	9.5	7.13	692	8.7 (?)	65.37
Arsenic.......	4.5	13.3	7.53	1073	1.6	74.96
Selenium.....	12.0	18.5	4.28	490	13.	79.2
Bromine......	52.	25.1	3.12	266		79.92
Rubidium....	40.	56.0	1.53	312	[30] (?)	85.45
Molybdenum .	0.46	11.1	10.23	2773	1.1	96.0
Palladium....	0.54	9.3	12.13	1822	3.8	106.7
Silver........	1.01	10.3	10.5	1134	5.7	107.88
Cadmium.....	2.1	13.0	8.60	594	7.4	112.40
Tin..........	1.9	16.2	7.29	505	6.7	119.0
Antimony....	2.4	17.9	6.71	903	3.3	120.2
Iodine........	13.	25.7	4.94	386	25.0	126.92
Cesium.......	61.	71.0	1.87	301	[33] (?)	132.81
Tantalum....	0.53	10.9	16.67	3123	2.3	181.5
Tungsten.....	0.27	9.6	19.23	3300(?)	1.1	184.0
Platinum.....	0.38	9.1	21.31	2020	2.7	195.2
Gold.........	0.66	10.2	19.24	1336	4.3	197.2
Mercury......	3.95	14.8	13.55	234		200.6
Thallium.....	2.3	17.2	11.83	574	9.0	204.0
Lead.........	2.33	18.2	11.37	600	8.8	207.2
Bismuth......	3.0	21.2	9.80	543	4.0	208.0

In the few cases where the compressibilities have been determined by others, especially by Grüneisen and Bridgman, the agreement with the results given above is good; and there is every reason to believe that these figures represent closely the true values of the constants in question. Therefore, they are worthy of further study, especially as regards their mutual relations and their relations to other properties.

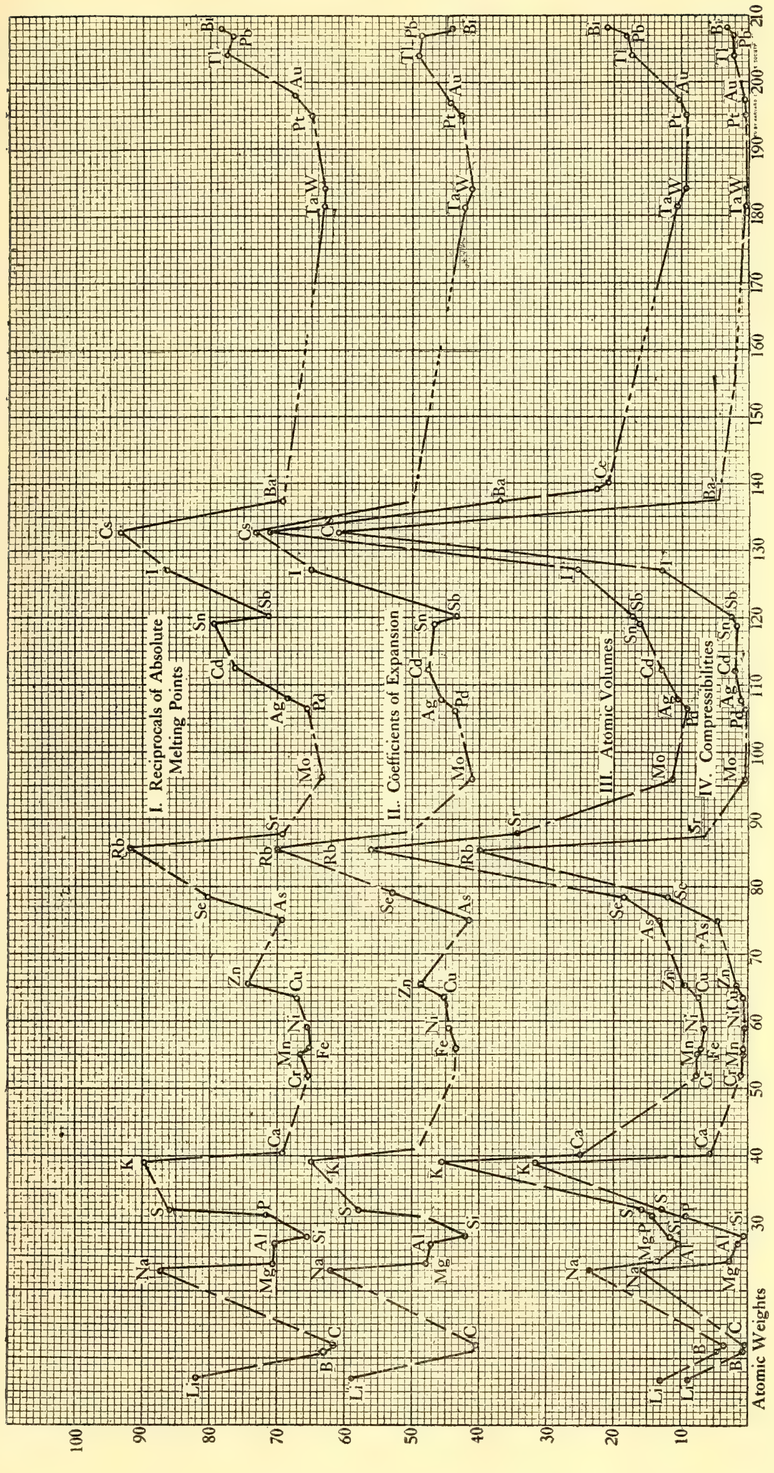

Diagram showing: I. Reciprocals of Absolute Melting Points; II. Coefficients of Expansion; III. Atomic Volumes; IV. Compressibilities of a Majority of the Commonly Solid and Liquid Elements between 100 and 500 Megabars.

Leaving out of consideration the behavior of the liquids, which doubtless is not fairly comparable with that of solids, we may compare these various quantities in parallel curves, each being plotted in relation to the increasing atomic weight. The accompanying diagram presents in the lowest line compressibilities; in the next above, atomic volumes; in the curve no. II, coefficients of expansion, and in the highest curve, the reciprocals of the absolute melting points.

That essential relations exist between these data cannot be doubted. In each case a maximum is seen to occur with the alkali metals; and although the rest of the curves are less similar, nevertheless, distinct connection is traceable. The compressibility curve is most like the atomic volume curve, and the coefficient of expansion curve most like that of the reciprocals of the melting points.

The attempt to find a mathematical relationship between these quantities is an interesting one. Such a relationship has already been sought by Grüneisen, Einstein, Debye, and others, upon the basis of a speculative theory, without much experimental support. With the wealth of data herewith presented, however, an empirical attempt is possible. It is soon evident on studying the figures that in cases where the melting points and atomic volumes are nearly similar, as, for example, in the cases of gold and silver, the denser substance is the less compressible. On the other hand, with similar density and melting point, as in the case of sodium or potassium, the substance with the greater atomic volume is the more compressible. Yet again, where the atomic volumes and densities are not very different, as in the case of sodium and calcium, the substance with the higher melting point has the lower compressibility. Taking account of these general tendencies, one sees that the equation should take this general form:

$$\text{Compressibility} = K\frac{f_1(V)}{f_2(D)f_3(T_m)}$$

in which K represents a constant, V the atomic volume, D the density, T_m the absolute melting point, f_1, f_2, f_3, various as yet undetermined functions.

The search for the nature of these respective functions is tedious and has not been carried very far; but the substitution in this equation of $K = 0.00021, f_1(V) = V, f_2(D) = D^{0.25}$ and $f_3(T_m) = (T_m - 50)$ gave results of about the right order with nearly all of the substances. For example, lithium is thus calculated to be 8 instead of 9, sodium, 15.6 (exactly its observed value), copper 0.66 instead of 0.75, silver 1.02 instead of 1.01, tungsten 0.29 instead of 0.27, etc. Some of the calculated values, to be sure, deviated much more widely than this, but none is of entirely a different order from the truth. It is possible to show a necessary relationship between this empirical equation and the outcome of recent work by Grüneisen, and there is also some relationship between this equation and the well-known formula of Dupré, but the full discussion of these relationships would occupy more space than is available in the present publication. It may be noted, however, that the equation of Dupré is less satisfactory than the empirical equation just given as regards the elements, for density plays too large a rôle in the older expression. Moreover, the equation of Grüneisen is designed only for presumably monatomic elements, and it gives a negative (and therefore absurd) result for bismuth, showing that its range is more limited than that of the equation just mentioned.

No pretense is made, of course, that the empirical equation herewith communicated is a final statement of the relation of the quantities concerned. Nevertheless, because it gives the approximate order of magnitude of the compressibilities of widely different substances, one has a right to believe that the basis of the equation really refers to some of the more important tendencies which are at work.

Attention may be called to the fact that these relationships are in accord with those which would be expected from the prediction of the theory of compressible atoms. An atom much distended because of its internal makeup might be supposed to be highly compressible, and so might one much distended because not greatly pressed upon by outside cohesive pressure. Both of these tendencies would cause small density, and, accordingly, large atomic volume, and the second of these tendencies would add low melting point. Therefore, the general form of the equation is plausible.

A more complete discussion of the data and equation will soon be given in the *Journal of the American Chemical Society.*

Summary.—This paper records all the recent work on the compressibility of the elements performed at Harvard, reduced to the best available standard—the newly determined compressibility of mercury. It is pointed out that the reciprocals of the melting points are very closely associated with the coefficients of expansion, and that both of these properties seem to be essentially connected with atomic volume and compressibility. A preliminary empirical equation is proposed, which shows roughly the nature of this relationship.

RADIAL VELOCITIES WITHIN THE GREAT NEBULA OF ORION

By Edwin B. Frost

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Presented to the Academy, June 9, 1915

The *Astrophysical Journal* for October, 1914, contained an important paper by MM. Bourget, Fabry, and Buisson of Marseille, summarizing the results of their application to the Orion nebula of the photographic interferometric methods of M. Fabry They found variations in the radial velocity from one point to another within the nebula, differences as large as 10 km. per second at points quite close to each other being indicated by local deformations of the interference rings. They also detected graat collective movements, the northeastern region receding, and the southwestern region approaching, relatively to the mean velocity at the trapezium, with velocities of about 5 km. per second.

It seemed of interest to repeat these determinations independently by the standard spectrographic method, and toward the close of the winter observations were begun at Yerkes Observatory with the Bruce spectrograph, arranged with a dispersion of one prism. The radial velocity was inferred from the displacements of the hydrogen lines β and γ and the lines of nebulium at λ 5007 and 4959. At the position of the western star of the trapezium, No. 619 in Bond's catalogue of stars in the nebula, the radial velocity of the nebula was found from measures of eight plates (taken occasionally during the past eleven years) to be $+15.6 \pm 0.5$ km. per second (recession). This is the mean of independent measures of each plate by the writer and Mr. C. A. Maney, and it agrees almost exactly with the value of $+15.8$ km. found by the observers at Marseille for the region of the trapezium.

For several other positions around the trapezium, and not over 2′ from it, we obtained values ranging from $+6$ to $+17$ km. The probable error for the velocity at one point, from the mean of measures by the two observers of a single plate, should be about ± 1.5 km., so that the reality of the difference may be regarded as fully confirmed. Exposures of from two to three hours were required at some of the positions, so that the accumulation of plates was slow, and additional observations will be required when the 'open season' for Orion returns.

We must accordingly alter our conceptions of the nebula as an enormous mass of quiescent gas, and regard it as seething with local whirlpools besides perhaps having a considerable motion of rotation as a whole.

THE RADIAL VELOCITIES OF THE MORE DISTANT STARS

By Walter S. Adams

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Presented to the Academy, June 8, 1915

As a result of the work of Kapteyn, Campbell, and Boss it has been recognized for some years that the linear motions of the brighter stars vary with their spectral types, the velocities of the solar type stars being higher than those of types B and A. It is equally well-known, however, that the solar type stars in general have larger proper motions and so are much nearer to the sun than those of earlier types. Hence the stars, for which the velocities have been compared, have quite different distances, and if, as was once suggested by Eddington,[1] the nearer stars move more rapidly than the distant ones this fact would account, in part at least, for the apparent variation of velocity with spectral type.

In the course of an analysis of the radial velocity results from the Lick Observatory and Mount Wilson, J. C. Kapteyn derived the relationship between radial velocity and proper motion for the K type stars,[2] and found that the velocity increased rapidly with the amount of proper motion. In this computation the effect of stream motion was eliminated to a large extent by a selection of stars nearly 90° from the vertices of the streams. The Mount Wilson observations of the spectra of stars having both small and large proper motions provide the material for a similar comparison for other types of spectra. This is given in Table I: the effect of stream motion, however, has not been eliminated. The average velocity v' is corrected for a solar motion of 20 km. directed toward the apex

$$\alpha = 17\text{ h }59\text{ m}. \quad \delta = +30^\circ.8$$

and no velocities exceeding 100 km. are included.

TABLE I

SPECTRAL TYPE	NUMBER OF STARS	μ	v'	NUMBER OF STARS	μ	v'
B	61	0″.016	8.2	52	0″.041	9.6
A...........	55	0 .019	10.0	104	0 .067	10.7
F............	20	0 .011	10.1	45	0 .530	24.6
G...........	63	0 .013	10.6	69	0 .670	24.9
M..........	27	0 .015	12.6	12	0 .170	17.6

To these we may add Kapteyn's value for the K stars with stream motion eliminated:

K...........	27	0.013	10.9	19	>0.30	26.7

It is clear from these results that the radial velocity increases rapidly with proper motion. Only a few A type stars of very large proper motion are known. Of those observed at Mount Wilson for which μ exceeds 0″.20, two have velocities of over 150 km.; one has a velocity of 87 km.; and the average for the other six is 20 km.

In view of this relationship between proper motion and radial velocity it becomes of especial interest to compare the Mount Wilson observations of stars of small proper motion with the results of W. W. Campbell for stars selected on the basis of apparent magnitude alone. Table II contains the average values of the radial velocities corrected for solar motion, of 1034 stars published in Campbells' first table.[3] No constant *K* has been applied to these results. The proper motions have been taken from Boss's catalogue for the individual stars appearing in Campbell's later lists, and hence the mean values are not strictly correct for the stars whose velocities are given. Since the number of stars is large, however, it does not seem probable that these values can be seriously in error.

TABLE II

SPECTRAL TYPE	CAMPBELL				MOUNT WILSON		
	No. Stars for v'	No. Stars for μ	μ	v'	No. Stars	μ	v'
O and B	141	224	0″.031	8.99	61	0″.016	8.23
A	133	206	0 .094	9.94	55	0 .019	10.04
F	159	192	0 .234	13.90	20	0 .011	10.14
G	529	549	0 .202	15.15	63	0 .013	10.60
K					56	0 .014	11.53
M	72	78	0 .074	16.55	27	0 .015	12.56

The omission of 5 stars of the A type would reduce the proper motion for Campbell's stars from 0″.094 to 0″.079. The principal feature of this comparison is the relatively close agreement of the Lick and Mount Wilson results for the B and A stars, and the large difference for those of other types. This difference appears to be associated directly with the great increase in proper motion between the A and F type stars in Campbell's list. With the aid of the relationship already referred to between proper motion and radial velocity deduced by Kapteyn for the K stars it is possible to apply corrections to Campbell's results to reduce to the proper motion 0″.031 of the B stars. The resulting values are shown under v' in Table III. In addition both the Lick and the Mount Wilson velocities have been corrected for the effect of stream motion by the approximate method devised by Eddington.[4]

TABLE III

SPECTRAL TYPE	CAMPBELL			MOUNT WILSON		
	v'	Corrected[1]	μ	v'	Corrected[1]	μ
O and B	9.0	9.0	0″.031	8.2	8.2	0″.016
A	8.6	6.8	0 .031	10.0	7.7	0 .019
F	9.0	7.8	0 .031	10.1	8.8	0 .011
G	11.0	9.6	0 .031	10.6	9.2	0 .013
K				11.5	10.0	0 .014
M	15.6	13.6	0 .031	12.6	10.9	0 .015

[1] v' corrected for stream motion.

In his later discussion of the solar motion as derived from the several spectral types Campbell has given the average radial velocities with a constant correction K applied to the velocity of each star. This constant has a value ranging from about zero for the F and G stars to over 4 km. for those of type B. If these results are treated in the same way as those of Table III we obtain the following values:

TABLE IV

TYPE	v'	CORRECTED	μ
O and B	6.5	6.5	0″.031
A	9.6	7.4	0 .031
F	9.5	8.3	0 .031
G	9.1	7.9	0 .031
K	13.2	11.5	0 .031
M	16.1	14.0	0 .031

The constant K as used by Campbell is the average velocity v' taken according to sign for the stars of each spectral class, and is dependent upon the value of the solar motion. For all of the Mount Wilson stars the same value, $V = 20$ km., has been used. This gives the following average velocities taken according to sign:

B +1.26; A −0.24; F −0.86; G +0.05; K −1.18; M +0.31.

A change in the value of V from 20 km. to 19 km. would reduce the value for the B stars from +1.26 km. to +1.06 km. These quantities must be regarded as very moderate in size.

It appears clear from the Mount Wilson results given in Table III that the variation in velocity with spectral type is very gradual for these distant stars; and except in the case of the M stars, which, because of their number, are of comparatively low weight, the same conclusion may be drawn from Campbell's values after allowance has been made for the effect of the large number of relatively near stars included among his

F, G, and K type spectra. This would be in agreement with the hypothesis of Eddington, already referred to, that the relation between velocity and spectral type might be a relation between velocity and distance, the stars nearest the sun, mainly the types F, G, and K, moving more rapidly than the distant stars. Eddington considered this hypothesis as disproved because an analysis of the A type stars indicated no increase of radial velocity with increasing proper motion. The fact that such an increase exists in the case of the later type stars, however, is shown clearly in Table I. Because of the slight range in proper motion a similar variation for the B and A stars is less certain, although indicated on the face of the results.

The principal feature of interest in this comparison of proper motion and radial velocity is the low average velocity found for the distant stars of types F to M. These stars are on the average stars of high absolute luminosity, and the possibility of a relationship between radial velocity and absolute luminosity has been considered in the communication by Kapteyn and Adams, to which reference has already been made. The observational material included here is much too limited to provide the basis for a discussion of this question. It may be noted, however, that the average radial velocity corrected for the solar motion of such absolutely faint stars as have been observed at Mount Wilson is exceptionally great. The average velocity of sixteen stars with absolute magnitudes below 8 on a scale for which the sun is 5.5 is 36 km. Eight have velocities exceeding 40 km. Since these stars are probably of small mass the evidence so far as it goes is in favor of Halm's hypothesis[5] of the equipartition of energy among the stars, their motions being a function of their masses.

[1] *British Association Report*, 1911.

[2] These Proceedings, **1**, 14 (1915).

[3] *Lick Obs. Bull.*, No. 196.

[4] *Stellar Movements and the Structure of the Universe*, p. 157.

[5] *London, Mon. Not. R. Ast. Soc.*, **71**, 634 (1911).

LOCALIZATION OF THE HEREDITARY MATERIAL IN THE GERM CELLS

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Presented to the Academy, May 17, 1915

It has come to be recognized that it must be more than a coincidence that in each animal and plant there are two representatives of each hereditary character (one derived from the mother and one derived from the father), and that there are two of each kind of chromosome (one derived from the mother and one derived from the father). Moreover this relation becomes much more impressive when it is found that in the formation of the germ cells the representatives of the different characters separate from each other, and that the chromosomes separate also, so that each germ cell has but one set of factors and but one set of chromosomes. The comparison does not stop even here, and I propose to review briefly the further evidence that leads to the conclusion that the chromosomes are the bearers of the hereditary characters and that the known chromosomal behavior suffices as a mechanism to explain Mendel's law.

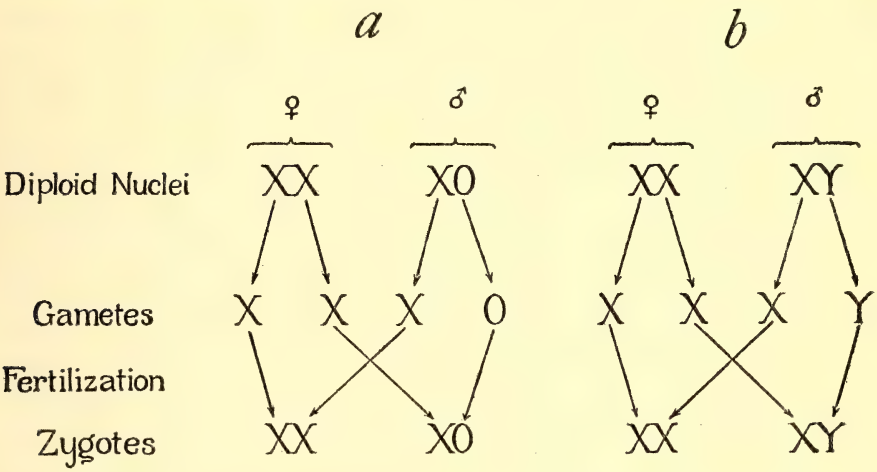

FIG. 1. DIAGRAMS OF SEX DETERMINATION; *A*, TYPE IN WHICH THE FEMALE IS XX AND THE MALE XO; *B*, TYPE IN WHICH THE FEMALE IS XX AND THE MALE XY.

Sex Linked Inheritance and the Sex Chromosomes. There is one exception to the rule of the dual nature of the chromosomes and at the same time an exception to the rule of the dual nature of the hereditary factors. In many species of animals it has been shown that while there are two sex chromosomes in the female, there is only one in the male (fig. 1 a). It is true in other animals often closely related to these, that the male may have one sex chromosome like the two in the female, and in addition a smaller Y chromosome, and in a few cases the Y may be as large as the X (fig. 1 b). In Drosophila there is an XY pair in the male, but the Y chromosome has been shown not to carry any factors that affect the characters due to factors carried by the X chromosomes. The influence of the Y is therefore nil, and the results are the same as though no Y were present.

The unequal distribution of the X chromosomes in the two sexes has furnished an opportunity to examine the theory that the chromosomes are the bearers of the hereditary factors, because it can be shown that the inheritance of a certain class of characters follows the known distribution of the sex chromosomes. For example, color blindness in man is inherited in the following way (figs. 2 and 3). A color blind man married to a normal woman (fig. 2) produces normal sons and daughters. If two individuals that have had this origin should marry each other, the daughters will all be normal, half of the sons will be normal but half will be color blind. In other words the color blind grandfather has transmitted his defect to none of his granddaughters, but to half of his grandsons. If we assume that color blindness is due to factors carried by the X chromosomes—in this case by the single X chromosome of the grandfather—the inheritance of the chromosome (to the right) and of the defect (to the left) is the same, as shown by the scheme (fig. 2).

Conversely if a color blind female is married to a normal male (fig. 3), all of the sons will be color blind, but all of the daughters will be normal, If such a color blind male (or any color blind male in fact) is mated with a female having this origin, half of the daughters will be color blind, half normal; half of the sons will be color blind, half will be normal. In other words the color blind grandmother transmits her defect to all of her sons, and to half of her granddaughters and to half of her grandsons. The inheritance of the chromosomes and of the defect is strictly parallel in this case also, as seen in the diagram (fig. 3). In the fruit-fly Drosophila ampelophila there have been found over 40 sex-linked characters all of which are inherited according to the scheme given above. Moreover all possible combinations and tests that have been made—and there are many ways of testing such a conclusion—have given results that are entirely consistent with this scheme.

Linkage. If the hereditary characters are carried by the chromosomes we should expect that there would be great groups of characters that go together in inheritance, because while the number of the chromosomes is limited the number of the hereditary characters must be very great.

In Drosophila the number of pairs of chromosomes is only four, while over one hundred mutant characters have been studied. These characters fall into four great groups. The names of some of these characters that involve nearly all parts of the body, are given in the following list:

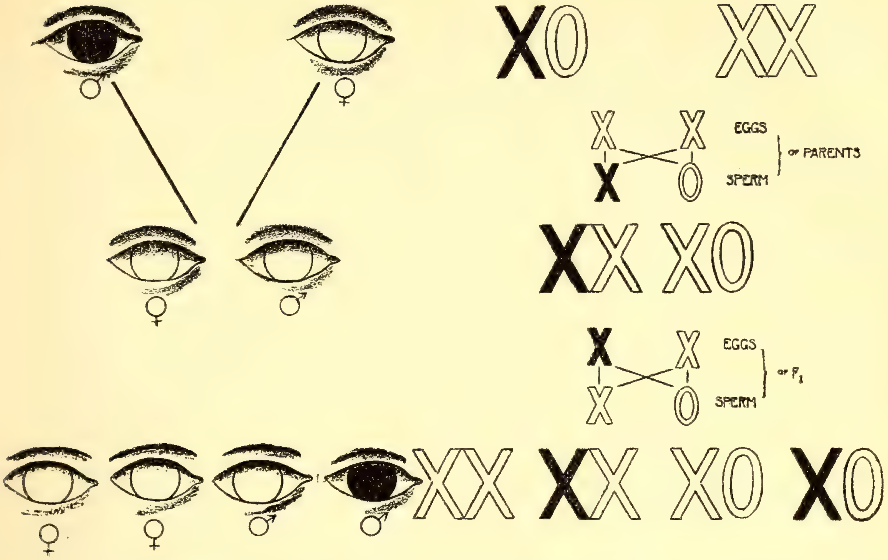
XO
XX
X X
X O
EGGS
SPERM
OF PARENTS
XX XO
X X
X O
EGGS
SPERM
OF F_1
XX XX XO XO

FIG. 2. DIAGRAM OF INHERITANCE OF COLOR BLINDNESS IN MAN. THE COLOR BLIND EYE AND THE X CHROMOSOME THAT CARRIES THE FACTOR FOR COLOR BLINDNESS ARE REPRESENTED IN SOLID BLACK. IN THIS DIAGRAM THE CHARACTER ENTERS THROUGH MALE.

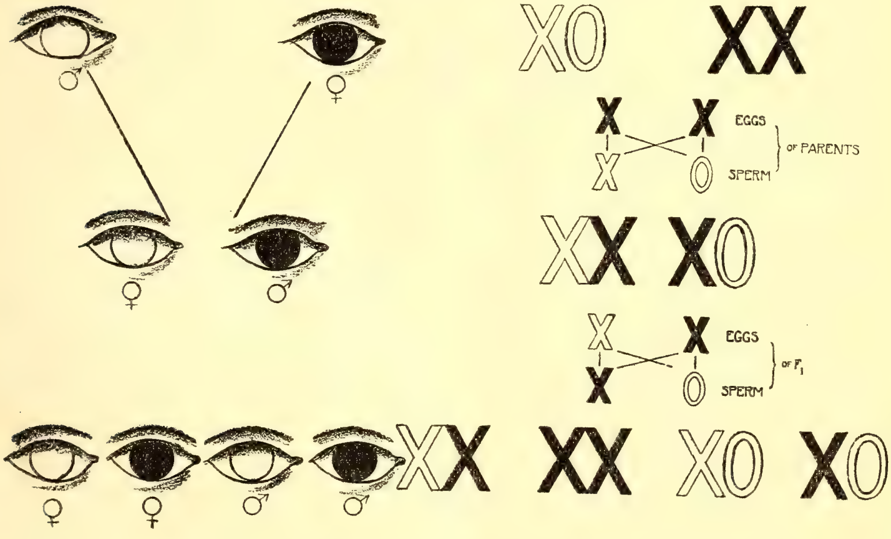
XO
XX
X X
X O
EGGS
SPERM
OF PARENTS
XX XO
X X
X O
EGGS
SPERM
OF F_1
XX XX XO XO

FIG. 3. AS IN FIG. 2, BUT HERE THE COLOR BLINDNESS ENTERS THROUGH THE FEMALE.

Group I	*Group II*	*Group III*	*Group IV*
Abnormal	Antlered	Band	Bent
Bar	Apterous	Beaded	Eyeless
Bifid	Arc	Cream III	
Bow	Balloon	Deformed	
Cherry	Black	Dwarf	
Chrome	Blistered	Ebony	
Cleft	Comma	Giant	
Club	Confluent	Kidney	
Depressed	Cream II	Low crossing over	
Dotted	Curved	Maroon	
Eosin	Dachs	Peach	
Facet	Extra vein.	Pink	
Forked	Fringed	Rough	
Furrowed	Jaunty	Safranin	
Fused	Limited	Sepia	
Green	Little crossover	Sooty	
Jaunty	Morula	Spineless	
Lemon	Olive	Spread	
Lethal's, 13	Plexus	Trident	
Miniature	Purple	Truncate intensifier	
Notch	Speck	Whitehead	
Reduplicated	Strap	White ocelli	
Ruby	Streak		
Rudimentary	Trefoil		
Sable	Truncate		
Shifted	Vestigial		
Short			
Skee			
Spoon			
Spot			
Tan			
Truncate intensifier			
Vermilion			
White			
Yellow			

A group consists of those characters that are inherited together, or rather *tend* to be inherited together. The chromosome groups of the male and of the female of Drosophila ampelophila are shown in figure 4. There is a pair of sex chromosomes, two pairs of large chromosomes, and a very small pair—four pairs in all. The coincidence between the number of hereditary groups and of the chromosomes is sufficiently evident to render comment unnecessary.

Crossing over. I have said that the characters that belong to the first group tend to be inherited together. Thus, if a fly with white eyes and yellow wings be crossed to the wild type, having red eyes and gray wings, the second generation of flies will be white yellow and gray red gray, *except* that once in a hundred times a fly will be found that is white gray or else red yellow. In other words an exchange of characters has in some way taken place, but not oftener than once in a hundred times. We can see the exchange taking place in the reverse way if we cross a white eyed, gray winged fly to a red eyed, yellow winged fly. In the second generation the flies will now be white gray and red yellow, except that once in a hundred times a fly will appear that is white yellow or red gray.

Now if at the time when the chromosomes conjugate the two like chromosomes should exchange pieces, a consistent explanation of the results described above can be given. As a matter of fact there are several stages before, during and after conjugation of the chromosomes when interchange of pieces might take place. One of the phases is illustrated in the following diagram (fig. 5). Two homologous chromosomes are represented as each made up of a single line of beads. During conjugation, like chromosomes come together, and as they unite they can be seen often to twist around each other in consequence of which the parts come to line up on opposite sides of each other, as shown in the figure. When the chromosomes move apart the beads on one side are represented as passing to one pole, and those on the opposite side to the other pole; this means that during conjugation the pairs of factors, here represented by the beads, segregate independently of each other, and that the forces that draw the beads apart are often stronger than those that hold consecutive beads in their original line or series.

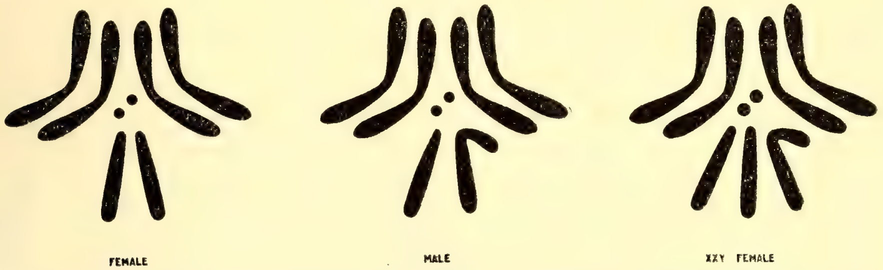

FIG. 4. DIAGRAMS OF THE CHROMOSOMES OF DROSOPHILA AMPELOPHILA: TO THE LEFT, NORMAL FEMALE; IN THE MIDDLE, NORMAL MALE; TO THE RIGHT, NON-DISJUNCTIONAL FEMALE.

It may then, I think, be fairly claimed that the facts of crossing-over which must be due to interchange between homologous chromosomes, if chromosomes be admitted as the bearers of the hereditary factors, can be accounted for without departing from the known behavior of the chromosomes at the time of conjugation. On the other hand it should be clearly understood that the conclusion as to the way in which the interchange takes place is not as yet on the same footing as are the other relations previously described.

Non-disjunction. In some experiments carried out by Mr. C. B. Bridges a race of flies was found that gave in 5% of cases exceptional results. There were 2½% of males that were like their father (patroclinous) when the expectation was that all of the males should be like their mother, in the sex-linked character in question; and there were 2½% of females that were like their mother (matroclinous) when the expectation was that they should all be like their father. From certain evidence that I can not now consider, Bridges was led to suspect that an additional sex chromosome was present in the mother that gave these results; and from certain genetic evidence it was predicted that this chromosome could be none other than the Y chromosome. A cytological study of these females has demonstrated that the inference was correct.

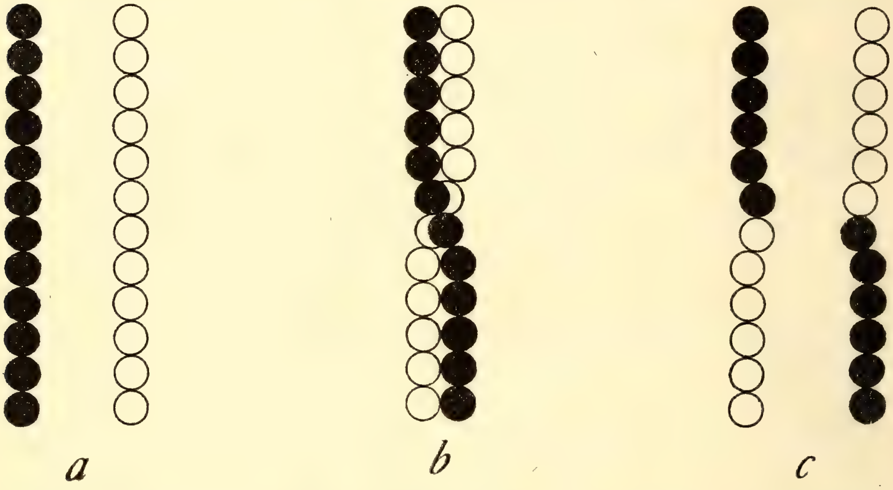

FIG. 5. DIAGRAM ILLUSTRATING CROSSING OVER OF THE MEMBERS OF A PAIR OF HOMOLOGOUS CHROMOSOMES REPRESENTED BY A SERIES OF BLACK AND A SERIES OF WHITE CIRCLES.

These XXY females contain three elements that might be expected to come together during conjugation. The following situation then arises: X and X may conjugate and later pass to opposite poles (one going into the polar body), and Y would then be left to go to either pole. Two classes of eggs would result namely XY and X eggs. On the other hand X and Y might conjugate and pass to opposite poles (one going into the polar body) and the other X would then be left to go to either pole. The two classes of eggs that would result are XX and XY. There would be expected then, in all, four classes of eggs from an XXY female. These might be fertilized by the female producing sperm of a male with the results shown in (1), (2), (3), (4) of figure 6; or by a male producing sperm with the results shown in the lower line of the same figure (5), (6), (7), (8). If the XXY female were a white eyed female (her two white bearing X's are represented by

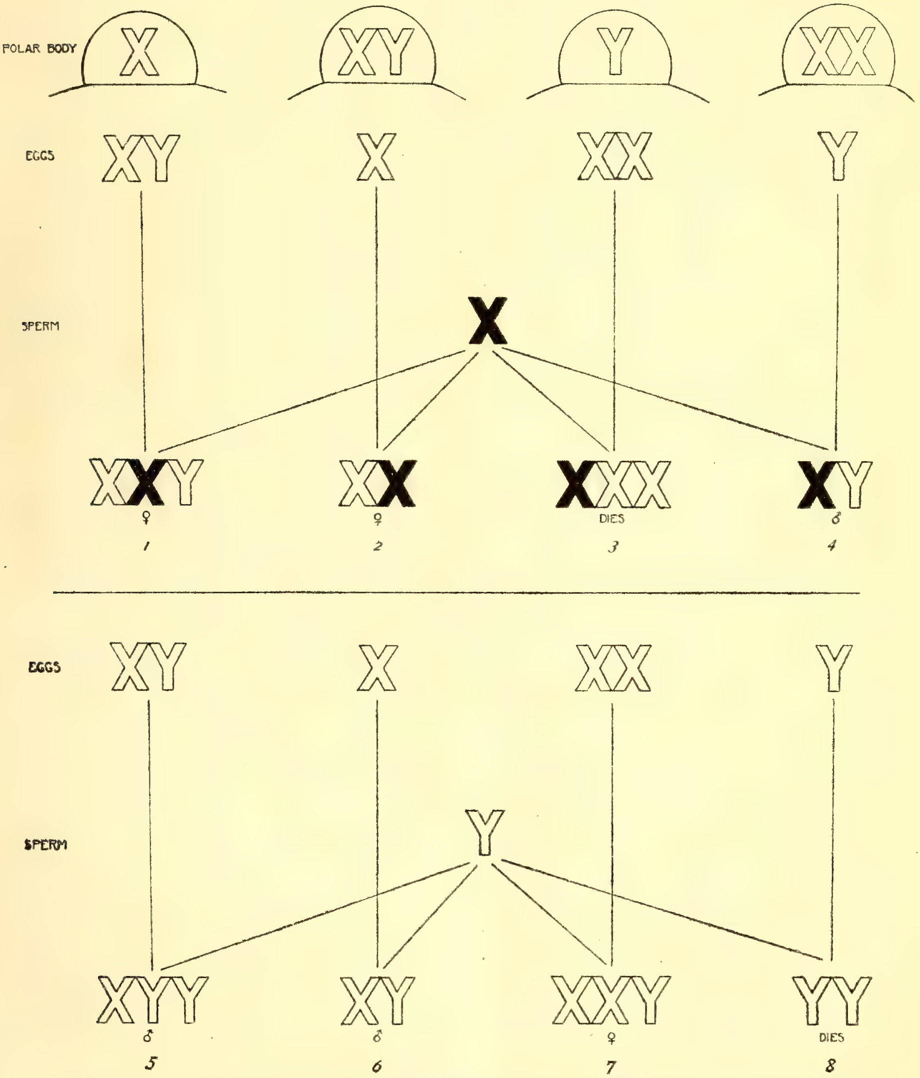
POLAR BODY
X
XY
Y
XX
EGGS
XY
X
XX
Y
SPERM
X
XXY
♀
1
XX
♀
2
XXX
DIES
3
XY
♂
4
EGGS
XY
X
XX
Y
SPERM
Y
XYY
♂
5
XY
♂
6
XXY
♀
7
YY
DIES
8

FIG. 6. DIAGRAM TO ILLUSTRATE THE PRODUCTION OF FOUR CLASSES OF EGGS OF A WHITE EYED FEMALE THROUGH NON-DISJUNCTION, AND THEIR FERTILIZATION BY AN X-BEARING SPERM, GIVING ZYGOTES 1, 2, 3, 4; AND BY Y-BEARING SPERM, GIVING ZYGOTES 5, 6, 7, 8.

the two open X's in the diagram) and if she were fertilized by a red eyed male (whose single X is represented by the black X in the diagram) the offspring will be expected to fall into the eight classes shown in the same diagram. The exceptional sons (4) arise when a Y bearing egg is fertilized by a female producing sperm (which here carries the factor for red). Thus a female producing sperm gives rise to a male, because maleness results from one X and not from anything else in the nature of the sperm that made him. The exceptional daughters (7) are the white eyed XXY females which arise when an XX egg is fertilized by a male producing sperm. Here again we see that femaleness is due to the occurrence of two X chromosomes, and it makes no difference whence these two X's have come.

There are many ways in which the hypothesis that non-disjunction is due to the presence of a Y chromosome, in the females that give these unique results, can be tested. For instance, the exceptional white females should repeat the non-disjunctional process. In fact, all such white females give non-disjunction. On the other hand the exceptional red males should neither give, nor transmit, non-disjunction, and this has been found to be true. The red females should be of two kinds; half (2) are expected to behave like normal females, and half (1) should show non-disjunction. These two kinds of females are found, and in equal numbers. There are two kinds of white males, XYY (5), and XY (6). The former should produce some XY spermatozoa. Such sperm would produce daughters, that would be XXY in composition, and from them non-disjunctional offspring are to be expected. This also has been tested and it has been found that half of the white males transmit through their daughters the peculiarity of non-disjunction. A diagram of a chromosome group of an XXY female is shown in figure 4.

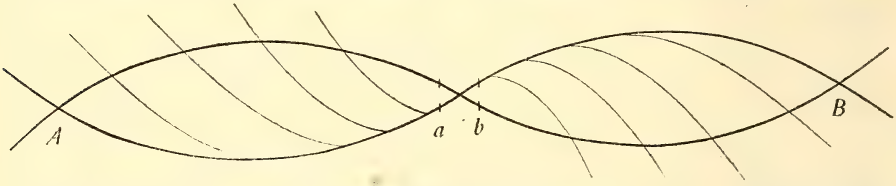

FIG. 7. DIAGRAM TO ILLUSTRATE INTERFERENCE: THE DISTANCE BETWEEN THE POINTS WHERE THE HEAVY LINES CROSS REPRESENTS THE AVERAGE DISTANCE BETWEEN TWO POINTS OF CROSSING OVER; THE LIGHTER LINES INDICATE THE LESS FREQUENT POINTS OF CROSSING OVER IN THE PROTECTED REGION.

Interference. If there is an average length of loop between crossing-over points, it would seem to follow that the region of the chromosomes on each side of the point where crossing over occurs would be protected, as it were, from another crossing over. This relation is illustrated in the diagram (fig. 7). If crossing over occurs at some point between two pairs of factors *a* and *b* indicated on the diagram, the next crossing over point of the chromosome would, if it occurred at all, lie at some distance away, rather than near by. This possibility can be tested in a case where several known points are present in a pair of chromosomes. If in such a case we determine how often crossing over occurs between *A* and *a* in general and then determine how often it occurs between *A* and *a* in those cases where it is known to have taken place between *a* and *b*, we find an enormous decrease in the number of times it occurs between *A* and *a* when at the same time it has occurred between *a* and *b*. In general one may say that crossing over at any level interferes with crossing over in the region of each side of that level.

Conclusions. The chromosomes not only furnish a mechanistic explanation of Mendelian heredity, but in the case of Non-disjunction and in the case of the point by point correspondence between the Linkage Groups and the chromosomes, furnish a *verifiable* explanation of the results. In the case of Crossing-over and of Interference the chromosomes give us the only objective explanation of the results that has been as yet offered.

RESEARCHES ON THE CHEMICAL AND MINERALOGICAL COMPOSITION OF METEORITES

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Presented to the Academy, June 1, 1915

The primary motive of these investigations was to test the authenticity of numerous reported occurrences of certain minor constituents, such as antimony, arsenic, gold, lead, tin, tungsten, uranium, zinc, etc., and incidentally to formulate the analyses in such a way that the results might be made comparable with those of terrestrial rocks. Upwards of twenty meteorites were subjected to searching chemical analyses, with the particular end in view stated above. The results were in part confirmatory and in part contradictory. In none of the samples tested, either metallic or stony, could any traces be discovered of antimony, arsenic, gold, lead, tin, tungsten, uranium, or zinc. The presence in traces of platinum, palladium, iridium, ruthenium, and vanadium was, however, proved beyond apparent question, ruthenium being noted for the first time, and vanadium having previously been reported but once.

Attention was also directed toward the possible occurrence of barium, strontium, and zirconium, particularly in the feldspathic forms. Only negative evidence was obtained. This is not regarded as conclusive, but the investigation was hampered by a paucity of material.

Incidental to the work, a determined effort was made to ascertain to what of the minerals these minor constituents should be relegated. These results have thus far been only partially successful. It is evident that the platinum, palladium, and ruthenium, and perhaps the vanadium as well, are constituents of the metallic portions. The same is true of at least a part of the nickel and cobalt, although determinations made on the silicate portions, after most careful work to remove all of the native metal, still showed traces of both of these elements, indicating that they were also constituents of the pyroxenes or olivines as is often the case in terrestrial rocks.

In attempting to ascertain the source of the phosphorus in the silicate portion, unexpected results were developed, the presence being shown of a phosphate of lime differing from normal apatite in being optically biaxial and with a somewhat lower index of refraction. This, together with the apparent widespread occurrence of the mineral oldhamite, a sulphide of calcium, as indicated by aqueous solutions, has been made the subject of a special paper in the May number of these PROCEEDINGS.

A table comprising fifty-nine selected analyses of stony meteorites was given, twelve of which were made during the course of the investigations here being considered, others being taken from published descriptions by other workers. This table, too large for reproduction here, will appear with the extended report in some future publication. The average composition of the stony meteorites, as shown by fifty-three of the analyses, is given in column I of the following table. In column II is shown, for purposes of comparison, the average composition of the terrestrial lithosphere as given by F. W. Clarke, while in columns III and IV are the meteoric averages presented in previous papers by the author and Dr. O. C. Farrington respectively.

	I	II	III	IV
SiO_2	38.68	59.85	38.732	39.12
TiO_2	0.18[1]	0.73	.....	0.02
SnO_2	none			0.02
ZrO_2	none	0.03		
Al_2O_3	2.88	14.87	2.7333	2.62
Fe_2O_3		2.63	0.835	0.38
Cr_2O_3	0.47[2]	0.05		0.41
V_2O_3	trace	0.02		
Fe	11.98		11.536	11.46
Ni	1.15[3]		1.312	1.15
Co	0.07[4]			0.05

	I	II	III	IV
FeO	14.58	3.35	16.435	16.13
NiO	0.48[5]	0.03		0.21
CoO	0.06[6]			
CaO	2.42	4.81	1.758	2.31
BaO	none	0.10		
MgO	22.67	3.77	22.884	22.42
MnO	0.29[7]	0.09	0.556	0.18
SrO	none	0.04		
Na_2O	0.87[8]	3.29	0.943	0.81
K_2O	0.21[9]	3.02	0.328	0.20
Li_2O	trace	0.01		
H_2O(Ign.)	0.75[10]	2.05		0.20
P_2O_5	0.26[11]	0.25		0.03
S	1.80[12]	0.10	1.839	1.98
Cu	0.014[13]			
C	0.15[14]	0.03		0.06
Cl	0.08[15]	0.06		
F	?	0.10		
CO_2	?	0.70		
SO_3		0.02		
Ni, Mn Cu, Sn				0.02
	100.044	100.00	100.00	99.82

[1] Average of 46 determinations

[2] Average of 42 determinations.

[3] Average of 50 determinations.

[4] Average of 41 determinations.

[5] Average of 19 determinations.

[6] Average of 6 determinations.

[7] Average of 33 determinations

[8] Average of 49 determinations.

[9] Average of 44 determinations.

[10] Average of 15 determinations.

[11] Average of 44 determinations.

[12] Average of 51 determinations.

[13] Average of 16 determinations

[14] Average of 8 determinations.

[15] Average of 5 determinations.

The detailed results of these investigations on the chemical and mineralogical constituents of meteorites begun in 1912 under a grant from the National Academy will be printed as a Memoir in the series of *Memoirs of the National Academy.*

ON THE REPRESENTATION OF ARBITRARY FUNCTIONS BY DEFINITE INTEGRALS

By Walter B. Ford

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Presented to the Academy, May 27, 1915

Let $f(x)$ be any function of the real variable x defined and with $|f(x)|$ integrable throughout the interval (a, b) and having limited total fluctuation in the neighborhood of the particular point $x = \alpha$ $(a < \alpha > b)$. Then, if $\varphi(n, x - \alpha)$ be a function of the parameter n and of $x-\alpha$ satisfying certain well known conditions the integral

$$I_n(\alpha) = \int_a^b f(x)\, \varphi(n, x-\alpha)\, dx \tag{1}$$

will have the property that

$$\lim_{n=+\infty} I_n(\alpha) = \frac{f(\alpha-0)+f(\alpha+0)}{2}. \tag{2}$$

An important special instance of an integral (1) having the property (2) is presented in the study of the convergence of the Fourier series for $f(\alpha)$, in which case the sum of the first $n+1$ terms of the series can be put into the form (1) with $a = -\pi$, $b = \pi$ and $\varphi(n, x-\alpha) =$ a certain trigonometric expression.

While the conditions upon $\varphi(n, x-\alpha)$ that will insure (2) have been extensively studied, especially by Du Bois Reymond, Dini, Hobson and Lebesgue, relatively little appears to have been done in the actual determination of such functions, the sole desideratum being the determination of the conditions themselves. In this connection the present paper would point out a noteworthy class of possible functions φ with special emphasis upon the corresponding integrals (1) to which they give rise. Four theorems are established, the second being especially noteworthy in that it shows that to every convergent improper integral of the form

$$\int_0^\infty p(x)\,dx = k \neq 0,$$

wherein $p(x)$ satisfies very simple conditions, there can be made to correspond a certain integral (1) having the property (2).

The theorems are as follows:

Theorem I: Let $F(x)$ be any single valued function of the real variable x defined for all finite values of x and satisfying the following three conditions:

(*a*) $$\lim_{x=+\infty} F(x) \text{ exists and } = k \neq 0$$

(*b*) $F(-x) = -F(x)$.

(*c*) The derivative $F'(x)$ exists and is such that if we exclude the point $x=0$ by an arbitrarily small interval $(-\epsilon, \epsilon)$, $(\epsilon>0)$, we shall have for all remaining values of x, $|x\ F'(x)| < A_\epsilon =$ an assignable constant depending only on ϵ. Then, if $f(x)$ be an arbitrary function of the real variable x defined throughout the interval (a, b), we shall have for any special value α $(a<\alpha<b)$

$$\lim_{n=+\infty} \frac{1}{2k}\int_a^b f(x)\frac{d}{dx}F[n(x-\alpha)]\,dx = \frac{f(\alpha-0)+f(\alpha+0)}{2}$$

provided merely that $f(x)$ satisfies suitable conditions (analogous to those under which the Fourier series for $f(\alpha)$ converges) in the neighborhood of the point $x=\alpha$.

Theorem II: Given any convergent improper integral of the form

$$\int_0^\infty p(x)\,dx = k \neq 0$$

wherein (a) the function $p(x)$ is even, i.e. $p(-x)=p(x)$, and (b) the expression $|x\,p(x)|$ for all values of x lying outside an arbitrarily small interval surrounding the origin remains less than a constant depending on the interval. Then if $f(x)$ be an arbitrary function of the real variable x defined throughout the interval (a, b), we shall have for any special value α $(a<\alpha<b)$

$$\lim_{n=+\infty} \frac{n}{2k}\int_a^b f(x)\,p[n(x-\alpha)]\,dx = \frac{f(\alpha-0)+f(\alpha+0)}{2},$$

provided merely that $f(x)$ satisfies suitable conditions in the neighborhood of the point $x=\alpha$.

Theorem III: Let $F(x)$ be any single valued function of the real variable x which, when considered for positive (negative) values only of x satisfies the following three conditions:

(a)

$$\lim_{x=+\infty} F(x) \text{ exists and } = k \neq 0 \quad \left(\lim_{x=-\infty} F(x) \text{ exists and } = -k = 0\right)$$

(b) $F(0)=0$.

(c) The derivative $F'(x)$ exists and is such that if we exclude the point $x=0$ by an arbitrarily small interval $(-\epsilon, \epsilon)$, $(\epsilon>0)$, we shall have for all remaining positive (negative) values of x, $|x\,F'(x)|<A_\epsilon=$ an assignable constant depending only on ϵ.

Then, if $f(x)$ be an arbitrary function of the real variable x defined throughout the interval (a, b), we shall have for any special value α $(a<\alpha<b)$

$$\lim_{n=+\infty} \frac{1}{k}\int_\alpha^b f(x)\,\frac{d}{dx}F[n(x-\alpha)]\,dx = f(\alpha+0)$$

$$\left(\lim_{n=+\infty} \frac{1}{k}\int_a^\alpha f(x)\,\frac{d}{dx}F[n(x-\alpha)]\,dx = f(\alpha-0)\right),$$

provided merely that $f(x)$ satisfies suitable conditions at the right (left) of the point $x=\alpha$.

Theorem IV: Given any convergent improper integral of the form

$$\int_0^{\infty} p(x)\,dx = k \neq 0 \qquad \left(\int_{-\infty}^{0} p(x)\,dx = k \neq 0\right) .$$

wherein $|x\, p(x)|$ for all positive (negative) values of x lying outside an arbitrarily small interval to the right (left) of the point $x=0$ remains less than a constant depending on the interval.

Then, if $f(x)$ be an arbitrary function of the real variable x defined throughout the interval (a, b), we shall have for any special value α $(a<\alpha<b)$

$$\lim_{n=+\infty} \frac{n}{k}\int_{\alpha}^{b} f(x)\, p[n(x-\alpha)]\,dx = f(\alpha+0)$$

$$\lim_{n=+\infty} \frac{n}{k}\int_{a}^{\alpha} f(x)\, p[n(x-\alpha)]\,dx = f(\alpha-0),$$

provided merely that $f(x)$ satisfies suitable conditions in the neighborhood at the right (left) of the point.

The proof of Theorem I follows directly from the fact (see for example, Dini's *Serie di Fourier* (Pisa, 1880), pp. 119–121) that if $f(x)$ satisfies the indicated conditions there exists the general relation

$$\lim_{n=+\infty} \int_{a}^{b} f(x)\,\varphi(n, x-\alpha)\,dx = \frac{f(\alpha-0)+f(\alpha+0)}{2},$$

whenever $\varphi(n, t)$ is any function of the independent variables n and t satisfying the following three conditions, ϵ always denoting an arbitrarily small positive quantity:

(I) $$\lim_{n=+\infty}\int_0^t \varphi(n,t)\,dt = \begin{cases} -\frac{1}{2} \text{ when } -\epsilon<t<0 \\ +\frac{1}{2} \text{ when } \quad 0<t<\epsilon \end{cases}$$

(II) $\left|\int_0^t \varphi(n,t)\,dt\right| < c_1$ when $-\epsilon<t<\epsilon$, c_1 being a constant (dependent only on ϵ)

(III) $|\varphi(n,t)| < c_2$ when $\left\{\begin{matrix} a-\alpha<t<-\epsilon \text{ or} \\ o<t<b-\alpha \end{matrix}\right\}$, c_2 being a constant (dependent only on ϵ)

Theorem II is a corollary of Theorem I.

Theorem III results from the fact (cf. Dini, l.c.) that if the conditions (I), (II), (III) above hold only for the positive (negative) values of t there specified, then, whenever $f(x)$ satisfies suitable conditions at the right (left) of the point $x=\alpha$, we may write

$$\lim_{n=+\infty} \int_{\alpha}^{b} f(x)\,\varphi(n, x-\alpha)\,dx = \tfrac{1}{2} f(\alpha+0)$$

$$\left(\lim_{n=+\infty} \int_{a}^{\alpha} f(x)\,\varphi(n, x-\alpha)\,dx = \tfrac{1}{2} f(\alpha-0)\right)$$

Theorem IV is a corollary of Theorem III.

While the forms of representation for an arbitrary function $f(x)$ afforded by the preceding theorems do not, strictly speaking, represent the function in terms of definite integrals, but rather in terms of the limits of such integrals as the parameter n increases to $+\infty$, it is to be observed that the first member of (2) may always be expressed as a convergent series, viz:

$$I_0(\alpha) + \sum_{n=0}^{\infty} [I_{n+1}(\alpha) - I_n(\alpha)]$$

and thus it appears that to every integral (2) obtained by any one of the preceding theorems there corresponds an actual representation of the arbitrary function in series of definite integrals.

THE LYMPHOCYTE AS A FACTOR IN NATURAL AND INDUCED RESISTANCE TO TRANSPLANTED CANCER

By James B. Murphy and John J. Morton

ROCKEFELLER INSTITUTE FOR MEDICAL RESEARCH, NEW YORK

Presented to the Academy, June 22, 1915

Histologically there is a striking resemblance between the series of phenomena which take place about a failing tissue graft in a host of a foreign species, and an homologous cancer graft in an animal with a natural or induced immunity to transplanted cancer. A constant finding in both cases is a local lymphoid reaction which appears early in the process, and lasts till the destruction of the tissue or cancer graft is complete. We have shown in previous communications that the lymphoid tissue is apparently the important factor in the destruction of a tissue graft in an animal of a foreign species. The facts which lead up to this conclusion are, that an organism like the chick embryo, which normally has no defensive agents against the cells of a foreign species, if supplied with adult lymphoid tissue becomes as resistant as the adult animal in this respect. Furthermore when the adult animal is deprived of the major portion of its lymphoid tissue by repeated small doses of X-ray, it loses the ability to destroy the cells of a foreign species and these will live and grow as well as they would in a native host. It seemed of importance from these observations to study the lymphocytes in animals immune to transplanted cancers.

There are two types of this so-called immunity; the natural, possessed normally by a certain variable proportion of animals inoculated; and the induced variety, obtained by a previous treatment of the animal with an injection of homologous living tissues. We have studied the circulating lymphocytes in these two types of immunity and for comparison in susceptible mice with growing tumors.

In the mice with induced immunity the circulating lymphocytes show no change either in actual numbers or relative proportion to the other white cells during the ten days which must elapse between the immunizing tissue injection and the cancer inoculation. Within twenty-four hours after the cancer graft is introduced, however, there is a sharp rise in the number of these cells amounting to an average increase for the group of 100% above the former level. This increase continues with slight variations for something over 50 days, with a maximum average for the series of between 200 and 300% above the normal. The other white cells of the blood retain their normal level.

The natural immune animals, those which without treatment were able to overcome the cancer graft, showed a similar response on the part of the lymphocyte to that seen in the preceding group. The period of increase, however, is not evident for several days or a week after inoculation and the average maximum is not so high, being between 100 and 200% above the normal. Like the first group, there is slight, if any change in the actual numbers of the other white cells of the blood.

Animals in which the cancer graft resulted in a take, showed no such lymphoid response as did the immune animals. A composit curve of the white cells plotted for a number of such animals showed a slight tendency on the part of the lymphocyte to increase during the first two weeks, but this was followed by a gradual decline in number as the cancer increased in size. The polymorphonuclear cells during this period showed an actual increase in numbers.

In order to ascertain the importance of the lymphoid reaction in the immunity process, we have destroyed the major portion of the lymphoid tissue in mice having one of the two types of immunity. This was done by giving the animal several small exposures to X-ray, previously estimated to be sufficient for the purpose. In the induced immunity the X-ray was given between the time of the immunizing tissue injection and the cancer inoculation. This treatment resulted in the complete destruction of the immunity and the inoculated cancers grew more readily than in the normal animals.

For testing the importance of the lymphoid tissue in natural immunity we can only compare the percentage of takes in X-rayed and normal animals inoculated with the same cancer. This has been done with a variety of different cancers and a large series of animals. The average number of takes in the X-rayed animals was 94%, while in the untreated animals only 32% of those inoculated grew the cancer. This shows a very considerable destruction of the natural immunity accompanying a destruction of the lymphocytes.

To summarize, we have shown that a marked increase in the circulating lymphocytes occurs after cancer inoculation in mice with either a natural or induced immunity. When this lymphoid reaction is prevented by a previous destruction of the lymphoid tissue with X-ray the immune states are destroyed. Hence it would seem fair to conclude that the lymphocyte is a necessary factor in cancer immunity.

SOME THEOREMS CONNECTED WITH IRRATIONAL NUMBERS

By William Duncan MacMillan

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Presented to the Academy, May 27, 1915

As is well known to those who have investigated the fields of celestial mechanics, the series which arise there from the integration of the equations of motion involve factors of the form $(i-j\gamma)$ in the denominators of the coefficients, where i and j run over the entire series of positive integers and γ is a positive number which may be rational or irrational. Previous to the time when Poincaré had shown the existence and construction of periodic solutions (in which γ is always rational) it had been the custom for the astronomers to regard γ as irrational since with this hypothesis the factors $(i-j\gamma)$ never vanish and consequently non-periodic terms did not arise in the solutions. The presence of these factors in the denominators naturally led to very grave doubts as to the convergence of these series since there are infinitely many such factors which are smaller than any assigned limit, and the convergence has never been proved.

Considerations of this nature have led me recently to examine the convergence of simple types of power series in which this phenomenon occurs, and it has been found that the series $\Sigma\Sigma\, a_{ij}/(i-j\gamma)\; x_1^i\, x_2^j$ has precisely the same domain of convergence as the series $\Sigma\Sigma a_{ij}\; x_1^i\, x_2^j$, provided γ is a positive irrational number which satisfies a rather mild condition, which is stated below.

Further investigations in this field have resulted in the three following theorems:

Theorem I. If γ is any positive number, rational or irrational, and if p_n/n is a rational fraction such that $|p_n - n\gamma| \leqq \frac{1}{2}$, and if $A_n = \frac{1}{n}\sum_{\kappa=1}^{n}(p_\kappa - \kappa\gamma)$ is the arithmetic mean of the first n of the quantities $(p_\kappa - \kappa\gamma)$, signs considered, then the limit of A_n, as n increases without limit, is zero.

If $\gamma = p/q$ is rational with an even denominator q then for certain values of κ there are two integers p_κ which differ by unity such that $|p_\kappa - \kappa\gamma| = \frac{1}{2}$; for one, the value is $+\frac{1}{2}$ and for the other it is $-\frac{1}{2}$. It is supposed that such terms are taken alternately $+\frac{1}{2}$ and $-\frac{1}{2}$.

Theorem II. If γ is a positive number, and if p_n/n is a rational fraction such that $|p_n - n\gamma| \leqq \frac{1}{2}$, and if $A_n = \frac{1}{n}\sum_{\kappa=1}^{n}|p_\kappa - \kappa\gamma|$ is the arithmetic mean of the first n of the quantities $|p_\kappa - \kappa\gamma|$, signs discarded, then the limit of A_n, as n increases without limit, is $\frac{1}{4}$ if γ is irrational or rational with an even denominator; but if γ is rational with an odd denominator, $\gamma = p/q$, then the limit of A_n is $(q^2 - 1)/4q^2$.

Theorem III. If γ is a positive number, and if p_n/n is a rational fraction such that $|p_n - n\gamma| \leqq \frac{1}{2}$, and if $G_n = \left(\prod_{\kappa=1}^{n}|p_\kappa - \kappa\gamma|\right)^{\frac{1}{n}}$ is the geometric mean of the first n of the quantities $|p_\kappa - \kappa\gamma|$, then the limit of G_n, as n increases without limit, is zero if γ is rational, and is equal to $1/(2e)$ where $e = 2.71828\ldots$ is the naperian base, if γ is an irrational number which satisfies the condition

$$a_{n+1} \leqq M\, q_n\, (q_{n+1}) \ldots (q_{n+s}),$$

where γ, expressed as a simple continued fraction, is

$$\gamma = a_1 + \frac{1}{a_2+}\,\frac{1}{a_3+}\cdots,$$

q_n is the denominator of the n^{th} principal convergent, and s any assigned positive integer independent of n. If γ is an irrational number which does not satisfy this condition then G_n for large values of n oscillates between zero and $1/(2e)$.

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WEBER'S LAW AND ANTAGONISTIC SALT ACTION

By Jacques Loeb
ROCKEFELLER INSTITUTE FOR MEDICAL RESEARCH, NEW YORK

Presented to the Academy, June 26, 1915

1. In a series of papers beginning in 1900 I have shown that: (*a*) It is necessary for the normal functions of living organs and organisms that the ratio of the concentration of antagonistic ions or salts $C_{\mathrm{I}}/C_{\mathrm{II}}$ of the surrounding solution be kept within certain limits; if the value of this quotient becomes either too high or too low, life phenomena become abnormal and finally impossible.[1] (*b*) The salts to be considered as antagonistic in this sense are in the first place those of univalent and bivalent metals and that therefore the most important critical quotient will generally be $C_{\mathrm{Na+K}}/C_{\mathrm{Mg+Ca}}$. (*c*) There is also an antagonism between the salts of bivalent ions such as Sr and Ca, and Mg and Ca.[2] In the following discussion the antagonism between Na and K on the one hand, and Mg and Ca on the other will be solely considered.

I further suggested that if the value of $C_{\mathrm{I}}/C_{\mathrm{II}}$, e.g., $C_{\mathrm{Na+K}}/C_{\mathrm{Mg+Ca}}$ had once reached the lower limit required for the normal process of life phenomena the alternate reversible replacement of one of the two groups of ions by their antagonists in combinations with certain colloids (proteins or fatty acids) of the cells determined the alteration between activity and rest of the organs or the organisms.

This idea was carried further by Lasareff, who showed that from these data not only Nernst's law of electrical stimulation could be derived[3] but also the phenomena of vision in weak light in which the effect depends solely on the bleaching of visual purple by the light.[4]

Life phenomena as a rule take place in a medium whose composition and concentration undergoes little or no variation, such as sea water or blood serum, and the majority of organisms cannot stand any wide variation from this fixed standard. For such organisms only one value for C_{Na+K} can be considered and hence only the lower and upper limit for C_{Mg+Ca} can be determined for this value. A few organisms can stand wide variations of osmotic pressure and hence in their case the value C_{Na+K} can undergo wide variations. The question arises: How do the minimal and maximal values of C_{Mg+Ca} change when C_{Na+K} varies? It is this question which will be discussed in this paper.

2. The animals used in these experiments were the newly hatched larvae of a barnacle (*Balanus iberneus*). These larvae upon hatching are positively heliotropic and form a dense cluster on the light side of the aquarium, so that they can be collected in a pipette with comparatively little sea water. They are able to live not only in normal sea water but also in sea water which is diluted with eight times its volume of distilled water, as well as in sea water whose concentration is raised almost 50%.

When such larvae are put into a mixture of NaCl, KCl and $CaCl_2$ of the concentration and proportion of sea water, the majority will not swim but fall to the bottom of the dish. If, however, we increase the amount of $CaCl_2$ in the solution the larvae instead of falling to the bottom will swim, as they would in normal sea water, and will collect at the side of the dish nearest to or most remote from the light. Instead of adding more $CaCl_2$ we may add $MgCl_2$ or $SrCl_2$.

3. If the bivalent cations contained in the solution are of one kind only, namely either $CaCl_2$ or $MgCl_2$ or $SrCl_2$, and if they are in sufficient concentration to allow the animals to swim, the latter will do so as a rule only for about five minutes. If we wish the animals to swim permanently we must add a mixture of two bivalent cations, e.g., Ca and Mg.

In the following experiments a mixture of $CaCl_2$ and $MgCl_2$ was always used to supply the bivalent cations. This mixture contained the two cations in that ratio in which they occur in the sea water, namely 1.5 molecules of $CaCl_2$ to 11.6 molecules of $MgCl_2$. The concentration of the mixture used was $\frac{3}{8}$ grammolecular.

4. When we put the animals into a mixture of 50 cc. NaCl + KCl (no matter what concentration) they will fall to the bottom and are unable to make sufficiently vigorous swimming motions, although they may live in such a solution for a day or longer. Various concentrations of the mixture of NaCl + KCl (in the proportion in which these salts exist in the sea water) were prepared and it was ascertained what was the minimum amount of $CaCl_2 + MgCl_2$ necessary to induce all (or practically all) the animals to make normal swimming movements and to collect in a dense cluster at the window side (or the opposite side) at the surface of the dish.

The absolute value of Ca + Mg required was not always identical for the same concentration of NaCl + KCl in different experiments; this was probably due to differences in temperature or in illumination or in the condition of the larvae. It was, therefore, necessary to compare only experiments made simultaneously with the same material. As a rule two sets of experiments were made simultaneously for two different concentrations of NaCl + KCl; the two concentrations of NaCl + KCl were usually chosen in the ratio of 1:2 or 1:4.

Table I gives some of the results. The figures for CaCl + $MgCl_2$ indicate the number of cc. $\frac{3}{8}$ m $CaCl_2$ + $MgCl_2$ required to just allow all the animals to swim and to collect in a dense heliotropic cluster; a diminution of this quantity by 0.1 cc. resulted, as a rule, in a diffuse and incomplete collection while with a further diminution by 0.1 cc. only a small percentage of larvae rose to the suface. The endpoint or threshold according to our criterion was therefore usually sufficiently sharp to allow different unbiased observers to select the same dish as representing the threshold in a series.

TABLE I

MINIMAL AMOUNT OF $CaCl_2 + MgCl_2$ REQUIRED TO ALLOW THE LARVAE TO SWIM IN 50 CC. (NaCl + KCl) OF THE FOLLOWING CONCENTRATIONS

NUMBER OF EXPERIMENT	CONCENTRATION NaCl + KCl	CC. OF 3/8 M $CaCl_2 + MgCl_2$ REQUIRED	VALUE OF $\frac{C_{Na+K}}{C_{Mg+Ca}}$
I.	m/16	0.3	27.8
	m/8	0.4 to 0.5	37.0
II.	m/8	0.5	33.3
	m/4	0.9 to 1.0	35.1
III.	3/16 m	0.7	35.7
	3/8 m	1.3	38.5
IV.	m/8	0.5	33.3
	m/2	1.8 to 1.9	36.0
V.	m/4	0.8 to 0.9	39.2
	m/2	1.6 to 1.7	40.3
VI.	5/16 m	0.9	46.3
	5/8 m	1.7	49.0
VII.	3/16 m	0.6	41.7
	6/8 m	2.4	41.7

It was necessary to wait four hours or still better a day, before the final decision was made, as in the very weak as well as in the strong solutions the larvae fall first to the bottom and will not rise until after some hours.

Other experiments gave similar results.

If we select for comparison the experiments with the moderate concentrations (between m/8 and m/2 NaCl + KCl) *we find that the value of* C_{Na+K}/C_{Mg+Ca} *varies little around 35.0 or in other words that the concentration of CaCl + MgCl required increases in direct proportion with the concentration of NaCl + KCl;* as far as the degree of accuracy of these experiments permits it to be stated. In concentrations above m/2 NaCl the animals will usually not all rise or if they do they soon fall to the ground again and this makes the determinations of the endpoint difficult. The value of $MgCl_2 + CaCl_2$ to be added to m/16 NaCl is higher than the theory demands and therefore the law does not hold strictly in this case. *The law of direct proportionality found for moderate concentrations is therefore Weber's law.* As is well known, Weber's law is not strictly correct for very low or very high intensities and the deviation is in the same sense as in our experiments, namely as if the weakest concentration tried were stronger than it actually is.

TABLE II

Maximal Amount of $CaCl_2 + MgCl_2$ Which Allows the Larvae to Swim in 50 cc. (NaCl + KCl) of the Following Concentrations

CONCENTRATION NaCl + KCl	MAXIMAL AMOUNT OF $CaCl_2 + MgCl_2$	VALUE OF $\frac{C_{Na+K}}{C_{Mg+Ca}}$
50 cc. m/16 NaCl + KCl	> 5 cc. 3/8 m $CaCl_2 + MgCl_2$	1.67
50 cc. m/8 NaCl + KCl	> 8 cc. 3/8 m $CaCl_2 + MgCl_2$	2.08
50 cc. m/4 NaCl + KCl	>14 cc. 3/8 m $CaCl_2 + MgCl_2$	2.38
50 cc. m/2 NaCl + KCl	>25 cc. 3/8 m $CaCl_2 + MgCl_2$	2.67

5. According to the writer's theory the swimming motions should also cease when the value of C_{Na+K}/C_{Mg+Ca} falls below a certain limit. It is more difficult to determine this endpoint than the other one, since with an increasing concentration of Ca + Mg the percentage of the larvae able to swim diminishes very gradually.

When the larvae are put into solutions with an excess of $CaCl_2 + MgCl_2$ they all swim at first but in a few minutes they fall to the bottom though they will live for a long time. The power of rising to the surface and forming a permanent dense heliotropic cluster no longer existed when more than the following quantities of 3/8 m $CaCl_2 + MgCl_2$ were added to 50 cc. of NaCl + KCl (Table II). The final observations were usually taken after two days.

Considering the difficulty in getting a sharp endpoint, these values are in good agreement with Weber's law of proportionality. It is, therefore, apparent that the upper limit of the value C_{Na+K}/C_{Mg+Ca}, which just annihilates the power of free swimming motions, remains also approximately constant, namely about 2, if the value of C_{Na+K} varies. Attention should be called to the fact that this value seems to rise slowly with the concentration of NaCl + KCl.

6. It is occasionally stated that the bivalent cations are the stimulating ions. This can be disproved if we prepare solutions of $MgCl_2$ + $CaCl_2$ in m/2 cane sugar or m/2 glycerine. It is impossible to induce the larvae to swim in such solutions. Neither can we induce them to swim if we add NaCl + KCl to a m/2 cane sugar or glycerine solution. This is in harmony with the law that the normal activity is only possible if the value of the quotient C_{Na+K}/C_{Mg+Ca} keeps within certain limits, but contradicts the idea that Ca or Mg stimulate the animals into activity.

7. It has been stated that in a m/2 solution of NaCl + KCl + $CaCl_2$, in the normal proportions, none or few of the larvae can swim, but that on addition of either more $CaCl_2$ or $MgCl_2$ or both the larvae will swim. We can bring about the same effect if we add instead of $MgCl_2$ or more $CaCl_2$ simply some alkali, e.g., 0.8 cc. N/100 NaOH or 0.5 cc. m/10 $NaHCO_3$ to 50 cc. of the solution of NaCl + KCl + $CaCl_2$. The larvae will after some time rise to the surface and gather permanently in a cluster although no $MgCl_2$ is present. This fact brings these phenomena into a close parallel with the phenomena of fertilization. I have shown that the eggs of sea urchins and other marine animals cannot be fertilized in a mixture of NaCl + KCl although the sperm remains active for a long time in such a solution; that the addition of $CaCl_2$ allows the fertilization of a few eggs, but that the addition of either $MgCl_2$ or NaOH (or $NaHCO_3$) may allow all the eggs to be fertilized.[5] This is mentioned to indicate that the phenomena discussed in this paper are of a wide biological application.

Summary. I had formerly shown that the normal functions of an organism are only possible if the value C_{Na+K}/C_{Mg+Ca} remains with certain limits *A* and *B*. In this paper this value has been investigated in an animal which stands wide variations of C_{Na+K} and it was found that the values *A* and *B* remain approximately constant if C_{Na+K} changes. This fact is the expression of the Weber-Fechner law. Since this law underlies many phenomena of stimulation it appears possible that changes in the concentration of antagonistic ions or salts are the means by which these stimulations are brought about, as suggested in my ion-protein theory and by the investigations of Lasareff.

[1] Loeb, *The dynamics of living matter*, New York, 1906. Physiologische Tonenwirkung, *Oppenheimer's Handbuch*, Vol. 2, Jena, 1909.

[2] Loeb, *Amer. J. Physiol.*, **3**, 434 (1899); **6**, 411 (1902); and *J. Biol. Chem.*, **1**, 427 (1906).

[3] Lasareff, *Arch. ges. Physiol.*, *Bonn*, **135**, 196 (1910).

[4] Lasareff, *Arch. ges. Physiol.*, *Bonn*, **154**, 459 (1913).

[5] Loeb, *Science*, **40**, 316 (1914); *Amer. Nat.*, *Boston*, **49**, 257 (1915).

THE POLARIZED FLUORESCENCE OF AMMONIUM URANYL CHLORIDE

By E. L. Nichols and H. L. Howes

PHYSICAL LABORATORY, CORNELL UNIVERSITY

Read before the Academy, April 20, 1915. Received June 2, 1915

The remarkable fluorescence spectrum of ammonium uranyl chloride ($UO_2Cl_2 2NH_4Cl+2H_2O$), which has been described in a recent paper read before the American Physical Society, consists of several equidistant groups of bands. Each group contains five nearly equidistant bands, *b*, *c*, *d*, *e*, and *a* (fig. 1) and the spacing repeats itself with such precision in successive groups that the homologous bands, b_1, b_2, b_3,—c_1, c_2, c_3 (see fig. 2) form series having a common and constant frequency interval.

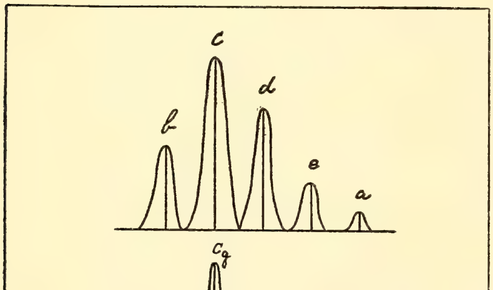

FIG. 1

Observations at the temperature of liquid air show that these bands are really doublets, unresolved at +20° but separated at low temperatures; a dim companion of the band as observed at +20° increasing greatly in brightness as the temperature falls while the dominant member becomes relatively feeble or in some instances disappears altogether. At low temperatures, as is usual with the uranyl compounds, all the bands are very narrow so that overlapping components which are entirely indistinguishable at the temperature of the room appear as separate bands. The relative brightness of the components, and also the effect of temperature upon them, varies greatly with different bands.

Ammonium uranyl chloride crystallizes in triclinic plates, some of which show a marked pleochroism. According to Mr. D. T. Wilber, to whom we are indebted for the preparation of this and the other uranyl compounds mentioned in this paper:

> The strong pleochroism is observed in crystals tabular parallel to the face c(001) the crystals being set to correspond with the isomorphous potassium uranyl chloride as described in Groth's *Chemische Krystallographie,* volume 1, p. 594. The transmitted light is polarized with one component vibrating nearly parallel to the edge between the c(001) face and the b(010) face. This is less absorbed, the transmitted light appearing nearly white in plates 1 mm. or less in thickness. The component at right angles to this is strongly absorbed in the blue and violet and appears deep greenish yellow even in much thinner layers.

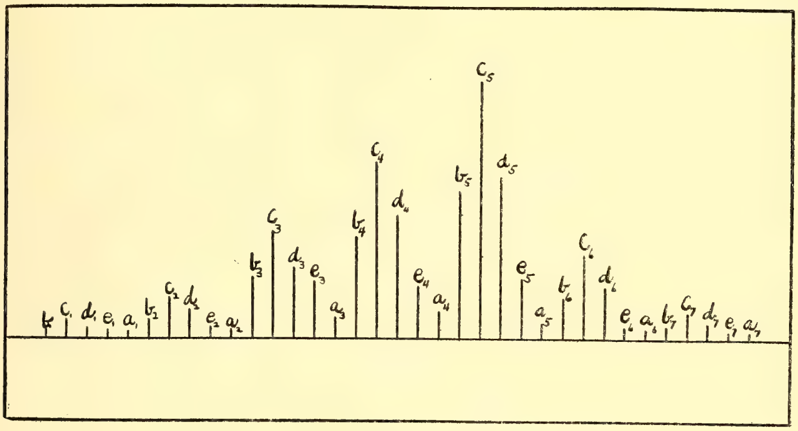

FIG. 2

We shall designate these directions of polarization simply as *white* and *green.*

When the violet rays from an electric carbon arc are transmitted through the slit of a spectroscope and the fluorescence and absorption bands are observed through a Nicol prism, marked changes are found to occur as the Nicol is turned with its plane polarization parallel successively to the green and white planes of the crystal.[1]

The two components of the fluorescence and absorption may be conveniently observed simultaneously by means of the following arrangement of apparatus which is that employed in our determinations.

In the collimator of a spectroscope a doubly refracting rhomb was so mounted as to give vertically displaced images of the slit, and the length of the slit was adjusted so as to render these images contiguous but not overlapping. Two spectra one above the other were thus produced polarized vertically and horizontally and when the crystal to be studied was mounted before the slit and carefully adjusted by rotation until its planes of polarization were likewise vertical and horizontal these spectra gave the fluorescence and absorption for the *green* and *white* components respectively. A suitable light filter of deep blue glass which was opaque to the entire region occupied by the fluorescence so that the bands appeared upon a black background was placed in the path of the incident light. It transmitted light of the wave-lengths necessary to excitation, however, and this included the region of the absorption spectrum, so that fluorescence bands and absorption bands were simultaneously visible in both spectra.

Complete separation of the two polarized components, even when the adjustments are accurately made, occurs only when the crystal is perfect and many specimens were tested and discarded before one suitable for study was found. The best results were obtained with small thin crystals newly formed and immediately protected from the air by a layer of Canada balsam.

Comparisons of the fluorescence bands by the method just described show that in the various groups each band consists of two components; thus confirming in general the conclusions reached by the study of the unpolarized fluorescence at +20°C. and −185°. These components moreover are polarized at right angles to each other. The stronger fluorescence, as might perhaps be expected, is polarized in the *green* direction, corresponding with the greater absorption.

The location and roughly approximate relative intensity of the fluorescence of each group is indicated in figure 1, in which for a single typical group the bands of the unpolarized spectrum are shown and, below, the *green* and *white* components. From this figure it will be seen that all except the band b appear as oppositely polarized doublets. This band in the *green* spectrum is absent or too dim for detection. We know however that at −185° a component at the position b' takes the place of b hence it is probably only a question of insufficient brightness. The component d_w is so dim as to be seen only with great difficulty and the same is true of a_w.

For purposes of measurement photographs were made of the spectra of this compound and also of the fluorescence and absorption of crystals of potassium uranyl chloride, caesium uranyl chloride, and rubidium uranyl chloride prepared for us by Mr. Wilber. In photographing absorption bands a nitrogen filled tungsten lamp was substituted for the arc and a light blue filter was sometimes used in place of the deep blue screen already described. Owing to the very great range of intensity in the fluorescence bands and the great difference in the transmission. within the region of absorption for the *white* and *green* directions respectively it was necessary to make many exposures of varying duration in order to obtain a complete record of the spectra.

When the transmitted light is polarized and parallel to one of the planes of vibration of the crystal, only one absorption spectrum is observed and the completeness of extinction of the other affords a delicate test for the adjustment of the apparatus and for the homogeneity of the specimen. It is however significant that the two polarized fluorescence spectra are always present, provided the emission is that leaving the crystal in the direction described in an earlier paragraph, and these spectra are the same whether the exciting light is polarized in the *white* or the *green* direction or whether unpolarized light is used for excitation. The exciting light, moreover, may enter the crystal in any direction without affecting the character of the fluorescence spectra.[2]

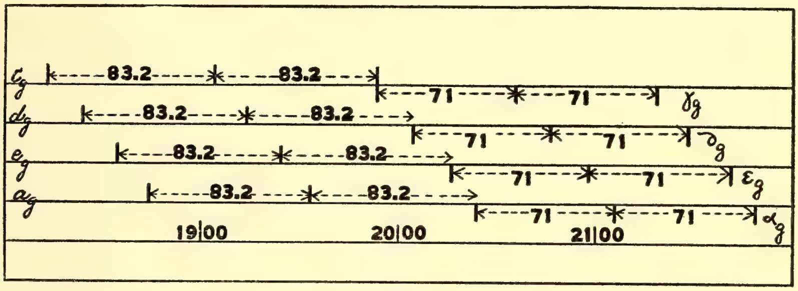

FIG. 3

This is in accord with the general principle established by the study of numerous cases of fluorescence,[3] i.e., that a fluorescence band is independent, as regards its location and character, of the nature of the excitation.

It follows moreover that a beam of plane polarized light is capable of exciting not only the polarized fluorescence which has its vibration in the same plane *but likewise that component of the fluorescence which is polarized at right angles to the exciting light.* This would seem to remove the fluoresence of these crystals and presumably the very similar fluorescence of the other uranyl compounds from the class designated by Wood as resonance spectra.

The absorption spectrum of ammonium uranyl chloride, observed under the conditions described above, is no less remarkable than the spectrum of the fluorescent light.

It extends from just about 0.5 μ into the ultraviolet and consists, like the fluorescence spectrum, of several series of equidistant bands having a common frequency interval.

This interval, expressed arbitrarily in the form $10^3/\lambda$ where μ is the unit of wave length, is approximately 71.0 whereas the interval for the fluorescence is 83.2.

Our earlier observations, without regard to polarization had led to the recognition of four series designated as α, β, γ, δ, and three of these appeared to consist of doublets.

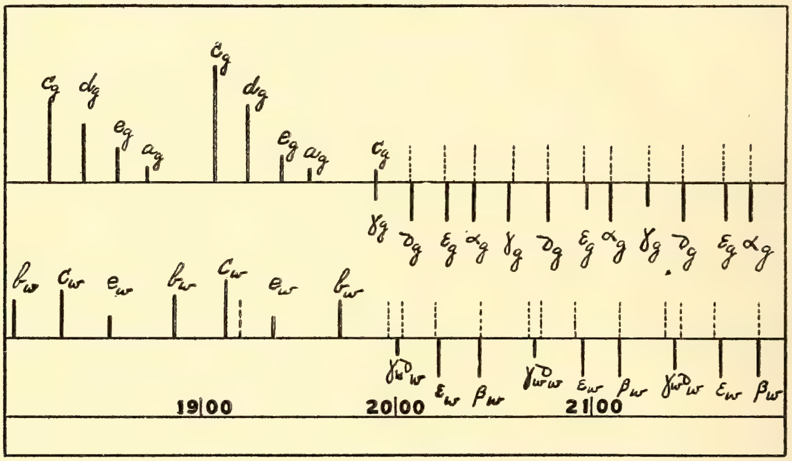

FIG. 4

The relation between absorption and fluorescence at +20° is briefly as follows:

In the region lying between 0.5075μ and 0.4900μ which is occupied by the fluorescence group of shortest wave length, and which we have termed the reversing region, there is an absorption band for each fluorescence band and coincident as to position. The reversing region is the most unsatisfactory for the study of fluorescence on account of this coincidence of fluorescence and absorption. It is however possible by the use of suitable screens, to almost entirely suppress fluorescence and to locate the absorption bands of this region. Towards the red all the fluorescent bands form series with a common constant interval of 83.2. Towards the violet all the absorption bands belong to similar series starting from these reversible bands with a common constant interval of 71.0. (See fig. 3.)

If these remarkable relations are fundamental rather than fortuitous they should be found to exist where the spectra are resolved into their polarized components: i.e., the absorption spectrum of each component should consist only of series corresponding to the several fluorescence series of that component and each series of the fluorescence spectrum should have its related series of absorption bands. The rigorous test is difficult because of the incomplete separation of the components in most crystals. That the relation holds however with slight discrepancies, obviously ascribable to the necessarily imperfect conditions as regards polarization, will be seen from figure 4.

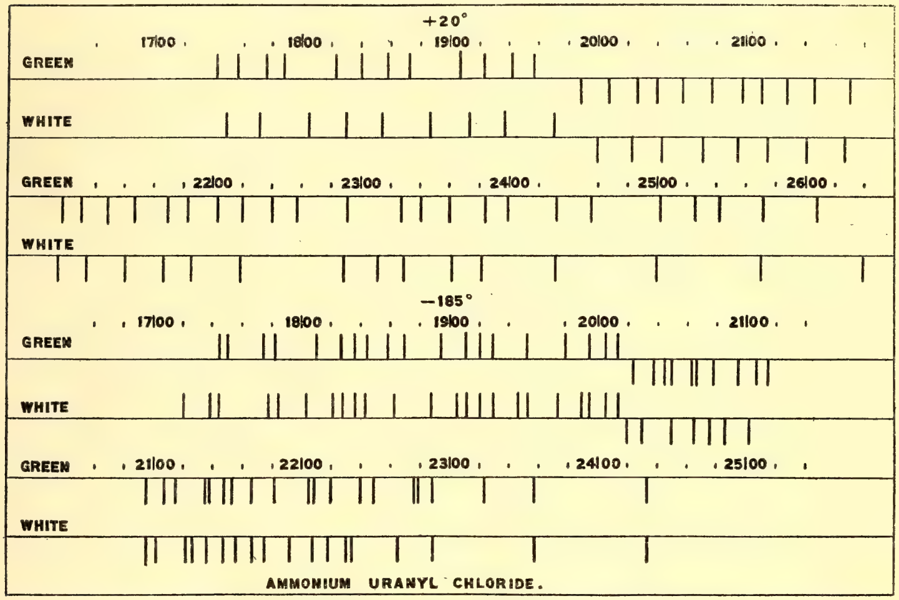
+20°
17100
18100
19100
20100
21100
GREEN
WHITE
GREEN
22100
23100
24100
25100
26100
WHITE
−185°
17100
18100
19100
20100
21100
GREEN
WHITE
GREEN
21100
22100
23100
24100
25100
WHITE
AMMONIUM URANYL CHLORIDE.

FIG. 5

In this diagram the reversing region, together with two groups of fluorescence bands, to the left, and of absorption bands to the right, is shown for the green and white components of the spectrum. The location of fluorescence bands is indicated by solid vertical lines above the base line, absorption by lines below. Fluorescence is designated by b_g, c_g, d_g, e_g, a_g for the green component and b_w, c_w, d_w, e_w, a_w for the white component. The absorption series related to the above are marked β_g, γ_g, δ_g, ϵ_g, α_g, and β_w, γ_w, δ_w, ϵ_w, α_w respectively.

It will be noted that each series of the green component has its properly related absorption series and that no other absorption bands occur; also that b_g, which is absent or too weak to be observed, is not represented by a related absorption series. The same is true of the white component excepting that the closely neighboring and feeble γ_w, and β_w are blended into a single intermediate band in each group. The ϵ_w series moreover is slightly shifted towards the violet. The positions given are the averages from five of seven negatives measured.

In figures 3 and 4 only the groups near the reversing region have been indicated. Figure 5 shows all the bands thus far located in the polarized spectra of ammonium uranyl chloride. The diagram is by no means complete however for in the previous studies of the spectrum of this salt, already referred to, at least seven groups of fluorescence bands and eight groups of absorption bands were found. Preparations are in hand for extending the study of the absorption spectra into the ultra violet by means of a quartz spectrograph.

Analogous relations between the two components of the fluorescence and absorption spectra of the other three uranyl double chlorides, $2KCl.\ UO_2Cl + 2HO$; $2RbCl.\ UO_2\ Cl_2 + 2H_2O$ and $2CsCl.\ UO_2Cl_2 + 2H_2O$, have also been determined. Full data concerning these spectra and that of the ammonium uranyl chloride at +20° and −185° will be published at an early day.

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[1] Polarized fluorescence, first described by Grailich (*Krystall-optische Untersuchungen*, Wien, 1858) has since been studied by Maskalyne (*London, Proc. R. Soc.* **28**, 479), V. Lommel (*Ann. Physik., Leipzig*, **8**, 634), E. Wiedemann (*Ibid.*, **9**, 158), Sohncke (*Ibid.*, **58**, 417); G. C. Schmidt (*Ibid.*, **60**, 740), and H. Becquerel, (*Paris, C. R. Acad. Sci.*, **144**, 671).

[2] Becquerel, in the paper already cited, has noted the fact that the fluorescence spectrum is not changed by polarizing the incident light. Sohncke obtained a similar result with calcite and some other crystals but found certain exceptions to the general rule.

[3] See Nichols and Merritt, *Physic. Rev.*, Ser. 1, 27, 373 (1908); also *Studies in Luminescence*, Carnegie Institution (1912).

THE LINGUISTIC CLASSIFICATION OF POTAWATOMI

By Truman Michelson

BUREAU OF AMERICAN ETHNOLOGY, WASHINGTON

Presented to the Academy, May 26, 1915

For a considerable period of time it has been commonly supposed that Potawatomi is very closely related to Ojibwa and Ottawa. The statement of William Jones in his *Some principles of Algonquian word-formation*[1] is the most authoritative one on this point. In my *Preliminary report on the linguistic classification of Algonquian tribes*[2] I concluded from my limited first-hand knowledge of Potawatomi that Potawatomi belonged to the Ojibwa group (comprising Ojibwa, Ottawa, Algonkin, Potawatomi; and somewhat removed from them Peoria, etc.) of Central Algonquian dialects, but that Potawatomi differed more from Ojibwa, Ottawa, and Algonkin than these from one another. But it should be carefully noted that thus far no data have been published to justify these opinions. In the course of the past winter the Bureau of American Ethnology had photostat copies made of three grammatical treatises on Potawatomi which were written by the Rev. Maurice Gailland years ago, but which exist only in manuscript form. In spite of the fact that these treatises are fundamentally arranged on the plan of Latin grammar which is wholly unsuited to the genius of Potawatomi, and that the phonetic system employed is patently inadequate, I have been able to construct nearly complete tables of the verbal pronouns of the independent, subjunctive, and subjunctive of the suppositive modes. From these, as well as from the fact that the negative of the independent mode is formed by means of a *ssi* suffix and that a sibilant is retained before *p*, it follows that the opinions expressed above are completely substantiated.

The personal subjective and objective pronouns which are incorporated in the verb are called 'verbal pronouns' in contradistinction to the independent personal pronouns which in Algonquian languages are used solely for emphasis. The highly specialized character of the verbal pronouns in the Central Algonquian dialects has made them thus far the most satisfactory classificatory criteria.

The decisive verbal pronouns showing the close relationship of Potawatomi to Ojibwa, Ottawa, and Algonkin are:

Independent mode: I—you, I—him, I—them (animate), we (exclusive)—thee, we (exclusive)—you, we (exclusive)—it, we (exclusive)—them (inanimate), we (inclusive)—it, we (inclusive)—them (inanimate), thou—him, thou—them (animate), ye (intransitive), ye—me, ye—him, ye—them (animate), he—you, he—him, they (animate)—us (exclusive), they (animate)—us (inclusive), they (animate)—you, they (animate)—him, they (animate)—them (animate).

Subjunctive mode: I—them (animate), we (exclusive)—thee, we (exclusive)—you, thou—them (animate), they (animate)—thee, they (animate)—him, they (animate)—them (animate), they (animate)—it, they (animate)—them (inanimate).

Subjunctive of dubitative mode: They (animate, intransitive).

The verbal pronouns showing the divergence of Potawatomi are:

Independent mode: we (exclusive)—him, we (inclusive) them (animate), we (inclusive)—him, we (inclusive)—them (animate), ye—it.

Subjunctive mode: I—you, we (inclusive)—him, we (inclusive)—them (animate), he—us (exclusive).

Subjunctive of the suppositive mode (corresponding to the Fox subjunctive of the interrogative mode): ten verbal pronouns.

By consulting the table of verbal pronouns on the folder at the end of my *Preliminary report*, the reader will at once see the importance of the pronouns mentioned above, in determining the linguistic classification of Potawatomi. Complete tables for Potawatomi will be published in the *American Anthropologist.*

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[1] *Amer. Anthrop., New York,* N. S., **6,** 369–411.

[2] *Smithsonian Inst., Rep. Bur. Amer. Eth.,* **28,** 221-290b.

THE LIGHT CURVE OF XX CYGNI AS A CONTRIBUTION TO THE STUDY OF CEPHEID VARIATION

By Harlow Shapley and Martha Betz Shapley

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Presented to the Academy, June 29, 1915

Probably as many as 90% of the 4500 stars whose light intensities are known to be variable may be placed in three well-defined classes. (1) The eclipsing variables are of various colors (or spectral types); their periods range from a few hours to a year or more in length, and their variation may be from less than one-tenth to several stellar magnitudes. (2) The long period variables are generally reddish stars; the cycle of their light-changes occupies from one hundred to six or seven hundred days, and the amount of their light variation is usually several magnitudes. (3) The largest class, however, since it includes the great number of variables in globular clusters and in other special regions of the sky, is that known as short period variables or Cepheid variables. Practically all types of spectra are represented, though types A and F predominate so far as now known. The periods in general are much less than 50 days, and for a very large subdivision (the cluster type variables) average about 12 hours. The variation, which is practically continuous, is nearly always of the order of one magnitude in range, and is characterized generally by a more rapid increase than decrease of brightness.

The cause and characteristics of eclipse variation are definitely known. But this can not be said of the other two main types of variable stars. Though many suggestions have been made, the true cause of long period variation remains more or less obscure. The interpretation of Cepheid variables has been much debated and much evidence in favor of various theories has been collected, but no explanation has as yet received general acceptance. The behavior of the spectral lines of Cepheids has led to the widely adopted assumption that they are spectroscopic binaries and that the light variations are in some way related to orbital motion. But the many serious obstacles in the way of the double star hypothesis have been pointed out in a previous paper,[1] and the suggestion made that the observed facts may accord much better with the assumption that the light and velocity variations are both due to disturbances on a large scale of the radiating surfaces of single stellar bodies. Such periodic disturbances, which have in sun-spots a not very distant analogue both as to spectral displacements and light variations, would have as an underlying cause the long-enduring free oscillations that may be set up in a gaseous mass in a variety of ways, such as the collision with or close approach to another body, the perturbations by an obscure companion, or the adjustment of mass or temperature dissymmetries.

Among the arguments against the double star interpretation of Cepheids is their property of definitely changing color (therefore, of changing surface temperature) with the variation in the brightness of the star, thus indicating that the phenomenon is peculiar to the radiating surface and not merely geometrical as with the eclipsing double stars. Another point is the lack of regularity in the time of the principal phases and in the nature of the light variations. It is in connection with these two arguments that the present investigation hopes to contribute to the explanation of the Cepheid variables. In particular the present discussion of the light variations of the star XX Cygni challenges the frequently iterated assertion that the short period variable, because of its supposedly great precision and regularity of performance, is a veritable timepiece. In fact the suggestion is on record that the unit of time might possibly be based on the periodic return to maximum brightness of certain short period variables rather than on the rotation of the earth. While in most cases so far as now known the *mean* periods of such variables have, to be sure, but little or no secular or periodic variation, the study of XX Cygni, in agreement with preliminary results for similar variables, has shown definitely that the light curves change rapidly and, it may be, erratically both in time of maximum and in character of variation. The details of this work will appear in the *Astrophysical Journal;*[2] some of the chief results are outlined below.

In a monograph on the light variations of XX Cygni, Kron gives for the visual range 0.76 mag., based on the observations of seven observers, and for the photographic range, from observations by Parkhurst and Jordan, 0.63 mag.[3] This result is unusual in that the visual variation is the larger; every other variable star for which we have such information has a greater range photographically than visually. The difference between the ranges is a direct measure of the change in color. That the star with the shortest known perod (3.2 hours) should show an abnormal color variation suggests that for some unknown reason it may be a real exception. To determine whether or not this is the case, observations were undertaken at Mount Wilson with the 60-inch reflector. Four series of plates were obtained (in all more than 300 exposures); each of the first two covers a complete period, and as Seed 27 plates were alternated with isochromatic plates (used with a yellow color filter), each series yields for the study of color phenomena complete, simultaneously determined photographic and photovisual light curves.

The photovisual curves thus obtained show no great deviation from Kron's mean visual curve. The ranges are 0.72 mag. and 0.85 mag., as compared with Kron's mean of 0.76 mag. The curves for the two nights differ in certain particulars from each other, and as both are smooth and well determined, the observed differences are probably real. This belief is supported by an investigation of the visual observations published by Kron, which show a much greater diversity in the shape of the nightly curves than can possibly be accounted for by errors of observation. There are, in general, two types of maximum; one is narrow and pointed, the other is broad, round-topped, and somewhat lower. The accompanying figure gives examples taken from the photometric observations by Guthnick at Berlin.

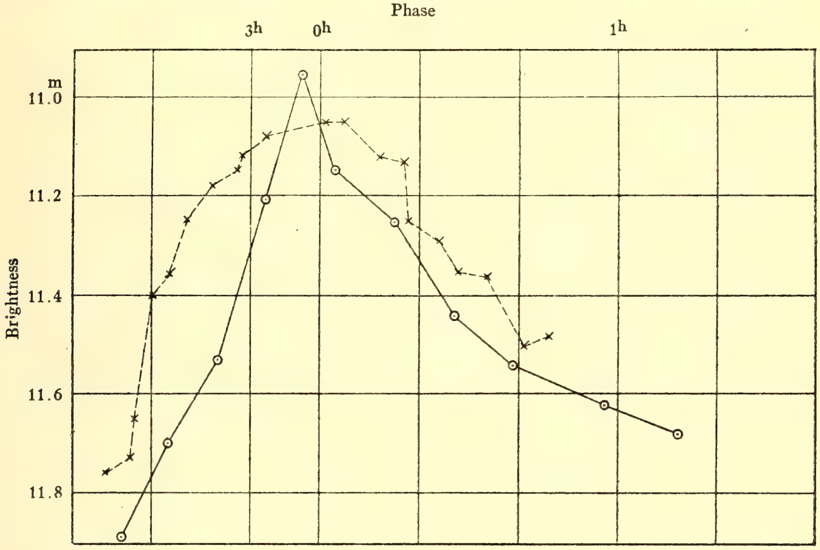
The Mount Wilson photographic curves differ from each other in the same way as the photovisual; for the same maximum the photographic and photovisual curves are very similar in form but differ from those of the other night.

A comparison of the Mount Wilson curves yields the following results: The photographic maximum occurs simultaneously with the photovisual within the errors of observation. The magnitudes at minimum phase agree for the two nights, being photovisually 12.17 and 12.17, and photographically 12.50 and 12.49, respectively. This gives a color index at minimum of +0.32, corresponding to spectral type A8 if the usual relation between color index and spectrum is adopted for this particular case. In the magnitudes at maximum, however, there is no such agreement. Photovisually they are 11.45 and 11.32, photographically 11.53 and 11.74, so that the color index at maximum varies from +0.08 to +0.42.

It is thus seen that so far as our data are decisive the photographic range is not constant. On one night it is 0.97 mag., on the other 0.75 mag. The smoothness and definiteness of the curves makes it impossible to attribute this large difference to observational errors entirely.

The constance of light at minimum and the evident variation at maximum leads to the conclusion that the minimum phase is the normal condition of the star, and suggests that the maxima represent fairly equally spaced but otherwise irregular and varied light disturbances.

As to whether the photographic or the visual range is the greater (the ratio of the two may not be constant), the data seem to indicate that the former is at least equal to the latter, if it does not, in fact, considerably exceed it, so that XX Cygni probably is not materially different in the matter of color change from other variables of the cluster or Cepheid type.



GUTHNICK'S VISUAL LIGHT CURVES OF XX CYGNI FOR SEPTEMBER 22, AND OCTOBER 5 (BROKEN LINE), 1908, SHOWING TWO EXTREME TYPES OF MAXIMUM.

Summary. (1) The cause of light variation has not been ascertained definitely for the variable stars known as Cepheids or short period variables, but in preference to the hypothesis that they are double stars an alternative explanation is proposed that seems to harmonize much better with observed data. (2) The new hypothesis would ascribe the periodic light and spectrum variations to vibrations in isolated stellar masses. (3) The present study of the variable XX Cygni is based upon 300 Mount Wilson photographic observations in which the variation for light of different wave-lengths is considered, and upon the analysis of 2700 visual measures by European astronomers. (4) It contributes to the solution of the problem by showing that a supposed exception to the normal color phenomena of Cepheids probably does not exist. (5) Of more importance to the Cepheid interpretation it shows that the periodic occurrence of maximum, far from being clock-like in its precision, is distinctly irregular in phase and in other details.

[1] *Mt. Wilson Contr.* No. 92; *Astrophys. J.*, **40,** 448 (1914).

[2] A Study of the Light Curve of XX Cygni, *Mt. Wilson Contr.* No. 104; *Astrophys. J.*, *in press* (1915).

[3] *Potsdam, Publ. Astrophysik. Obs.*, **22,** Part III (1912).

THE FEEBLY INHIBITED. III. INHERITANCE OF TEMPERAMENT; WITH SPECIAL REFERENCE TO TWINS AND SUICIDES

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Presented to the Academy, June 29, 1915

Mood or emotional tone depends largely upon temperament; and temperament, it is commonly recognized, has a constitutional basis—is hereditary; the temperament is as little alterable as stature. However, there is a great diversity in temperaments; some persons are prevailingly gay; others prevailingly somber; and still others pass through alternating cycles of elation and depression. Of so complex a phenomenon the explanation cannot be simple. It must account for the following three states:

(*a*) the hyperkinetic state—i.e., a state of overactivity. Of this it will be convenient to distinguish two grades, a lesser and a greater. Following the terminology of an old psychology we may call the lesser grade of hyperkinesis *nervous*. The nervous person is active, energetic, irritable, excitable, ambitious, given to planning, optimistic; usually talkative and jolly. The greater grade of hyperkinesis is the choleric. A choleric person is apt to start a new line of work before he completes the old, brags, is usually hilarious, hyperotic, often profane, liable to fits of anger, brutal, destructive, assaultive, and even impulsively homicidal and suicidal.

(*b*) The opposite state is the hypokinetic or depressed. Here too we may distinguish two grades. The phlegmatic person is quiet, serious, conservative, pessimistic. The melancholic person is unresponsive (often mute), lachrymose, given to worry, weak and incapable, feels life a burden, often longs for death as a relief.

(*c*) The normal mood is shown in that the possessor is cheerful without being boisterous, calm, well balanced, and *en rapport* socially; he works and plays moderately, laughs quietly, does not weep easily, feels little drive, and is always responsive and coöperative. This state may be due either to the absence of excitation or the absence of depression.

Now each of the three states of hyperkinesis and its absence may be combined with any of the three states of hypokinesis and its absence, making nine combination-states. And since there are two parents in any mating the number of different matings possible will be the sum of the series 9 + 8 + ... + 1, or 45.

To account for this variety of temperaments, hyperkinetic, hypokinetic, normal and mixed, various hypotheses were tried and finally the following selected as most fitted to explain the facts on an hereditary basis. *There is in the germ plasm a factor E which induces the more or less periodic occurrence of an excited condition (or an exceptionally strong reactibility to exciting presentations) and its absence, e, which results in a calmness. There is also the factor C which makes for normal cheerfulness of mood, and its absence, c, which permits a more or less periodic depression. Moreover, the factors behave as though in different chromosomes, so that they are inherited independently of each other and may occur in any combination.*

What the nature of these factors is, whether they affect primarily the development of certain parts of the nervous system or the secretions of certain glands, is not known and is not involved in the hypothesis. It is even conceivable that each state may be due to more than the pair of factors here suggested; but if the hypothesis fits the facts it would indicate that in the factors *E* and *C* we have the predominating influences that control mood.

For the test of the hypothesis 89 carefully described family histories were available, and these afforded 147 matings in which the mated pair, their parents (usually), and certain of their offspring were sufficiently described for the purposes of the test.

The test of the hypothesis is found in a comparison of the expected and actual distribution of temperaments in the children of each sort of mating. Of the 45 possible matings 29 were realized. The relations of the sums of the observed to the sums of the expected distributions among the nine classes of temperaments is shown in the following table.

choleric cheerful	*choleric phlegmatic*	*choleric melancholic*	*nervous cheerful*	*nervous phlegmatic*	*nervous melancholic*	*calm cheerful*	*calm phlegmatic*	*calm melancholic*
36	25	30	128	149	63	77	79	46
41–6/8	46–7/8	19–5/8	98–4/8	154–6/8	72–6/8	51–4/8	97–3/8	49–7/8

There are several reasons for not expecting a very close relation between observed and expected; but the chief one is that there are sometimes several 'expected' results; and in these cases the most varied result was taken as the 'expected.' Nevertheless, the relation between the two series is fairly close—the greatest difference being an unexpected excess of choleric-melancholics and corresponding deficiency of choleric-phlegmatics; also an excess of calm-cheerful and deficiency of calm-phlegmatic. These discrepancies imply a rather slight error in the classification of the observed cases.

We conclude, then, from our own data that the hypothesis is confirmed. In addition, an examination of the literature reveals clear evidence that a difference in the inheritance of extreme hyperkinesis (a dominant) and extreme hypokinesis (a recessive) has unconsciously been observed. And the differences in the conclusions of Rosanoff and Orr[1] and Rüdin[2] concerning the inheritance of manic-depressive insanity—one regarding it as recessive and the other regarding it as *sometimes* dominant—are easily explained on the ground of its complex hereditary nature.

The hereditary nature of temperament is demonstrated by the facts of the personal history of identical twins, as given by Galton.[3]

There is reason for thinking that the different zygotic combinations of temperamental factors occur with different frequencies. A comparison of the relative number of matings in equally frequent kinds of zygotic combinations shows that some are much more common than expected, others much less. The mated pair rarely have the same zygotic temperamental formula. Two choleric or 'nervous' persons frequently do not marry each other. Two melancholics rarely intermarry. There is, in marriage, a selection against similar temperaments, i.e., a preference for those of more or less markedly dissimilar temperaments.

An analysis of the temperaments of suicides shows that they fall into the two types of the hyperkinetic (or impulsion) suicides and the hypokinetic (or depressed) suicides. Also, a family tendency to suicide by the same method is evident; but it is uncertain whether this is due chiefly to subconscious suggestion or to an hereditary bias.

This study throws light upon the nature of the 'functional insanities' and lends strong support to the view that they are syndromes whose elements are separately inheritable.

Finally the study throws light upon the 'springs of conduct.' Just what we shall, in any situation, do is determined by numerous factors, but the general nature of our reactions, whether violent or repressed—this is determined by the hereditary nature of our temperaments. The romantic and the classic type of reacting, the hyperkinetic and the hypokinetic, the radical and the conservative, the feebly inhibited and the strongly inhibited constitute a dualism that runs through our whole population.

It is expected that the complete paper will appear as a publication of the Carnegie Institution of Washington.

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SECOND TYPE STARS OF LOW MEAN DENSITY

By Harlow Shapley

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Presented to the Academy, June 29, 1915

The rôle played by stellar densities in the theories of the order of stellar evolution is easily recognized. If the development of a star uniformly follows the spectral sequence B, A, F, G, K, M, that is, if in contracting through the agency of its own gravitation its light changes in color steadily from bluish white to red, then the mean density of the B and A type stars must naturally be less than that of the redder stars. But, on the other hand, if the earlier as well as the later stages in a star's evolution are characterized by the second and third type spectra (F, G, K, M), the density of the redder stars should have both small values and large values relative to the bluer stars.

There are two principal lines of observational evidence that indicate the existence of two orders of densities for second type stars:

1. The recognized existence of enormous differences in the absolute brightness of red or yellow stars of essentially identical spectra leads immediately to the conclusion that the volumes also differ greatly. Now since stellar masses so far as known have not very greatly differing values, we are lead to believe that the disparity in size must be accounted for by large differences in mean density. For instance, the density of our sun, which on this interpretation must be considered a body relatively advanced in age, may be from a thousand to a million times greater than that of a star of great intrinsic luminosity, such as Alpha Aurigae, which has a similar spectrum and presumably therefore a comparable suface temperature. An objection to this argument in favor of very low densities for some of the redder stars may be that our information relative to stellar masses is insufficient to prove that the large volumes are not to be attributed to large masses with normal densities.

2. The densities of eclipsing binaries, however, can be determined directly, independently of the unknown masses; and recent studies of

this class of variable stars have given results for different spectral types with considerable certainty. The average density of first type stars (B, A) is about 0.15 on the solar standard, while none exceeds the sun in density and none is less than one-hundredth as dense. Also a number of second type stars are found to be much denser than the average first type star, even exceeding the sun in rare cases. This condition fits in with either theory of the order of stellar evolution. Hence for the purpose of differentiating between the rival hypotheses it is of importance to know how definitely we can say that there are also stellar densities much lower than any known for A and B type stars. The present communication outlines the data now in hand relative to low-density eclipsing stars of the second spectral class.

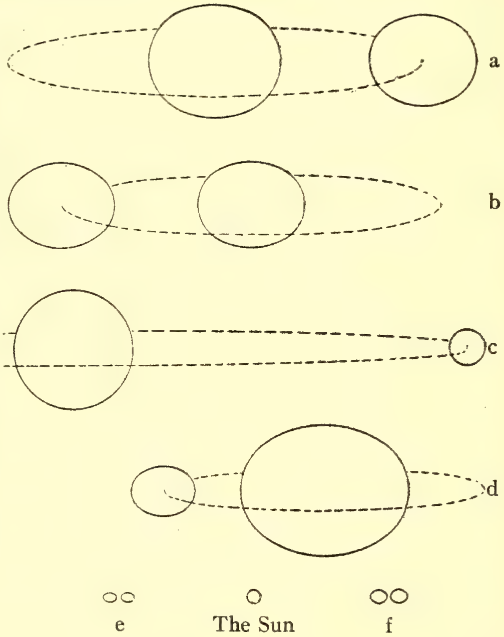
The values of the densities already derived for the stars considered in this note refer to the individual components of each system, and therefore, since for such purpose knowledge of the relative dimensions and masses is necessary, the results are involved with rather complicated computations of the orbital elements. It is possible, however, to derive upper and lower limits of the mean density of each system without the necessity of any data relative to the stars or their light curves other than the period of revolution and the duration of eclipse, which are the two quantities most commonly and most easily obtained for eclipsing variables. In the more detailed discussion of this subject the following simple relations are obtained, which give in a circular orbit the mean density, d_0, in terms of the sun:

$$d_0 \leqq \frac{0.054}{P^2 \sin^3 \frac{2\pi t}{P}}, \qquad d_0 > \frac{0.216}{P^2\left(1+3\sin^2\frac{2\pi t}{P}\right)^{3/2}}.$$

The quantities P and t are the period of revolution and the semi-duration of eclipse. In very exceptional cases the second relation may give too high a lower limit, but never a value as much as twice too large.

The above formulae are especially suitable for the easy computation of the order of the density in systems of long period, which are usually also the cases where very low densities are found. With their aid results for five low density binaries of the second spectral type were readily determined and the values found to correspond closely to those computed rigorously from the orbital elements. The light curves on which the results are based are from various sources, and, while not of the first order of accuracy, are probably sufficient in all cases to leave no doubt that the low densities really exist. Three independent classifications of the spectra were made at the Harvard College Observatory.

In the accompanying figure is given a comparative diagrammatic representation of some of these giant eclipsing double stars and also of the two systems of highest known density. The key to the diagram gives the data relative to densities and spectra. Making the reasonable assumption that the mass of the sun is sensibly equivalent to the mass of one component of any of the double stars, then the sun can be drawn to scale, as at the bottom of the diagram. If its mass is but one eighth that of a single component, its diameter in the diagram should be one-half as great; if eight times as massive as assumed, its diameter should be doubled. The true dimensions of the stars relative to the sun are probably well within these limits. It is to be noticed that the components of five of the six binaries are elongated in the line of centers.



Eclipsing binaries of low and high density. The mass of each component is assumed equal to that of the sun.

a. SX Cassiopeiae; period $36^{d}.572$; spectrum G3; density 0.0003. Reduced to one-half.

b. RX Cassiopeiae; period $32^{d}.316$; spectrum Ko; density 0.0005. Reduced to one-half.

c. RZ Ophiuchi; period $261^{d}.9$; spectrum F8; density 0.001. Reduced to one-fourth.

d. W Crucis; period $198^{d}.5$; spectrum Gp; density 0.000002. Reduced to one-eighth.

e. W Ursae Majoris; period $0^{d}.334$; spectrum G; density 1.84.

f. U Pegasi; period $0^{d}.375$; spectrum F?; density 0.75.

Summary. Because of its bearing on the question of the order of stellar evolution, the density of stars of the second spectral type has been discussed from the standpoint of the dependability of the observational and theoretical work that is the basis of the derivation of occasional extremely low values. The evidence discriminating between the two principal theories seems very definite, but further observational work would materially strengthen the case. The light curves of the variable stars involved, the relevant tabular data, and the derivation of certain density relations will appear in an early number of the *Astrophysical Journal.*

ON THE PATHOLOGICAL ACTION OF ARSENICALS UPON THE ADRENALS

By Wade H. Brown and Louise Pearce
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Presented to the Academy, July 3, 1915

The fact that arsenicals of diverse chemical constitution exert a pronounced pathological action upon the adrenals has not been generally recognized. The importance of this adrenotropic action was first impressed upon us while carrying out the routine biological tests of compounds of arsenic in our chemotherapeutic studies. Observations on more than sixty compounds including such substances as arsenious acid, arsenic acid, sodium cacodylate, atoxyl, arsacetin, arsenophenylglycine, salvarsan, and neosalvarsan have shown that, without exception, toxic doses of all of these arsenicals produce definite lesions of the adrenals.

The adrenotropic action of all compounds of arsenic is not equally great or identical in character but the lesions produced by a given compound in a given animal species are quite constant and in some instances are the dominant pathological manifestations of the toxic action of the compounds. The essential features of this action concern vascular changes in the adrenal, alterations in the lipoid content, cellular degeneration, and the effect upon the chromaffin.

In general, arsenical intoxication in the guinea pig produces an acute enlargement of the adrenals with some congestion and hemorrhage. The lipoid granules, normally demonstrable in the outer half of the cortex with Herxheimer's Scharlach R., first appear as larger droplets. Later, the demonstrable lipoids increase in amount and are spread over the entire adrenal cortex. This stage of lipoid increase is succeeded by one of diminution which with some compounds progresses almost to exhaustion.

The cells of both the cortex and the medulla show a variety of degenerative changes and even necrosis; colloid degeneration of the medulla is particularly striking. While the cortex is slightly infiltrated with leucocytes the accumulation of both leucocytes and polyblasts in the medulla is especially marked with such substances as arsenophenylglycine.

Regeneration of cortical cells by mitosis is very active after forty-eight hours and mitotic figures are occasionally seen in the cells of the medulla.

The effect of arsenicals upon the chromaffin is of especial interest. Some compounds seem to exercise but slight influences upon the chromaffin content of the adrenals while others, such as sodium cacodylate, salvarsan, and neosalvarsan, cause a rapid and marked decrease in this substance, as judged by the color of the medullary cells after fixation in Müller's fluid.

From these observations it appears that the adrenotropic action of arsenicals is one of the most constant and important features of arsenical intoxication and we suggest that therapeutic doses of some arsenicals may be found to produce definite stimulation of the adrenal glands.

VARIATIONS IN THE CHARACTER AND DISTRIBUTION OF THE RENAL LESIONS PRODUCED BY COMPOUNDS OF ARSENIC

By Louise Pearce and Wade H. Brown
ROCKEFELLER INSTITUTE FOR MEDICAL RESEARCH, NEW YORK

Presented to the Academy, July 3, 1915

The chemical agents employed in the production of experimental nephritis are usually divided into two classes, those producing tubular lesions and those producing vascular lesions, to the latter group of which the compounds of arsenic have been assigned. From a study of the renal lesions produced by a large number of arsenicals, however, we have been led to question the validity of such a classification and to view the pathogenic action of such substances from the standpoint of their chemical constitution as well as their arsenic content.

The classical hemorrhagic kidney of arsenious acid is by no means constant for all compounds of arsenic. Grossly, the kidneys of dogs given lethal doses of such substances as arsenious acid, salvarsan, neosalvarsan, galyl, arsenophenylglycine, atoxyl, and arsacetin are separable into two extreme groups, the red and the pale kidneys, with transitional types in which the predominating changes ally them more closely with the one group or the other. In the group of red kidneys, congestion and hemorrhage are the dominant features of the arsenical action, while in the pale kidneys, the dominant lesion is tubular.

Upon closer analysis of the gross and microscopic changes we can make a further differentiation of the action of compounds that produce kidneys of the one or the other of these types. For example, arsenious acid, salvarsan, neosalvarsan, and galyl all produce red kidneys, but the congestion and hemorrhage produced by arsenious acid is diffuse in character with but slight tubular necrosis, while the vascular injury of salvarsan, neosalvarsan, and galyl is more pronounced in the cortex and the boundary zone and is accompanied by much more marked tubular necrosis and interstitial edema. Again, atoxyl causes extensive hemorrhage extending from the boundary zone through the medulla and only slightly into the cortex, the outer rim of which remains pale; there is marked degeneration and necrosis of tubular epithelium. These atoxyl kidneys which are outwardly pale combine in a peculiar form the essential features of both the red and the pale kidneys. In like manner, arsenophenylglycine produces kidney lesions of a combined type and while tubular degeneration and necrosis are dominant there is usually some congestion and hemorrhage in the boundary zone and medulla—more rarely in the cortex.

The other extreme in the action of arsenicals upon the kidney is exemplified by arsacetin which produces a typically pale kidney. While congestion and hemorrhage are still apparent to a minor degree in the boundary zone of these kidneys the vascular injury is so completely overshadowed by the injury to the tubular epithelium as to leave no doubt as to the dominance of tubular injury. Further, the prompt and vigorous regeneration of the tubular epithelium indicates that the extensive necrosis produced by arsacetin can not be regarded as a secondary anemic phenomenon.

It is certain, therefore, that all arsenicals do not produce renal lesions that are identical either in character or distribution but that this group of substances includes agents producing a so-called tubular nephritis as well as those producing a vascular nephritis, and that these wide differences in the pathogenic action of different compounds of arsenic are explainable only upon the basis of their chemical constitution.

SEVEN POINTS ON A TWISTED CUBIC CURVE

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Preseented to the Academy, August 2, 1915

Six points in space, barring special situations, determine a twisted cubic curve. From any seventh point of the curve those six are projected by six generators of a quadric cone. For any seven points of a cubic curve there is accordingly a symmetric set of seven cones; and it is well known that seven points giving rise to two such cones are on a cubic curve, and so give rise to five more cones. This is the only current theorem on seven points of a twisted cubic. Concerning eight points there is the elegant theorem of von Staudt, that if two tetraedrons have eight points of a twisted cubic for vertices, their eight faces osculate a second cubic curve. I propose to demonstrate a theorem whose formulation resembles the latter, while like the former it relates to the fewest possible points, seven. It is this: *If seven points on a twisted cubic be joined, two and two, by twenty-one lines, then any seven planes that contain these twenty-one lines will osculate a second cubic curve.*

The proof to be given is analytic, consisting in a repeated application of Pascal's theorem concerning a hexagon inscribed in a conic, adapted to the six projecting lines from a seventh point by means of Clebsch's so called *Ränderungs-princip*.

Pascal's theorem may be expressed in notation readily understood, for six points a, b, c, d, e, f of a conic (coordinates (a_1, a_2, a_3) etc.) by the vanishing of the determinant

$$\begin{vmatrix} ((ab), (de))_1 & ((bc), (ef))_1 & ((cd), (fa))_1 \\ ((ab), (de))_2 & \cdots\cdots & \cdots\cdots \\ ((ab), (de))_3 & \cdots\cdots & \cdots\cdots \end{vmatrix}$$

or any one of the 59 equivalent forms. This is reducible to the equation, in three-rowed determinants of point-coordinates,

$$(abe)\,(cde)\,(adf)\,(bcf) = (abf)\,(cdf)\,(ade)\,(bce). \tag{1}$$

(See *Encyklopädie der mathematischen Wissenschaften*, 3_2, §7.)

Clebsch's *Ränderungs-princip* applied to this equation gives the condition that six points in space (a_1, a_2, a_3, a_4), etc., or briefly a, b, c, d, e, f, shall be projected from a seventh point $(g_1, g_2, g_3\ g_4)$ by six generators of a quadric cone, viz.,

$$(abeg)\,(cdeg)\,(adfg)\,(bcfg) = (abfg)\,(cdfg)\,(adeg)\,(bceg). \tag{2}$$

If the seven points are on a twisted cubic, this relation may be written in $7 \cdot 15 \cdot 3$ different ways, according to the choice of the seventh point, g, the two points e and f, and the two ways of forming pairs from the remaining four points a, b, c, d. Let equation (2) denote all of this type.

Seven planes, 1, 2, 3, 4, 5, 6, 7 may be determined in 30 different ways so as to contain the 21 lines that join pairs of the points a, b, c, d, e, f, g. Any one such choice constitutes a *triad system* on those seven letters, since its seven sets of three points in a plane must contain every pair of points once and only once. Fix one set thus, planes, 1, 2, 3, 4, 5, 6, 7 containing the points ade, afg, bdf, beg, cdg, cef, abc, respectively. These seven planes, in order to osculate a twisted cubic, must satisfy conditions on planes precisely dual to the 315 conditions of type (2) on points. On account of their similarity we need to verify only one. This is a slightly tedious reckoning, which we give *in extenso*.

The question shall be: Does the plane 7 intersect the six planes 1, 2, 3, 4, 5, 6 in six tangents to a conic? Is the following condition verified?

$$(1257)\,(3457)\,(1467)\,(2367) = (1267)\,(3467)\,(1457)\,(2357). \tag{3}$$

Now it is necessary to express the coordinates of each plane as three-rowed determinants in the coordinates of the three points which it contains, to simplify, and compare the left hand product of four determinants in (3) with that on the right, and to show that equations (2) above will suffice to prove the equality so reduced. On the left:

$$\begin{aligned}
(1257) &\equiv (\,(ade)_1\,(afg)_2\,(cdg)_3\,(abc)_4) \\
&\equiv (adeb)\,(afgc)\,(cdga) - (adec)\,(afgb)\,(cdga) \\
&\equiv (cdga)\,\{\,(adeb)\,(afgc) - (adec)\,(afgb)\,\}, \\
(3457) &\equiv (cdgb)\,\{\,(bdfc)\,(bega) - (bdfa)\,(begc)\,\}, \\
(1467) &\equiv (adeb)\,(begc)\,(cefa) + (adec)\,(bega)\,(cefb), \\
(2367) &\equiv (afgb)\,(bdfc)\,(cefa) + (afgc)\,(bdfa)\,(cefb).
\end{aligned}$$

On the right:

$$\begin{aligned}
(1267) &\equiv (cefa)\,\{\,(adeb)\,(afgc) - (adec)\,(afgb)\,\}, \\
(3467) &\equiv (cefb)\,\{\,(bdfc)\,(bega) - (bdfa)\,(begc)\,\}, \\
(2357) &\equiv (afgb)\,(bdfc)\,(cdga) + (afgc)\,(bdfa)\,(cdgb), \\
(1457) &\equiv (adeb)\,(begc)\,(cdga) + (adec)\,(begc)\,(cdgb).
\end{aligned}$$

Notice that on the left two factors in the {} are identical with two in {} on the right. Exclude these, when multiplication gives in each member four terms. Two terms on the left are identical with terms on the right. Remove these, and transpose so as to exhibit the factor

$$(cefa)\,(cdgb) - (cefb)\,(cdga)$$

on each side. Neglect this, and our question is reduced to the following. Is it true that

$$\begin{aligned}
&(abfg)\,(abde)\cdot(bcdf)\,(bceg)\cdot(acdg)\,(acef) \\
&\qquad = (abdf)\,(abeg)\cdot(bcdg)\,(bcef)\cdot(acde)\,(acfg)\ ? \qquad (4)
\end{aligned}$$

Two applications of relations like (2) prove the truth of this, for example we use first b, then c, as seventh point like the g of (2);

$$(abfg)\,(abde)\,(bcdf)\,(bceg) = (abdf)\,(abeg)\,(bcfg)\,(bcde),$$

and

$$(bcfg)\,(bcde)\,(acdg)\,(acef) = (bcdg)\,(bcef)\,(acfg)\,(acde).$$

These two Pascalian relations upon the seven points are part of the hypothesis, hence the Pascalian equation (3) upon the seven planes is verified and the theorem is proved.

It is interesting to restate this theorem somewhat more fully. The converse is true by duality, from seven planes to any seven points that lie on all 21 of their lines of intersection. Consequently

If seven planes intersect three and three in seven points of a twisted cubic, each of the 29 other sets of seven points that contain all the 21 lines of intersection of those planes is upon another twisted cubic curve.

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THE INDIAN AND NATURE

By Alice C. Fletcher
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Presented to the Academy, August 3, 1915

From an extended study of the Siouan tribes of the Plains Indians, it is evident that their tribal organization and rites are based on concepts derived from observations of Nature.

These Indians belong to an observant, thoughtful, out-of-door people who for generations have lived on intimate relations with an unmodified environment. No animal but the dog was domesticated, all creatures pursued their own mode of life. With few exceptions plants were uncultivated, undisturbed in their manner of growth. No highways broke through the prairies or woodlands, the winding trails of animals served as footpaths for man. There was nothing visible to suggest any break in the continuity of the natural relation between man and his surroundings. It was amid such untouched, unforced conditions that these people attentively watched the various phases of life about them and pondered deeply on what they saw.

The Indian discerned that everywhere dual forces were employed to reproduce and so perpetuate living forms. The fructifying power of the sun was needed to make the earth fruitful and only on the union of the two, the sky and the earth, was life in its various forms made possible. Upon these two opposites, he projected human relations and made them, to a degree, anthropomorphic, the sky became masculine, the earth, feminine. Finally, he was led to conceive of the cosmos as a unit, permeated with the same life force of which he was conscious within himself; a force that gave to his environment its stable character; to every living thing on land or water the power of growth and of movement; to man it gave not only his physical capacities but the ability to think, to will, to bring to pass. This unseen, undying, unifying force is called by the Omaha and cognate tribes, Wakonda. Through Wakonda all things came into being, are ever related, and, more or less interdependent. Consequently, Nature stood to the Indian as the manifestation of an order instituted by Wakonda wherein man was an integral part. To this order he turned for guidance when establishing those means, religious and secular, that would insure to him, individually and socially, safety and continuous life.

Finding himself to be one of a wide reaching family, the Indian planned his tribal organization upon the type of that family. He divided the people into two great sections, one to represent the sky, the other, the earth. Each of these sections was composed of a number of kinship groups, called by the Indian, To^n-wo^n-gtho^n, meaning, village. (These villages are spoken of as clans or gentes by students of our race.) Each village stood for some one of the forms of life seen in Wakonda's instituted order. The sky was the abode of the sun, the moon, the stars, the storm cloud with its thunder and lightning. The earth, with its land and water was the abode of the trees, the grasses, and the various animals so closely allied to man and his needs. The tribal organization aimed to mirror man's environment. The tribal rites were instituted to provide a means by which the people could approach the invisible power, believed to abide in Nature, for help, to secure food, safety and long life.

Each village (gens) had its own ceremony which was also a component part of the tribal rites, wherein all the villages (gentes) were thus represented. The ceremony of each village (gens) had a central subject, some form or force, having its abode in the sky or on the earth, and represented by a symbol. The name given to this symbol by the Omaha and cognate tribes, is Í-ni-ka-shi-ki-the, a term composed of, I, by which; ni-ka-shi, a part of ni-ka-shi-ga, people; ki-the, make themselves; and means, that by which they make or designate themselves a people. (Students of our race have applied to this symbol the term 'totem.') As has been shown, this symbol referred to one of the forms or forces belonging to Nature as instituted by Wakonda, therefore represented in the tribal organization and the tribal rites. The symbol had a sacred significance to the people of the village (gens) in whose ceremony it held the central place. It bound the people of the village (gens) together by a sacred tie, made them distinctive among the other villages (gentes) that composed the tribe, and, it was a link between the people of the village (gens) and the invisible Wakonda. The symbol belonging to a village (gens) is always referred to metaphorically in the name by which the village (gens) is known, and, the symbol is also treated in the same manner in the personal names ceremonially given to every child born within the village (gens). The symbol may be an animal, as, the buffalo, or a force, as the wind, and the people be spoken of by the names of the symbol of their village, as, the buffalo people, or the wind people. There are certain articles that are regarded as associated with the different symbols. The people of a village (gens) treat with marked respect and never taste or touch such articles as are supposed to be associated with their sacred symbol.

Those villages (gentes), whose symbols are of some form or force that has its abode in the sky, have their unchangeable place in the sky section of the tribe, and those, having symbols that pertain to either the land or water, have their fixed place in the earth section. During the time when tribal rites are performed, the tribe is oriented, that the people may face the ever recurring day, a symbol of life. These rites may be grouped into three classes: Those which are an appeal for the securing of food; to this class belong those rites that relate to the maize and to hunting. Those that pertain to unity and peace; to this class belong the ceremonial giving of a tribal name, the Wa-wan ceremony and certain social customs. Those which relate to war, the defence of the tribe, both as to its food supply and the life of the people. The prayer for long life, that occurs under various forms throughout the rites is understood as an epitome of these essentials to individual and to tribal life.

The Omaha distinguishes tribal rites from other ceremonies by applying to the former the term Wé-wa-çpe. The word is compound; we, signifies an instrument, a means by which something is done or brought to pass; wa-çpe, means, orderly conduct, thoughtful composure. The word, according to its context can mean, religion, law, or any similar institution. As here used it signifies a means to bring the people into order, into thoughtful composure. This term applied to tribal rites, bears testimony to a discriminating observation of the social value of religious observances, not only as a power to hold the people together by the bond of a common belief, but, as a means to augment the importance of self control and, of submission to authority. Rites designated as wé-wa-çpe, were believed to open a way between the people and the mysterious, unseen Wakonda, and, any careless or irreverent act toward them, subjected the offender to supernatural punishment.

These rites are composed of dramatic acts, the recitations of rituals and the singing of ritualistic songs. In these are embodied the myths, and allegories in which the genesis of man and his relation to Nature are set forth. In the stories, symbols and metaphors are freely used, often in a highly imaginative manner and not infrequently touched with poetic feeling. By these means, the Indian's mind sought to bridge the gulf he recognized as stretching between him and the forms and forces of Nature that had so direct and yet so subtle a relation to his existence. These myths, allegories and metaphors form a nimbus about these rites that both illumines and yet makes elusive their meaning.

The Omaha, on his entrance into life is met by one of the tribal rites. He is introduced to the cosmos by the priest, standing outside the tent there, raising his right hand to the heavens, palm outward, he intones in a loud voice the following ritual hymn:

Ho! Ye Sun, Moon, Stars, all ye that move in the heavens;
    I bid ye hear me!
  Into your midst has come a new life!
    Consent ye, I implore!
Make its path smooth, that it may reach the brow of the first hill.

The Winds, Clouds, Rain, Mist, that move in the air;
The Hills, Valleys, Rivers, Lakes, Trees, Grasses of the earth;
The Birds of the air, the Animals of the forest, the Insects that creep among the grasses and burrow in the ground are addressed in the same manner. Finally he cries:

Ho! All ye of the heavens, all ye of the earth,
    I bid ye hear me!
  Into your midst has come a new life!
    Consent ye, consent ye all, I implore!
Make its path smooth, then shall it travel beyond the four hills.

Infancy, Youth, Maturity, Old Age, are the four hills across which lies the rugged pathway of life.

In the social life of the Indians many little dramatic acts occur significant of beliefs, that are difficult for a stranger to understand correctly. For example: A relative comes to the home of an infant and presents it with a tiny pair of moccasins with a hole cut in each sole. The Indian mother understands the tender wish conveyed by the act. The baby is thus recognized as an Omaha child, for the moccasins anticipate the ceremony in which the 'new life' is proclaimed a member of the tribe. The holes are a sign of usage, they express the giver's prayer for long life to the child. A person might enter the tent, see the tiny moccasins with the holes and exclaim: "What a long way the little one has traveled!" This too would be a prayer for long life to the child. If an unseen messenger from the spirit world should approach the infant to bid it come with him, the child would be able to say, "No, I can't go with you, look, my moccasins are worn out!" and so, the baby would not be taken away from its mother.

Both garments and the manner of wearing them ceremonially are by the Indians invested with symbolic meanings. For instance: The Robe is significant of a man's duties or purposes according to the manner in which it is worn or adjusted about his person. The position of the eagle feather on a man's scalp-lock indicates the class of act which brought to the man the right to this war honor. Other regalia made up of different articles, each one of which has its special significance, present to the Indian warrior a picture, as of the battle field where he fought, defending his tribe, and won his honors. None of the articles employed to represent war honors or a special part taken by a man in any of the tribal rites are allowed to be used as mere adornments. A war honor can not be worn by a man until he has won the right to wear it, by the performance of a valorous act, that has been publicly recounted, approved by witnesses, in the presence of the tribe, at which time the honor appropriate to his act is accorded him, and he is authorized to wear the insignia belonging to the grade of his act.

Moccasins have a significance. Formerly each tribe had its own style of moccasin, so that a person's tribe would be indicated by the kind of moccasin he wore.

In the ceremony that marks the birth of the 'new life' into the tribal organization, the dual forces are present, the masculine sky and the feminine earth; the former, represented by the 'Four Winds' invoked to 'come hither' in the opening ritual song; and, the latter, by the stone placed in the center of the ceremonial tent. The time when this tribal rite took place was in the spring, "when the grass was up and the meadow lark singing." The child was about four years old and must be able to go about alone and unassisted. A tent was set up and made sacred, therein the priest awaited the children brought thither by their mothers, each child carried a new pair of moccasins. As the mother approached the tent with her child, she addressed the priest, saying: "Venerable man, I desire my child to wear moccasins!" and, the little one, carrying its moccasins, entered the tent alone. According to the Omaha rite and that of some of the cognates, the priest, after summoning the 'Four Winds' lifted the child upon the stone, where it stood in its bare feet facing the east, then the priest lifted it and placed it on the stone facing the south, again he lifted it and on the stone it stood facing the west, lifting it again, its feet rested on the stone as it faced the north, lastly, the priest lifted the child and it stood on the stone with its face to the east. The priest sang the following ritual song. A free translation is given.

Turned by the Winds, goes the one I send yonder,
Yonder he goes who is whirled by the Winds,
Goes where the four hills of life and the Four Winds are standing,
There into the midst of the Winds, do I send him,
Into the midst of the Winds, standing there.

The priest then puts upon the child's feet the new moccasins, makes it take four steps, and says: "Go forth on the path of life!" A personal tribal name was now given the child, one that belonged to its father's village (gens) and referred to the second symbol of its rite. This name was then proclaimed by the priest to the "Hills, Trees, Grasses, and all living creatures great and small!" in the hearing of the assembled members of the tribe.

In connection with the part symbolically taken by the Winds in this ceremony, it is interesting to note, that it was the duty of the 'Wind people' to put moccasins on the feet of the dead, that they might enter the spirit land and there be recognized and able to rejoin their kindred.

After a boy had ceremonially received his tribal name, on his return home, his father cut the child's hair in an established manner which was meant to typify the sacred symbol of his village. This manner of cutting a boy's hair was kept up until the child was about seven years old. The queerly cropped heads of the boys fixed in the minds of the children the symbols belonging to the different villages (gentes).

The symbolism attached to garments and the manner of wearing them, already mentioned, runs through the myths, allegories and metaphors, and figures extensively in the tribal rites.

A detailed presentation of the subject of this paper is impossible within the accorded limits, but from what has been given, glimpses have been obtained of the line the Indian has pursued in his endeavor to express his view of Nature and of the relation he believed to exist between its various forms and forces and himself.

In the tribal rites can be traced the gropings of the Indians' mind to find that power, greater than man, which was the source of visible Nature; to discover a way for man to approach that power so that he could receive help from it; also to search for the meaning of the activities that were everywhere apparent. The religious and social ideas developed through this search, extending, through generations, as the rituals give evidence both directly and indirectly, were gradually evolved and formulated in the tribal rites, wherein was set forth, with unmistakable clearness, to the people, the importance of the perpetuation of human life upon the earth, and, of the recognition, that the life-giving power of Wakonda is ever present in all things that surround man.

THE MECHANISM OF ANTAGONISTIC SALT ACTION

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1. The work on antagonistic salt action has shown that we must discriminate between two distinct groups of such phenomena. The first group is represented by the counteraction of the toxic effects of a salt with a univalent cation by a salt with a bivalent cation. I showed in 1901 that all salts with a univalent cation rapidly kill the newly fertilized eggs of the marine fish *Fundulus* when the concentration of these salts exceeds a certain limit, while the addition of a very small (though definite) quantity of a large number of salts with a bivalent cation prevents or retards this injurious action.[1] Salts with bivalent anions had no such effect. The rôle of the valency of the cation in these phenomena of antagonism was unmistakable and was pointed out in the same papers as was also the relation to certain rules in the precipitation of colloids, but it was not possible to indicate an antagonistic action on colloids between salts with univalent and bivalent cations. I suggested later that the rapid death of the eggs in the salts with univalent cations was due to a diffusion of the salts into the eggs, while the addition of the salt with a bivalent cation prevents or retards this rapid diffusion,[2] and this suggestion was supported by later experiments.

It therefore appeared from these observations that the salts with monovalent cations increase the permeability of the membrane when their concentration exceeds a certain limit, and that the addition of a trace of a salt with a bivalent cation, e.g., $CaCl_2$, diminishes the permeability. This idea received support in the floating experiment of the writer with *Fundulus* eggs[3] and in Osterhout's experiments on the galvanic resistance of *Laminaria* in NaCl and $CaCl_2$ solutions.[4]

The second group of antagonistic phenomena is represented by the following experiments. In 1911 Loeb and Wasteneys found that a KCl solution in the concentration in which this salt is contained in the sea water is toxic for the marine fish *Fundulus*, while the addition of NaCl in a definite ratio (17 molecules or more of NaCl to 1 molecule of KCl) annihilates the toxic effect of KCl. Na_2SO_4 was about twice as efficient for this purpose as NaCl.[5] I showed later that the poisonous effects of NaBr, $NaNO_3$, and other salts, could also be annihilated by the addition of NaCl[6] and the same was found to be true for the effect of acids.[7] Since a pure solution of NaCl does not diminish but increases the permeability of the membrane it was not possible to explain these cases on the assumption of an opposite influence of the antagonistic electrolytes upon the permeability of the cell wall. I have recently made some experiments in collaboration with Mr. McKeen Cattell which have led me to a theory which seems also to furnish the physico-chemical analogue to this second group of phenomena of antagonism.

We found that the same concentration of KCl in distilled water causes the standstill of the heart of the embryo much more quickly than when contained in sea water. This experiment (which we will call experiment I) is simply a confirmation of the previous experiments of Loeb and Wasteneys on the adult fish, and it might be interpreted on the assumption that the permeability of the membrane for KCl is greater in distilled water than in sea water. But this explanation of experiment I is rendered impossible by experiment II which follows. Eggs were exposed to a KCl solution until the heart stopped beating. They were then equally distributed in distilled water and in sea water and it was observed in which of the two solutions the hearts began to beat first. The result was very striking. While the hearts of the eggs in the sea water began to beat again in a few hours, or in less than a day, those in distilled water often did not recover in a number of days. When their recovery failed to occur within reasonable time in distilled water the heart beat could be called forth in less than a day by transferring the eggs to sea water.

The recovery of the hearts poisoned with KCl depends upon the diffusing out of the KCl from the egg. If it is legitimate to assume in experiment I that in distilled water the KCl causes the heart to stop more quickly because the membrane is more permeable for KCl in distilled water than in sea water, we should expect that in the recovery experiment the hearts would also begin to beat sooner in distilled water than in sea water; since in the latter the membrane is assumed to be less permeable than in distilled water. Hence in sea water it should require more time for the excess of KCl to diffuse out of the egg than in distilled water, which is the reverse of what we observed. It is, therefore, impossible to explain these observations on the assumption that in distilled water the permeability of the cell membrane for KCl is greater than in sea water.

3. We next tried whether other substances acted like sea water, and

found that a great number of salts could replace sea water in the two above mentioned experiments. Thus m/2 NaCl or m/2 LiCl or $NaNO_3$ or many other salts retarded the poisoning effect of KCl upon the heart of the embryo and accelerated the recovery about as much as did sea water. We are, therefore, dealing here with a general salt action which increases in certain limits with the concentration of the salt and which varies for different salts. Thus it was found that for equal concentrations the citrates are much more efficient than the sulphates or tartrates and these are more efficient than the chlorides or nitrates; which indicates a valency effect of the anion.

A trace of acid when added to distilled water may also accelerate the recovery of the hearts poisoned with KCl. Bases have no such effect. Sugar solutions act like distilled water.

4. The question arises: How is it possible for salts (or acids) to retard the entrance of KCl into the egg or to facilitate the diffusion of KCl out of an egg which has been poisoned with this salt? All previous theories of antagonistic salt action (with the exception of the one proposed by Loeb and Wasteneys for the case of KCl and NaCl) have only considered three quantities: the concentration C_{I} of the poisonous salt (in our case KCl) in the outside solution, the concentration C_{II} of the injurious salt inside the membrane, and the "permeability" of the membrane. I am of the opinion that it is necessary to introduce another quantity, namely the concentration C_{III} of the poisonous salt at the boundary between the membrane and the outside solution. I assume, therefore, that there are forces at work (chemical or kindred) between the membrane and the poisonous salt in the surrounding solution (e.g., KCl) whereby this salt adheres or is attached to the external surface of the membrane in a concentration C_{III} which is different from C_{I}; and that C_{III} and not C_{I} determines the rate at which the salt diffuses into the egg. It is further assumed that the presence of other salts influences the (chemical or kindred) forces acting on the surface of the membrane whereby the concentration C_{III} at the surface of the egg is different from what it would be if the injurious salt (e.g., KCl) were alone in solution. *Whenever the influence of another salt is such as to diminish* C_{III} *we are dealing with a case of antagonistic salt action.*

5. In order to test this idea the writer made experiments in which he substituted a dye, namely neutral red, for KCl. He found that in the same concentration of neutral red the eggs of *Fundulus* are stained red more rapidly if the dye is contained in distilled water than in a salt solution or in a solution containing acid. He found, moreover, that the more dilute the solution of a given salt the less its antagonistic effect to

neutral red; and that sulphates inhibit the staining of the egg more powerfully than equal concentrations of chlorides. The same agencies which prevent or retard the entrance of KCl into the egg of *Fundulus* prevent or retard the staining of the membrane with neutral red.

If we stain eggs of *Fundulus* in neutral red and put them afterwards into distilled water or salt solutions free from stain, the eggs will be decolorized rapidly in salt solutions or in acid solutions, while they will be decolorized very slowly in distilled water. The decoloration occurs the more rapidly the higher the concentration of the salt, and more rapidly in sulphates than in chlorides.

The antagonism between the staining effect of neutral red upon the eggs of *Fundulus* and salts or acids is, therefore, parallel to that between the injurious effect of KCl upon the same eggs and salts or acids. In the case of neutral red we can see directly that the salt or acid diminishes the concentration C_{III} of the neutral red on the surface of the egg. When an egg stained in neutral red is put into a m/2 solution of NaCl or of $NaCl + CaCl_2$ or a m/8 solution of Na_2SO_4 streaks of the dye are seen to stream from the surface of the egg into the surrounding solution, while nothing of this kind occurs if the eggs are put into H_2O. Here we can ascertain by direct observation that the antagonistic action of the salt on the dye consists in diminishing the concentration C_{III} of the dye on the external surface of the membrane. The diminution of C_{III} on the external surface of the membrane diminishes the rate with which the dye diffuses into the egg and accelerates the rate with which the dye can diffuse out of the egg.

If it is legitimate to apply this reasoning to the explanation of the second group of cases of antagonistic salt action, the observed facts on the antagonism between KCl and other salts or acids could be expressed as follows: In distilled water the attractive forces acting between the outer surface of the membrane of the egg and KCl are very strong and hence the concentration C_{III} of this salt at the outer surface of the membrane reaches a high value; while these forces of attraction between KCl and the outer membrane are diminished when salts or acids are added to the outer medium in the proper concentration. This explains why the recovery of the embryo poisoned in KCl is very slow when the egg is put into distilled water, since in this case the concentration of the KCl (or other K salts) at the outer surface of the membrane remains very high and thus prevents the diffusion of KCl from the interior of the egg into the distilled water; while if a salt in a sufficiently high concentration is added to the distilled water the value C_{III} of KCl on the outside surface of the membrane is diminished and the barrier to the diffusion of the KCl from the interior of the egg to the outside solution is removed.

This theory holds probably for all cases of the second group of phenomena of antagonism, namely where salts (and possibly acids) in general antagonize the injurious action of an electrolyte. It must remain for further investigations to decide whether it holds also for the first group of cases of antagonism where the injurious action of high concentrations of a salt with a monovalent cation (e.g., NaCl) is inhibited by traces of a salt with a bivalent cation (e.g., $CaCl_2$). The two groups of phenomena are in one respect the converse of each other, since in the first group the efficiency of the antagonistic action increases with the valency of the cation, while in the second group the antagonistic action increases with the valency of the anion of the antagonistic salt.

[1] Loeb, *Archiv ges. Physiol., Bonn,* **88**, 68 (1901); *Amer. J. Physiol.*, **6**, 411 (1902).

[2] Loeb, *Archiv ges. Physiol. Bonn,* **107**, 252 (1905).

[3] Loeb, *Biochem. Zs.*, **47**, 127 (1912).

[4] Osterhout, *Science,* **35**, 112 (1912); *Bot. Gaz., Chicago,* **59**, 317 (1915).

[5] Loeb and Wasteneys, *Biochem. Zs.*, **31**, 450 (1911).

[6] Loeb, *Ibid.*, **43**, 181 (1912).

[7] Loeb and Wasteneys, *Ibid.*, **33**, 489 (1911); **39**, 167 (1912).

THE NITROGEN PROBLEM IN ARID SOILS

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Standing eminent, if not preëminent, everywhere, in considerations of soil fertility, the nitrogen problem is especially so under arid soil conditions. The acuteness of the situation in the latter has been recognized, however, by neither the scientist nor the practical man until recently when certain investigations on the one hand, and certain field manifestations on the other, have caused to stand out in sharp relief the nitrogen question from among others in California's soil puzzles. It is with reference to some of these recent findings, and their bearing on problems of soil fertility in California, that this brief paper is written as a forerunner of more detailed discussions soon to appear elsewhere.[1]

Considering only the average nitrogen-content of California soils, as based on a thousand or more analyses, the student of the subject does not obtain a true picture of the paucity in nitrogen which characterizes our truly arid soils. For many of our soils are situated in regions of heavy winter rainfall and produce a luxuriant spring growth;

hence their nitrogen-content, owing to the large supply of decaying organic matter, may compare very favorably with that of an average soil of the humid region. In our truly arid soils, however, which receive fifteen inches of rainfall per year or less, it is quite the usual thing to find the total nitrogen supply below 0.05% in the air-dry surface soil. Frequently, indeed, under the conditions of the San Joaquin Valley the percentage of total nitrogen in the surface soil may be no more than 0.01% or 0.02%. Therefore, even if all of this nitrogen could be rendered available for assimilation by the plant, the soil could not be expected to produce profitable crops for more than a few years. Fortunately, the roots of plants can draw more or less freely on the nitrogen supply of the soil below the first foot in depth, and thus crops have been produced at times on soils manifestly deficient in nitrogen. It must be remembered, however, that, even in arid soils in which we commonly find nitrification proceeding at the remarkable depth of six feet below the surface, nitrification and hence the available nitrogen supply decreases in intensity rapidly with increasing depth. As a result of a total supply of nitrogen which is too meager, coupled with the relatively small fraction thereof which is rendered available as is pointed out below, nitrogen starvation with its various manifestations in different plants is one of the prominent problems of soil fertility in California, and particularly in case of nonleguminous perennial plants. To illustrate this, it may be mentioned that it has frequently been found impossible to carry young fruit trees through one season of growth in the San Joaquin Valley on soils which are otherwise well supplied with plant food elements, for lack of a proper nitrogen supply.

More frequent even than the total starvation of crop plants on our typically arid soils is the occurrence of plants which languish for several years because of an insufficient supply of available nitrogen. With my coworkers I have obtained experimental and observational data, to appear in detail later as above indicated, which point significantly to a probable causal relationship between the lack of usable nitrates in the root zone and many features of backwardness or disease in our crop plants. These data indicate, almost without exception in the soils studied, that the lack of available nitrogen referred to is to be accounted for in one or more of four ways: 1st, a lack of sufficient nitrogen in the soil in toto; 2d, a feeble nitrifying power of the soil; 3d, accumulation of nitrates in the dry surface crust of the soil in which they can not be used by the feeding roots; and 4th, denitrification of nitrates produced within or added to the soil.

Our investigations point to the conclusion that the second cause, with certain qualifications soon to be indicated, is the most prominent of the four mentioned in connection with the nutritional problems of our crops. We are therefore in a position to confirm, as a result of our studies on truly arid soils, Stewart's[2] statement with respect to the intensity of nitrification in them, which was based on studies of the more distinctly semi-arid soils of Utah. While we possess ample evidence in support of Stewart's contention in the respect noted, I feel constrained to state that Stewart's criticism of Hilgard's explanation on the humus and humus-nitrogen question as between humid and arid soils is unwarranted by the facts, as we hope to show in detail in our forthcoming publications. The qualification which needs to be made with reference to our assertion as to the feeble nitrifying power of arid soils is that this power is more distinctly relatively rather than absolutely feeble. Thus we have found that some forms of nitrogen are very readily nitrified by certain arid soils which are not capable of nitrifying other forms of nitrogen at all. For example, the nitrogen of steamed bone-meal or cotton-seed meal or even of sulphate of ammonium is efficiently transformed into nitrate by many of our soils, which will not only produce no nitrate in a month's incubation period out of dried blood or high-grade tankage, but will even cause a loss of nitrate from that already contained in the soil. It appears further that the forms of nitrogen which nitrify most readily in humid soils give the most unsatisfactory results in arid soils as a general rule.

The nutritional factor contributing to the unsatisfactory growth of our crop plants is evidently then, in general, the soil's lack of power to transform enough of its own nitrogen supply or of the supply added in fertilizers or manures into a usable form. We must now give consideration to the theoretical aspects of the reasons underlying the condition just mentioned. There can be but little question that the feeble powers of the nitrifying flora of arid soils is primarily to be attributed to a deficiency in the supply of readily decayed organic matter in such soils. Inasmuch as the organic matter serves as a source of energy for the microörganisms, its initial small supply in virgin soils of this region coupled with the readiness with which it is depleted by oxidation, must operate to enfeeble and perhaps destroy the nitrifying bacteria. When we remember how small a supply of organic matter we start with in our soils and further that long hot dry seasons are the best of conditions for its dissipation through oxidation, it is small wonder that soil management by methods intended for application under eastern conditions should so far intensify the process, by constant tillage, that the necessary energy supply for the nitrifying bacteria should soon be so low as to render impossible the proper activation of those organisms. To all such direct damage to the soil must be added the indirect damage to the water and air supply necessary for the bacteria which follows the loss of organic matter from soils.

Still other considerations of a theoretical nature enter into the problem. These are concerned with the causes for the unsatisfactory nitrification of the nitrogen in dried-blood, for example. We have noted in all of our experiments that ammonification of dried-blood nitrogen may proceed with vigor in the soils in question while no nitrate is produced. It appears now that in soils which produce ammonia most energetically from dried-blood nitrogen that the nitrifying bacteria are deleteriously affected by the ammonium carbonate and gaseous ammonia which the soils in question liberate in large quantity. Whether this speculation be correct or incorrect will appear from experiments which are now in progress. It appears certain however, regardless of the outcome of these experiments, that the fermentation of dried blood proceeds very differently in the 'normal' and the 'abnormal' soils which represent respectively those which nitrify the nitrogen of dried blood and those which do not. In the former soils no odor of ammonia is ever noted in the soil cultures and no other odors but those of active soil are encountered. In the latter soils not only is ammonia given off in large quantities, but it is accompanied by ill-smelling gases resulting from putrefaction.

The brief space of this paper precludes the possibility of a more detailed discussion of the large amount of experimental data which we have accumulated in our experiments with both humid and arid soils; but it may suffice here to point out some of their practical bearings.

1. The addition and maintenance of a good supply of organic matter by green manuring or by the use of barnyard manure must be practiced on all soils deficient in nitrogen and organic matter.

2. Nitrogenous fertilizers when employed on such soils must be either of the low-grade organic variety such as steamed bone meal, cottonseed meal, and sewage sludge, or else sulphate of ammonium must be used.

3. The overheating of the soil, excessive evaporation of moisture, and oxidation of organic matter should be prevented through the use of some kind of straw or manure mulch. This is to be regarded as one of the most important measures for present and future soil management in California orchards and vineyards, when nitrogen and organic matter are deficient.

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[1] *Univ. Cal. Pub., Bull. Cal. Agric. Exp. Sta.*

[2] The Intensity of Nitrification in Arid Soils, *Proc. Amer. Soc. Agron.* 4. 132 (1912).

A NOTATION FOR USE IN THE DISCUSSION OF STAR COLORS

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The extension of absolute scales of photographic and photovisual magnitudes to the fainter stars provides a method of determining the colors of objects at present beyond the reach of spectroscopic investigation. For the statistical discussion of such color results it is convenient to adopt, tentatively at least, in place of the conventional color index, a notation similar to that used for spectral classification. Where for the latter we employ the letters B, A, F, G, etc., to represent various spectral types, we choose b, a, f, g, etc., to indicate different classes of color. When a more exact specification is required, a decimal subdivision of the classes may be employed.

Spectral type is determined by the number, character, and distribution of the lines in the spectrum. Color class depends upon the relative intensity of the continuous spectrum background in two different regions whose location varies somewhat with the instrumental equipment and the method of observing. Since both spectral type and color class stand in an intimate relation to the temperature, they are necessarily connected with each other, and when properly defined the color class indicates at once the general character of the spectrum. To secure this advantage, the classes b, a, f, etc., are assumed to represent the colors corresponding to the mean for typical spectra of the classes B, A, F, etc., respectively.

We cannot assume that there is an invariable correspondence between b and B, a and A, f and F, etc., for the observed color C, which specifically is represented by some one of the letters b, a, f, etc., may consist of at least three elements, namely,

$$C = C_S + C_{M,S} + C_\pi \tag{1}$$

The terms in the right member are functions of the quantities represented by the subscript letters. Thus C_S depends on the spectrum S; it is the average color index corresponding to S, a mean result derived from a large number of stars. Three or four determinations are available, for example, those of King, Parkhurst, and Schwarzschild. For the Mount Wilson photometric system we may adopt provisionally

$$C_S = 0.40\,S, \tag{2}$$

in which S has the values -1, 0, $+1$, etc., according as the spectrum is $B0$, $A0$, $F0$, etc. C_S is thus a part of the observed color index C and is expressed in magnitudes.

$C_{M,S}$ depends upon the absolute magnitude M; provisionally, at least, it must also be regarded as a function of the spectrum. Differences in absolute brightness for the same spectral type doubtless are accompanied by differences in stellar dimensions, and hence also by variations in the extent of the stellar atmosphere. These may produce modifications of intensity in the continuous spectrum, which possibly are also dependent upon the spectral type.

The last term C_π, provides for a scattering of light in its passage through space. It is a function of the parallax π or the distance Δ, but is independent of the spectrum. It may be expressed in the form adopted by Kapteyn, namely,

$$C_\pi = d\Delta \tag{3}$$

in which $\Delta = 0''.1/\pi$, while d is the change in the color index produced by an increase of one unit in the distance.

Finally, for the more precise definition of the color symbols it is convenient to suppose that the observed color index C is connected with the color classes b, a, f, etc., by the relation

$$C = 0.40s \tag{4}$$

or

$$s = 2.5C, \tag{5}$$

in which s is specifically denoted by $b0$, $a0$, $f0$, etc., when its numerical values are -1, 0, $+1$, etc., respectively. The relation is analogous to that connecting the average color index with spectral type.

Equation (1) can therefore be written in the form

$$s = 2.5\,C = S + 2.5\,C_{M,S} + 2.5\,d\Delta, \tag{6}$$

from which it appears that s differs from the spectrum S only in so far as the object in question deviates in luminosity and distance from the mean conditions underlying the color index and spectrum relation defined by equation (2). The color symbols b, a, f, etc. are thus intimately related to the spectrum symbols B, A, F, etc., and it will often be convenient to refer to them as *hypothetical spectra.*

As a matter of further notation, we may define the difference $C - C_S = C_E$ as the *color excess*, that is, the excess of the observed color over the mean color index corresponding to the spectrum S.

We have

$$C_E = 0.4\,(s - S) = C_{M,S} + d\Delta. \tag{7}$$

The color excess therefore represents the combined effect of the luminosity and distance terms, and is useful in investigations undertaken for the determination of the reality and magnitude of the phenomena giving rise to these terms.

DISTRIBUTION OF COLORS AMONG THE STARS OF N. G. C. 1647 AND M 67

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Recent investigations have given the colors of a considerable number of stars in two of the moderately large and open clusters, and it is of interest to examine the results for peculiarities of distribution and frequency. Although the data are slender, several points are revealed with some clearness.

N. G. C. 1647

The data for N. G. C. 1647 consist of effective wave-lengths for 184 stars determined by Hertzsprung.[1] Directly observed color indices[2] are also available for about 50 of these objects, but, as the two series of results are in close agreement, the discussion is based upon the effective wave-lengths alone. The following correspondences between color class[3] and effective wave length were established:

Effective Wave-Length	4190	4260	4330	4400	4470	4540A
Color Class	*b*0	*a*0	*f*0	*g*0	*k*0	*m*0

Counts within six 5′-zones gave the results in Table I, which shows the number of stars of each color within each zone, and also the totals, the relative areas of the zones, and the densities per area of 25 π sq. min. of arc.

TABLE I

Color and Distance—Number of Stars in N. G. C. 1647

COLOR	ZONE						TOTALS
	0′–5′	5′–10′	10′–15′	15′–20′	20′–25′	25′–30′	
b	2	4	5	1	1	1	14
a	8	15	16	4	9	12	64
f	12	10	14	14	7	13	70
g	0	3	4	3	4	3	17
k	0	0	4	1	7	0	12
m	1	1	1	1	1	2	7
Totals	23	33	44	24	29	31	184
Relative area	1	3	5	7	9	11	36
Density	23.00	10.99	8.80	3.43	3.22	2.81	—

A comparison of the totals for the three inner zones (N_1) with those for the three outer (N_2) suggests that the condensation may vary with the color (second, third, and fourth columns, Table II). But the background stars, as well as the members of the cluster, are included, and it does not follow that for the cluster itself the condensation is variable.

Let us assume for a moment that the ratio n_1/n_2 for the cluster alone does not vary with the color. Its value must then be approximately that of N_1/N_2 for the b and early a stars, for it is probable that all these belong to the cluster.[4] As a round number, we may assume $n_1/n_2 = 3$.

TABLE II

Percentage of Colors in N. G. C. 1647 and Background

COLOR	TOTAL NO. STARS		N_1/N_2	n	TOTAL NO. STARS		PERCENTAGES		
	0′–15′	15′–30′			Cl.	Bkgd.	Cl.	Bkgd.	Other Reg.
b....................	11	3	3.7	−0.25	15	−1	14	−1	1
a....................	39	25	1.6	4.50	46	18	43	24	13
f....................	36	34	1.1	8.25	37	33	34	43	47
g....................	7	10	0.7	2.88	5[5]	11[5]	5	15	30
km....................	7	12	0.6	3.62	4[5]	14[5]	4	19	9
Totals...............	100	84		19.00	108	76	100	100	100

The number of background stars, n, can then be calculated for each color by

$$\frac{N_1 - n}{N_2 - 3n} = 3,$$

for which the unit of area is 225 π sq. min. The results are in the fifth column of Table II. The numbers for the cluster and the background can now be separated; their values are in the sixth and seventh columns. The percentage distribution of colors is in the two following columns. The last column shows the corresponding mean distribution for two other regions of the sky, namely, the North Pole and that of S Cygni.[4]

The background percentages were calculated on the assumption that the condensation within the cluster is the same for all colors; their agreement with the values in the last column of Table II is close enough to give some validity to that assumption. At any rate, there is no evidence for supposing that the distribution within the cluster depends upon the color.

An examination of the distribution from the standpoint of distance and brightness suggests a relatively greater condensation of the brighter stars; but from 10.5 downward the distribution seems to be the same for all magnitudes.

The apparent magnitude corresponding to maximum frequency is, however, quite different for the different colors, as Hertzsprung has already pointed out[1] and as appears from the results in Table III. From these it is seen that the magnitude at which maximum frequency occurs increases by about two magnitudes for an increase in the color of one color class. If an extrapolation of this result is to be permitted, one would infer that the majority of the *g*, *k*, and *m* stars belonging to the cluster are so faint that they do not fall within the limits of the available data. This probably accounts for the relatively small percentage of these colors found among the cluster stars. (See eighth column, Table II.)

Inasmuch as there are no reliable data for the distribution of the background stars, with respect to color and magnitude, these objects are necessarily included in the conclusions stated in the two preceding paragraphs.

TABLE III

Color and Magnitude—Number of Stars in N. G. C. 1647

MAG.	COLOR						TOTAL
	b	*a*	*f*	*g*	*k*	*m*	
<9.5	5	0	0	0	2	0	7
9.5–10.5	8	11	0	1	1	0	21
10.5–11.5	0	13	3	0	0	1	17
11.5–12.5	1	25	6	1	0	1	34
12.5–13.5	0	8	26	3	0	2	39
13.5–14.5	0	8	34	12	9	2	65
> 14.5	—	—	—	—	—	1	1
Totals.......	14	65	69	17	12	7	184

MESSIER 67

The data consist of 231 color indices derived by Shapley with the 60-inch reflector. The distribution for color and distance is shown in Table IV. The densities in the last line indicate that here the counts have been extended beyond the boundary of the cluster, and that from 7′.5 outward the tabulated stars belong to the background.

We have, therefore, in this instance, an approximation for the constitution of the background in the direction of the cluster. After deducting the background stars and combining the results for the cluster into two zones, we find the numbers in the second and third columns of Table V. The totals for the cluster and the background, and the percentage distribution are in the remainder of the table. The percentages for the background are in reasonably good agreement with those for the North Pole and the region of S Cygni given in the last column. There is no evidence of a variation of the condensation with color either before or after the deduction of the background stars.

The distribution with respect to magnitude and distance, and magnitude and color, is similar to that for N. G. C. 1647, although the color and magnitude relation is less clearly marked than was the case with that cluster.

TABLE IV

Color and Distance—Number of Stars in M 67

COLOR	ZONE						TOTALS
	0–1′.5	1′.5–3′.5	3′.5–5′.5	5′.5–7′.5	7′.5–9′.5	9′.5–11′.5	
a.............	0	2	2	0	1	1	6
f.............	5	10	9	15	11	24	74
g.............	8	39	34	22	12	14	129
k.............	2	7	9	2	1	1	22
Totals.........	15	58	54	39	25	40	231
Relative area...	2.25	10	18	26	34	42	132.25
Density........	6.67	5.80	3.00	1.50	0.73	0.95	

TABLE V

Percentage of Colors in M 67 and Background

COLOR	CLUSTER ALONE		TOTALS		PERCENTAGES		
	0′–3′.5	3′.5–7′.5	Cl.	Bkgd.	Cl.	Bkgd.	Other Regions
a	2	1	3	3	3	3	13
f.............	9	4	13	61	11	54	47
g	43	42	85	44	71	40	30
k.............	9	9	18	4	15	4	9
Totals.......	63	56	119	112	100	101	99

SUMMARY OF RESULTS

1. Neither N. G. C. 1647 nor M 67 show any dependence of condensation upon color which cannot be explained on the basis of included background stars.

2. With the possible exception of an excess of a few bright stars in the center of N. G. C. 1647, there seems to be no dependence of condensation upon magnitude.

3. There is a marked relation between color and magnitude in N. G. C. 1647; the connection is also evident, though less pronounced, in the case of M 67.

4. The probable distribution of colors in the background of N. G. C. 1647, and that actually observed for M 67, is similar to that for the North Pole and the region of S Cygni.

5. In neither case is the limiting magnitude low enough to include all the cluster stars; but as far as these limits, the percentage distribution of the different colors in the cluster is notably different from that in the background; moreover, there is a marked difference for the two clusters. N. G. C. 1647 contains a considerable number of *b* and *a* stars, and practically none of the *g*, *k*, and *m* classes, except as they may be fainter than the limiting magnitude. M 67, on the other hand, shows no *b* and only 3% of *a* stars. The maximum frequency of 71% is for *g* and, curiously enough, no *m* stars appear.

[1] *Mt. Wilson Contr.*, No. 100; *Astrophys. J.*, 42, (1915).

[2] *Mt. Wilson Contr.*, No. 102; *Astrophys. J.*, 42, (1915).

[3] A Notation for Use in the Discussion of Star Colors, these PROCEEDINGS, 1, 481 (1915).

[4] *Mt. Wilson Contr.*, No. 81; *Astrophys. J.*, 39, 361 (1914).

ON THIELE'S 'PHASE' IN BAND SPECTRA

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In the emission spectra of all compounds and of many elements the lines start abruptly at a certain wave-length, near which they are very close together, and separate more and more as the distance from the beginning or 'head' is increased. Such spectra are called band or channeled spectra because, with relatively small dispersion, they have the general appearance of the chiaro-oscuro of a channeled column illuminated laterally. One of the problems of spectroscopy consists in finding the law governing the relative frequencies of the lines of one band or of a group of bands. In other words, the problem is to arrange the lines in series and to express their frequencies by a mathematical formula, as has been done for a fairly large number of line spectra. The analysis of a line spectrum into series is greatly facilitated and made secure by the fact that lines of the same series show their functional relationship by the similarity of the changes which they undergo when the source is subjected to pressure, or is placed in a magnetic field, etc. On the other hand, the frequencies of the lines of the majority of well-developed band spectra do not exhibit the pressure and Zeeman effects. Consequently, since physical criteria are lacking, the grouping into series of the lines of band spectra depends wholly upon the apparent regular sequence of the lines and the degree of accuracy with which their wave-lengths or their frequencies fit a smooth curve or satisfy an empirical equation. It is therefore evident that any formula proposed for band spectra must be subjected to very severe tests before it can be accepted as an expression of a law of nature.

A very general law for all series spectra was proposed by Thiele in the year 1897. It was expressed by the equation

$$\lambda = f[(n+c)^2], \tag{1}$$

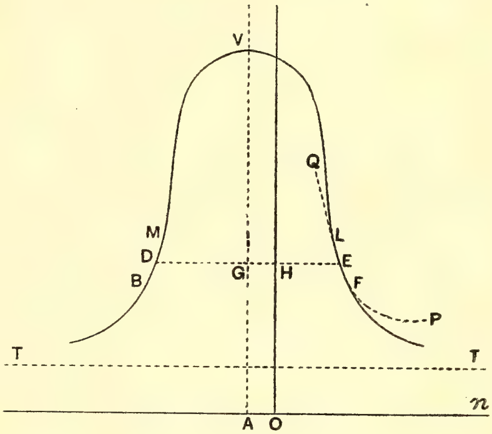
where λ denotes the wave-lengths of a line whose ordinal number is n, and c is a constant for any one series. Thiele styled c the "phase" of the series. The integer n is supposed to assume negative as well as positive values. Consequently a series must in general be composed of two groups of lines each of which would ordinarily be called a series. More precisely, the positive branch of each series must be accompanied by a negative branch of the same series, having the same head ($n = 0$) and the same 'tail' ($n = \infty$) and being represented alternately by a line in each interval of the other branch. In the case of band spectra Thiele highly recommended the special form of function (1) which he employed with apparent success in his computations of the wave-lengths of the lines of the carbon band at λ 5165 and which he wrote as

$$\lambda = \lambda_0 - \kappa \left(\frac{n+c}{10}\right)^2 \cdot \frac{1 + s_1\left(\frac{n+c}{10}\right)^2 + \cdots + s_{r-1}\left(\frac{n+c}{10}\right)^{2r-2}}{1 + t_1\left(\frac{n+c}{10}\right)^2 + \cdots + t_r\left(\frac{n+c}{10}\right)^{2r}} \tag{2}$$

The objects of the present paper are to outline a practical method for calculating c *without assuming a special form* of function (1) and to give briefly the results obtained by applying the new method to the λ 5165 carbon band and the third cyanogen band at λ 3883.

It should be remarked in advance that it is not feasible to solve equation (2) for c when a sufficiently large number of wave-lengths are used to ensure a satisfactory degree of accuracy. The adjacent figure shows the form of curve representing equation (1) in rectangular coördinates. The even power of $n + c$ involved in (1) causes the graph to be symmetrical with respect to the line AV whose equation is $n = -c$. $AV = \lambda_0$. The branches of the curve are asymptotic to the line TT' whose constant ordinate, $\lambda_0 - \kappa\, s_{r-1}/t_r = \lambda_t$, equals the wave-length of the tail of the series. The points of inflection express the hypothesis that the intervals between consecutive lines of the same series attain a maximum, as n increases arithmetically, and then decrease indefinitely. The curve also brings out two more properties of function (1) which are that the number of lines is finite and infinite respectively in the regions of the head and tail of a band.

A straight line parallel to the axis of n, and at a suitable distance therefrom, will intersect the curve in two points, such as D and E. Hence, if n and n' denote (the algebraic values of) the abscissae of any two points on the curve which have the same value of λ, it follows at once that $c = -\frac{1}{2}(n + n')$. [$GH = c$, $HE = n$, $DH = -n'$.] In the case of any series whose wave-lengths have been accurately determined there is no inherent difficulty associated with the calculation of n and n' corresponding to a chosen numerical wave-length. It is only necessary to evaluate the coefficients of any simple, convenient interpolation formula which represents a curve PQ fitting the locus of actual wave-lengths sufficiently closely over a limited range of spectral lines, such as FL or BM. In other words, the value of c may be obtained by taking an adequate number of terms of the power polynomial $\lambda = a_0 + a_1 n + a_2 n^2 + \ldots + a_k n^k$, determining the coefficients $a_0, a_1, \ldots, a_k$ from the known wave-lengths, substituting for λ an arbitrary wave-length (OH), and solving for n. Then using the same value of λ and repeating the process for the negative quadrant, the corresponding value of n' is computed. Knowing n and n', c follows immediately from the relation $c = -\frac{1}{2}(n + n')$. In practice I have found that three coefficients (a_0, a_1, a_2) are sufficient for all cases, that the method of least squares can be used to great advantage, and that certain transformations of coördinates simplify the computations enormously.



The accompanying table gives the values of c computed by the above method for two of the series of the λ 5165 carbon band. This band is the one investigated arithmetically by Thiele who obtained mean values of c by applying the laborious method of trial and error to formula (2). The tabulated values of the phase were calculated from the more recent and accurate wave-lengths published by Leinen in the year 1905.

The numbers in the fourth column show conclusively that c is not strictly constant for either the α or the δ series of the λ 5165 carbon band. In the case of the α series the phase increases from about 0.263 to 0.268 at least, while for the δ series it decreases from 0.250 to 0.235 or less. The values of c used by Thiele were 0.266 and 0.2445 for the α and δ series respectively.

The fact that the variations of the phase are real and not fictitious may be established by the following considerations. Each of the seven lines +36 to +42 would have to be displaced in the same direction, relatively to the corresponding group of lines (−37 to −43), by 0.104 A to change the value of c from 0.2686 to 0.2629. This shift is too great to be admitted, for, the *arithmetical sum* of all the fourteen residuals ($\lambda_{\text{calc.}} - \lambda_{\text{obs.}}$) divided by 14 only equals 0.007 A. In like manner, for the δ series, the average residual is 0.016 A whereas the displacement required to change c from 0.2344 to 0.2499 would be 0.355 A. By applying the same method and tests to the two apparent series of doublets which start at the first head of the λ3883 cyanogen band I have been able to show that these series cannot be combined as the positive and negative branches of a single Theile series with a constant phase.

SERIES	+ BRANCH . n .	— BRANCH . n'.	c	λ INTERVAL
α.........	+ 3,...,+ 9	− 3,...,− 9	0.2632	5164.4−5153.4
α.........	+ 3,...,+ 9	− 4,...,−10	0.2633	
α.........	+ 9,...,+16	− 9,...,−16	0.2626	5155.7−5129.8
α.........	+ 9,...,+16	−10,...,−17	0.2625	
α.........	+20,...,+26	−20,...,−27	0.2666	5116.0−5075.5
α.........	+36,...,+42	−36,...,−42	0.2684	5008.0−4944.9
α.........	+36,...,+42	−37,...,−43	0.2686	
δ..........	+ 8,...,+18	− 8,...,−18	0.2499	5156.3−5119.4
δ..........	+23,...,+29	−23,...,−29	0.2417	5095.4−5052.8
δ.........	+35,...,+41	−35,...,−41	0.2375	5009.6−4951.6
δ.........	+52,...,+60	−52,...,−60	0.2344	4843.2−4746.6

In conclusion, it may not be superfluous to remark that equation (2) is unsatisfactory in two respects: (a) as an interpolation formula it is too complicated, since modern wave-lengths require the retention of at least eight parameters, and (b) it predicts tails much too far away from the heads and in a region of the ultra-violet where no tails have been found experimentally. For additional details and comments reference must be made to the *Astrophysical Journal* in which the present paper will be presented in full.

WHY POLAR BODIES DO NOT DEVELOP

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Since the work of O. Hertwig (1890) and Boveri (1891) it has been known that the two small cells which are formed at one pole of the egg during its maturation divisions and which have long been known as 'polar bodies' since they mark out the 'animal' or ectodermal pole of the egg are homologous with cells which in the male give rise to functional spermatozoa, so that the polar bodies are generally recognized as small and non-functional egg cells, a view which was first set forth by Mark (1881). Before the two maturation divisions which lead to the formation of eggs or spermatozoa the sex cells are called 'oöcytes' in the case of the female, 'spermatocytes' in the case of the male; after the first maturation division they are called 'second oöcytes' or 'second spermatocytes' and after the second maturation division they are known as 'oötids' or 'spermatids.' In the case of the male both of these divisions are equal and all of the spermatids may become functional spermatozoa, in the female both maturation divisions are usually very unequal the smaller division product in each case being the 'polar body,' and the larger the 'egg.'

In many animals the maturation divisions of the egg may be made approximately equal by pressure or by centrifugal force so that the difference in size between the polar body and the egg largely disappears. Thus if the eggs of the gastropod *Crepidula plana* are subjected to a centrifugal force of about 600 times gravity during the first or second maturation divisions some of the eggs in which the axis of centrifuging coincides with the axis of the division figure, or spindle, divide into approximately equal halves. In my experiments on the eggs of Crepidula the giant polar body is usually the second one since the eggs were centrifuged in most cases during the second maturation division; when the centrifuging occurred during the first maturation the first polar body is the giant one; when it occurred during both maturation divisions both polar bodies are abnormally large. Nevertheless only one of these cells develops.

In this animal the spermatozoon always enters the egg during the first maturation division and usually at the pole opposite that at which the polar bodies normally lie. In normal cases therefore the spermatozoon is always found in the 'egg' or larger daughter cell and never in the polar body; and even when the two daughter cells are made equal by pressure or by centrifugal force the spermatozoon always lies in one daughter cell and not in the other. The same is true if the centrifugal force is applied during the second maturation division; although the daughter cells may be equal in size the spermatozoon is found in only one of these cells. Subsequent events show that only that cell develops which contains the spermatozoon.

In Crepidula and many other mollusks as well as in annelids and ascidians the spermatozoon normally enters the egg at the beginning of the first maturation division and always before the first polar body is cut off. Indeed the first maturation spindle will usually remain in the metaphase, or middle stage of division, until a spermatozoon enters or until the egg is stimulated by other means (artificial parthenogenesis) to begin development. As soon, however, as a spermatozoon enters an egg the maturation division proceeds and the whole process of development is set in motion in the cell which contains the spermatozoon. But the polar bodies, which do not contain the spermatozoon, never develop even though they may be as large as, or even larger than, the egg which does develop.

The giant polar bodies of Crepidula resemble unfertilized eggs in that the cytoplasm remains diffused throughout the whole cell whereas in eggs after fertilization there is a fairly sharp separation of cytoplasm and yolk. Associated with this lack of segregation of cell substances there is also a lack of distinct polar differentiation in giant polar bodies. The intra-cellular movements which lead in the fertilized egg to the segregation of cytoplasm at the animal pole and of yolk at the vegetative pole do not take place in giant polar bodies.

These giant polar bodies invariably contain a nucleus and they may contain samples of all the substances found in the egg; they may contain most of the protoplasm of the egg; they may be larger than the cell which does develop but the one thing which they lack is a spermatozoon, whereas the egg cell which does develop invariably contains a spermatozoon. We must conclude therefore that the giant polar bodies of Crepidula do not develop because they do not contain a spermatozoon.

The failure of normal polar bodies to be fertilized and to develop is generally held to be due to their small size, but even when these polar bodies are large as is sometimes the case in mollusks, polyclads and nematodes they do not undergo fertilization and do not delevop though they sometimes divide once or twice. In one case only has the development of a polar body, or rather of both second oöcytes, been observed.

Francotte (1898) discovered in the polyclad Prostheceraeus that at the first maturation the egg sometimes divided into two nearly equal cells; each was then entered by a spermatozoon and normally fertilized and at the second maturation division each formed a small second polar body and the larger cells then underwent normal cleavage and developed to the gastrula stage. In a few other instances the entrance of a spermatozoon into a polar body has been reported though some of the cases are not entirely convincing and need verification. Thus Platner (1886) described the entrance of a spermatozoon into a polar body of Arion; he maintained that the polar bodies are formed before the entrance of the sperm, which would make this case similar to that of Prostheceraeus, but the evidence is by no means conclusive. Sobotta (1895) calls special attention to the large size of the polar bodies in the mouse and suggests that they may be capable of being fertilized but offers no evidence in favor of this view. Kostanecki (1897) has observed a spermatozoon with its head penetrating the second polar body of Physa, a thing which he regards as merely a 'curiosity.'

Lefevre (1907) observed that the polar bodies of Thalassema undergo several cleavages resulting in the formation of a morula-like cluster of minute cells when they are exposed to weak solutions of HCl;" thus they respond to the same divisional stimulus supplied by the acid solutions as does the egg cell itself."

The most striking difference between Prostheceraeus and other animals in which giant polar bodies have been reported is to be found in the fact that in the former fertilization does not take place until after the first maturation division is completed and then each of the daughter cells is fertilized, whereas in the latter the entrance of the spermatozoon occurs before the completion of the first maturation division with the result that one of the daughter cells contains a spermatozoon and the other does not.

In this fact is to be found the explanation of the different behavior of the giant polar bodies of Prostheceraeus and of Crepidula, for it is well known that one of the first effects of the entrance of a spermatozoon into an egg is the prevention of other spermatozoa from entering. If the spermatozoon enters the egg before the first polar body is cut off that polar body as well as other cells which are formed later from the egg are rendered "immune" to other spermatozoa.

But although the influence of the entering spermatozoon spreads so rapidly over the egg that within a few minutes at most it renders all portions of the egg surface 'immune' to other spermatozoa and thus prevents the fertilization of polar bodies which are formed after fertilization, this influence does not go so far as to cause the polar bodies to develop, even though such polar bodies may be formed several hours after the spermatozoon enters the egg. In Crepidula the second polar body is formed about three hours after the entrance of the spermatozoon into the egg, and during this time the sperm head has grown into a vesicular nucleus and the sperm aster has become quite large, but in spite of this the spermatozoon has not sufficiently affected the egg substance to cause the second polar body to develop even though that body may contain the larger part of the egg protoplasm. Only that portion of the egg develops, in such cases, in which the sperm nucleus and aster are present.

This conclusion is similar in many respects to that reached by Zeigler (1898) who found that when eggs of the sea urchin, *Echinus microtuberculatus*, were constricted by cotton fibers under pressure only that portion of the egg which contained the sperm segmented while the portion containing the egg nucleus never divided, though its nucleus frequently went through the division phases, but without any division resulting. In this case the portion of the egg containing the sperm might remain for some time connected with the other portion by a narrow neck, and yet the influence of the sperm in the one half did not cause the other half to develop.

These facts are of interest because of their bearing on the nature of one of the processes concerned in fertilization. In a series of important and extensive works on artificial parthenogenesis and fertilization which he has summarized in a recent book Loeb (1909) has shown that at least two factors are involved in artificial parthenogenesis, (1) an external factor, such as butyric acid, which causes a cytolysis of the cortical layer of the egg followed by increased oxidation and which leads to the rapid disintegration of the egg at normal temperatures, and (2) an internal factor, such as hypertonic solutions, lack of oxygen, etc., which inhibits this disintegration. Loeb concludes that in normal fertilization also both of these factors are present and that the spermatozoon carries substances into the egg which (1) cause cytolysis of the cortical layer and increased oxidation and also other substances which (2) inhibit this cytolysis before it leads to the disintegration of the egg.

My experiments on the giant polar bodies of Crepidula show that changes in the cortical layer which prevent the entrance of a second spermatozoon take place very rapidly over the entire egg, but that the spermatozoon which enters does not cause any portion of the egg to develop except the cell in which it lies. Although the spermatozoon enters the egg of Crepidula about three hours before the formation of the second polar body the influence of the spermatozoon on the egg protoplasm during this time is not sufficient to start development in the second polar body even though it may contain the greater part of the egg substance.

This indicates that the second factor concerned in the process of normal fertilization is not to be found in the diffusion through the egg of some chemical substance carried in by the spermatozoon but is some non-diffusable substance, probably an organic structure.

Long ago Boveri (1887) showed that under certain circumstances the egg of Ascaris may divide at the first cleavage so that half of the egg nucleus passes into each daughter cell while the sperm nucleus does not divide but goes entire into one of the first two cells. Such a condition he called 'partial fertilization,' and in such cases he found that both halves of the egg develop, thus showing that the activating influence of the spermatozoon has affected both halves. Since in this case the centrosome is the only structure derived from the spermatozoon which is known to go into both cleavage cells he reached his well known conclusion that the essential thing in fertilization is the addition of a centrosome to the egg cell.

It is possible of course that other unrecognized structures are introduced by the spermatozoon and serve to activate the egg. Meves (1911) found that the spermatozoon of Ascaris introduces into the egg a number of coarse granules, the 'plastochondria,' which he thinks unite with similar granules in the egg and are then distributed to the cleavage cells. However, in one of the Echinids he finds that the large granule or 'plastosome' which is derived from the middle-piece of the spermatozoon goes into one only of the first two cleavage cells and yet both develop. I have found that the 'plastosomes' in the eggs of gastropods and ascidians may be distributed very unequally to the first two cleavage cells without interfering with the further division of both cells, and there is no evidence whatever that the activating influence of the spermatozoon is due to these granules.

On the other hand many investigators have held that fertilization is essentially a chemical process and that the activation of the egg depends upon the introduction by the spermatozoon of certain chemical substances which diffuse through the egg.

The observations recorded in this paper indicate that the second or internal factor in normal fertilization is a non-diffusable substance which is introduced by the spermatozoon, and they strongly suggest that this factor is the sperm centrosome, a position which Boveri has long maintained and which I have hitherto contested.

In conclusion, giant polar bodies do not develop because they are not fertilized and they are not fertilized because they are generally formed after a spermatozoon has entered the egg and has rendered it impervious to other spermatozoa.

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RADIAL VELOCITIES OF THE PLANETARY AND IRREGULAR NEBULAE

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LICK OBSERVATORY, UNIVERSITY OF CALIFORNIA

Presented to the Academy, September 10, 1915

In a former number of the PROCEEDINGS[1] one of the authors presented results on the radial velocities of 54 gaseous nebulae determined by spectrographic methods at the Lick and D. O. Mills Observatories, working respectively in the northern and southern skies. It was there shown that the planetary nebulae, or those of regular form, are rapid travelers in comparison with the stars, a fact which casts serious doubts upon the generally accepted hypothesis that the stars have been formed from planetary nebulae by processes of evolution.

During the past year observations at both institutions have been extended to fainter members of this class of objects, and there are now available some 348 measures of the velocities of approach and recession of 92 gaseous nebulae, or those whose spectra are composed of bright lines.

Before proceeding to a discussion of the results, those for 12 nebulae situated in the Magellanic Clouds are set aside for special consideration, since, as will be shown later, they are attended by conditions which are not representative of nebulae in other portions of the sky. There remain 80 observed objects, of which 7 are extended or irregular in form, and 73 have the forms characteristic of planetary and ring nebulae. When all the observed velocities have been freed from the effects of the solar motion, the average velocities of approach and recession of the various groups are found to be:

Of 7 extended nebulae	10 km./sec.
Of 73 planetaries or regular form	39 km./sec.
Of 34 'stellar' (less than 5″ diameter)	50 km./sec.
Of 39 non-stellar (disks and rings)	29 km./sec.

The velocities of the few extended nebulae thus far observed are low and of the order of the average velocity of stars of Class B, a result which is not surprising in view of the intimate relationship known to exist between these two classes of objects. The average velocity of the 73 planetaries is, on the other hand, more than six times that of the Class B stars. That planetary nebulae will eventually become helium stars can scarcely be questioned, but the old hypothesis that helium stars have in general evolved from planetary nebulae hardly appears tenable.

A division of the 73 planetaries into two groups according to their apparent size brings out a further relation of great interest The 34 nebulae which are described in the catalogues as 'stellar,' or those whose apparent diameter is less than 5″, are found to be traveling almost twice as fast as the 39 nebulae of greater apparent diameter whose forms are those of hazy disks or of concentric or superimposed rings. If these stellar nebulae are small objects their higher velocities may be in harmony with recent indications concerning stellar motions to the effect that the stars of small mass are traveling more rapidly than those of great mass. On the other hand, if they appear smaller on account of their greater distance, an analogy is suggested to the recent results obtained by Adams, that the more distant stars of certain spectral classes are traveling more rapidly than those which are nearer to us.

Attention was called in the earlier paper to the fact that the nebular velocities are distributed with more or less equal frequency for speeds of all observed magnitudes, a circumstance in marked contrast with the helium stars, whose peculiar motions follow pretty closely the 'probability curve.' Similar conclusions even more strongly marked may be drawn for the distribution of the velocities of the 34 stellar nebulae.

For an assumed apex of the sun's way at Right Ascension 270° and Declination +30°, the 73 planetaries give a solar velocity of 2.01 km./sec., while the 7 extended nebulae give 20.7 km./sec. These values are in remarkable accordance with that derived from 225 Class B stars, namely, 20.2 km./sec.

When the results for the 73 planetary nebulae are examined for the existence of preferential motions, in accordance with Kapteyn's two star-stream hypothesis, it is found that they show a marked preference for motions making small angles with the line joining Kapteyn's two vertices. A similar indication is shown when the velocities of these nebulae are grouped with reference to their distances from the central line of the Milky Way, a conclusion which is entitled to less weight, since only 10 of these objects are situated more than 20° from the galactic plane.

The fact that the gaseous nebulae have motions which are characteristic of the stars, as shown by the value of the solar motion derived from them, and by their exhibiting the phenomena of star streaming, taken together with their strong concentration in the Milky Way, affords a reliable basis for the view, frequently expressed, that these nebulae are members of our stellar system.

Since the announcement[2] by Dr. Wilson of the high velocities of 5 nebulae in the Magellanic Clouds, 7 more of these objects have been observed. The only known nebula in the Lesser Cloud has a velocity of recession of 149 km./sec., while 11 nebulae observed in the Greater Cloud have velocities of recession ranging between +237 and +287 km./sec.

In as much as gaseous nebulae are unknown in the surrounding regions of the sky, it is a fair assumption that all of these bright-line objects are within the structure of the two Clouds. It is therefore probable that the velocity of the Greater Cloud with reference to the stellar system is approximately the average velocity of the 11 nebulae observed within it, or 262 km./sec. recession. The observed velocity for only one object in the Lesser Cloud is hardly sufficient to justify an analogous hypothesis for it. However, the similarity in the appearance of the two clouds and their proximity to each other lead to the suspicion that a more or less intimate relationship may exist between them. Furthermore, the high galactic latitudes of these objects, coupled with their high velocities with reference to the centroid of stars, lends some support to the hypothesis that the Magellanic Clouds are isolated cosmic units, systems which have no apparent connection with our own stellar system.

[1] These Proceedings, **1,** 8 (1915).

[2] *Ibid.,* **1,** 183 (1915).

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Volume 1 OCTOBER 15, 1915 Number 10

THE OCTOPUS MOTIVE IN ANCIENT CHIRIQUIAN ART

By George Grant MacCurdy

PEABODY MUSEUM, YALE UNIVERSITY

Presented to the Academy, June 23, 1915

The ancient art of the one small province of Chiriqui in the diminutive Republic of Panama is perhaps better known than that of any region of like size in the New World. The material for study is abundant, consisting largely of ceramic products, both painted and unpainted. These have become more or less familiar to students of archeology through two large monographs, one by Professor W. H. Holmes[1] and the other by myself.[2]

It was found that the pottery could be readily divided into about a dozen rather distinct groups, depending largely on the nature of the paste and other materials used as well as the method of producing the dominant decorative features. For example, symbolism and ornament in the unpainted ware find expression in plastic forms and incised patterns. Both are traceable to zoömorphic originals, as are the plastic and painted motives in the painted ware. The motive may represent the entire animal in fairly realistic fashion or it may consist of almost any part of the animal, as for example the head, foot, tail, eye, appendage, or some characteristic body marking. Certain animal motives are always represented in the round or in relief; others appear only as incised patterns and still others predominantly as painted forms.

Thus we find the plastic armadillo dominating the great group that might appropriately bear that name; the incised serpent goes with a distinctive group of black ware; the plastic fish in the guise of tripod supports runs through another group; while the painted alligator is supreme in two closely related groups of painted ware.

One soon learns by experience to associate a given motive with a given paste, slip, quality of modeling, and the character and number of the colors employed, as well as the method of their application. After the armadillo ware, perhaps the largest group is the one to which Holmes gave the name *lost color* ware, the designs being produced by the removal of color rather than by its direct application. In addition to the process of negative painting employed, this group is also characterized by distinctive forms as well as the nature of the paste and the colors, also the degree of finish (or absence of it) to which the modeling was carried.

In the lost color ware a majority of the designs consists of rhomboidal figures, triangles, associated bands composed of groups of straight lines, and designs in the shape of fronds and waving arms. They are found alone as well as in combination. The fronds, waving arms, triangles, and straight bars, as well as the lozenge-shaped designs are often associated with series of dots. What is the meaning of all this? At first glance they seem far removed from the motives derived from animal forms so characteristic of other groups of Chiriquian pottery. Could they be plant derivatives? Are they perhaps simply the products of uncontrolled fancy?

A key to the mystery recently came to light in the shape of a more realistic rendering of the motive than had been known hitherto. It consisted of a lozenge-shaped body to which was attached eight waving arms. It filled a circular panel on the two sides of a round-bodied lost color vase collected by Mr. George G. Heye while on a trip to Chiriqui in 1913. This vase to which Professor Marshall H. Saville had called my attention was recently published by me.[3] The design represents an octopus (fig. 1). At that time, I pointed out its kinship to many designs previously published by Holmes and myself, but the significance of which had not been understood.

A further study tends not only to confirm what was said in my last note but also to emphasize the importance of this newly discovered motive as a special feature of lost-color symbolism in ancient Chiriquian art, and as another example of how a whole group of related motives in primitive art may be traced to a single zoömorphic prototype.

Recently in going over some duplicate Chiriquian pottery with a view to making an exchange, I came across several new variations of the octopus motive. In one shapely vase the eight octopus appendages are evenly distributed over the upper zone, being attached to a line just below the neck of the vessel (fig. 2). The neck and aperture thus take the place of the octopus body and mouth. The artist's point of view can perhaps be better appreciated by looking down on the vase from above rather than by a side view.

A similar idea but expressed in a different way is shown in figure

3 (fig. 167 of Holmes's monograph). Here the arms are attached to the equatorial zone (or slightly lower) and rise like short-based triangles to the neck of the vase. They are dotted to represent the succers. In looking down upon this vase the arms converge toward the narrow neck of the vessel, which thus becomes the mouth opening of the octopus. The body of the vase therefore becomes the body of the octopus, which is represented not only in painted design but also in the round.

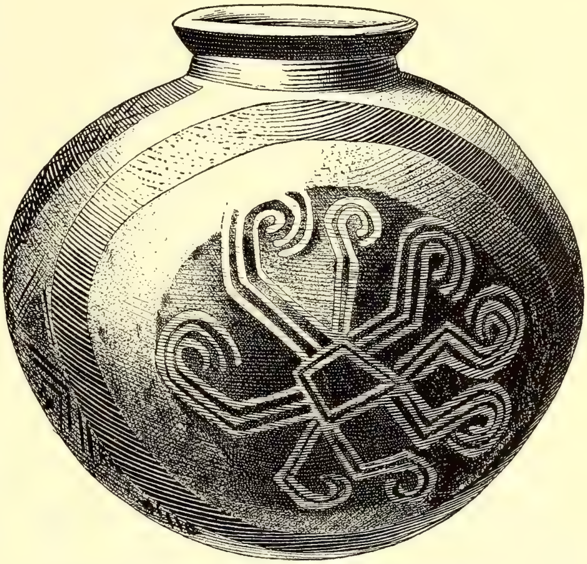

FIG. 1.—OCTOPUS MOTIVE AS A CIRCULAR PANEL DECORATION. HEYE COLLECTION. CAT. NO. $\frac{3}{3511}$.

Thus the octopus appendage may be represented in two ways; as a frond-like arm or as a short-based triangle. Each type is often (but not always) accompanied by dots representing the succers. Both types are seen in figure 4. Four of one kind alternate with four of the other kind converging toward the neck of a small globular vase with black ground and cream pattern. The four fronds are straight, each consisting of two lines enclosing a row of dots. Each triangular arm is set in a series of paired converging lines the outer ones being accompanied by succer dots. An identical octopus design is repeated on the lower half of the vase, beginning midway and with the eight arms converging toward a central point on the bottom.

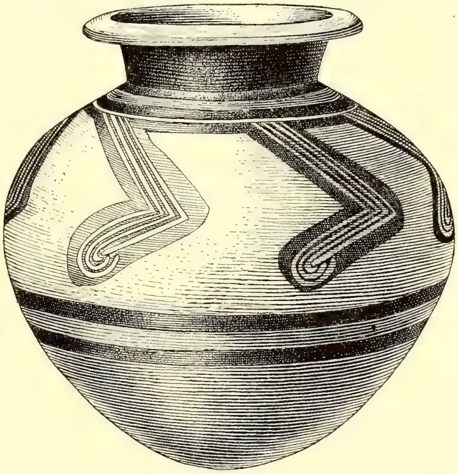

FIG. 2.—OCTOPUS MOTIVE, THE EIGHT APPENDAGES FORMING THE SHOULDER DECORATION, THE NECK OF THE VASE REPRESENTS THE OCTOPUS BODY. YALE COLLECTION. $\frac{1}{3}$.

Akin to all the foregoing is an octopus motive produced by shifting the mouth opening from the neck of the vase to an equatorial point on the side of the body and representing it by means of a painted circle. In order to eliminate as it were the neck and aperture of the vase from consideration, a large circular panel is formed about this make-believe mouth opening as a center. The eight arms converging from the periphery of the panel toward the central mouth opening produce the same effect as though one were looking down on the vase in figure 3. This gives the design that in earlier publications was called the rosette. It is simply a variety of the octopus motive and is met with perhaps oftener than any other single variety. In order the more easily to arrive at a circular panel, the body of the vessel was made spherical and the neck small, two characters common to lost color vases. It is therefore probable that the exigencies of the design tended to control the shape of the vessel, and vice versa. A good example just rescued from the duplicate material in the Yale Museum is reproduced in figure 5.

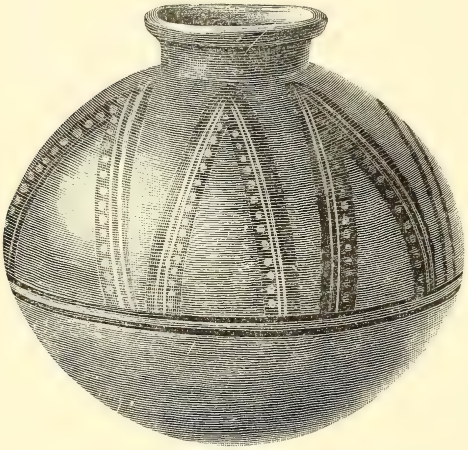

FIG. 3.—OCTOPUS MOTIVE. THE SUCCERED ARMS RISE LIKE SHORT-BASED TRIANGLES TOWARD THE NECK OF THE VASE WHICH SERVES AS THE OCTOPUS BODY. FROM HOLMES.

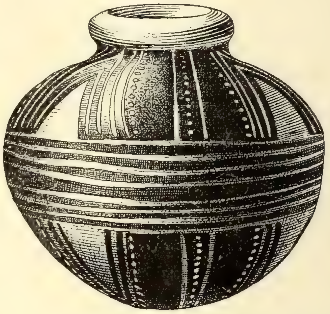

FIG. 4.—TWO VARIETIES OF THE OCTOPUS APPENDAGE MOTIVE. FOUR ARMS OF EACH KIND RISE TOWARD THE NECK OF THE VASE; THIS DESIGN IS REPEATED ON THE LOWER HALF OF THE VASE. HEYE COLLECTION. CAT. NO 7461. $\frac{1}{2}$.

Referring back to the key specimen (see fig. 1 , we find the octopus body represented by a rhomboidal or lozenge-shaped figure. In some realistic examples showing appendages attached to the body, the dots representing succers are placed within the field of the body rather than on the appendages. Remembering the freedom with which the ancient Chiriquian artist suppressed or transposed parts, one would expect to find cases where the body is represented and the appendages omitted. This would give an octopus body motive. The body motive, as was the case with appendage motives, is repeated to form zonal or other ornaments (fig. 6). As might be expected, it is not limited to the lozenge form. Any four-sided, perhaps even rounded or triangular design would answer the symbolic requirements, especially if it contained dots to suggest succers and, by inference, the appendages on which they grow.

With such an exuberant proliferation of motives derived from a single zoömorphic original, there is of course ever present the possibility of the overlapping of motives that started from wholly different originals. I have already referred to the occurrence of the dorsal-view motive of the alligator on lost color ware (*Op. cit.*). It is highly probable that the overlapping of this motive (perhaps also the scale-group and spine motive) and the one derived from the succers and appendages of the octopus has taken place to some extent, due to the convergence toward a common type, of scale-spine symbols of the alligator on the one hand and appendage-succer symbols of the octopus on the other. In so far as ancient Chiriquian art may serve as a guide, however, such overlappings instead of accounting for the evolution of the various motive groups, are rather to be considered as exceptions that prove the rule.

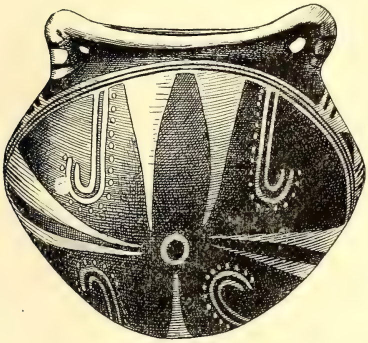

FIG. 5.—OCTOPUS MOTIVE. THE TWO KINDS OF APPENDAGES CONVERGE TOWARD THE CENTER OF A CIRCULAR PANEL. YALE COLLECTION. ½.

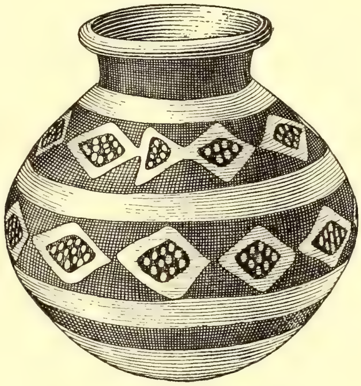

FIG. 6.—ZONAL DECORATION CONSISTING OF THE OCTOPUS BODY MOTIVE ENCLOSING SUCCER DOTS. YALE COLLECTION. ½

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[1] Ancient art of the province of Chiriqui, *Smithsonian Inst., Rep. Bur. Amer. Eth.* 6 (1888).

[2] A study of Chiriquian antiquities, *Mem. Conn. Acad. Arts Sci.*, 3 (1911).

[3] Note on the archeology of Chiriqui, *Amer. Anthrop., N. S.*, **15**, 661–667 (1913).

THE LIFE CYCLE OF TRYPANOSOMA BRUCEI IN THE RAT AND IN RAT PLASMA

By Rh. Erdmann

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Presented to the Academy, August 5, 1915

It is well known that Trypanosoma brucei does not appear in the peripheral circulation until about forty-eight hours after inoculation. The actual history of the organism during this period has not yet been established for Trypanosoma brucei; and in other trypanosomes, in spite of numerous investigations, it has not been fully elucidated (cf. Minchin,[1] and Carini[2]). It has been impossible to study the uninterrupted life history of the trypanosome in the mammalian host.

The method of cultivating Trypanosoma lewisi *outside* the mammalian host was first discovered by Novy and McNeal in 1903.[3] McNeal[4] states that Trypanosoma brucei changes its form in the culture medium into round bodies which are not capable of causing infection in the rat. Laveran and Mesnil[5] show four stages of this transformation of trypanosomes into round bodies in a medium of serum and horse blood. These authors term these forms "mis en boule" and believe that they are capable of starting a new infection and that they are comparable, when submitted to unfavorable conditions, to the 'latent bodies' found by More and Breinl[6] in Trypanosoma gambiense and later in Trypanosoma equiperdum and Trypanosoma lewisi.[7] These mammalian trypanosomes showed, according to these authors, cyclical changes in the number of trypanosomes in the host and the periodic appearance of the 'latent bodies.' Thus there are periods in which many trypanosomes are in the peripheral *blood* and periods in which few or none can be discovered. These round forms have never been seen in the mammalian *blood* in Trypanosoma brucei, but only in the above mentioned culture media.

For the purposes of the present study, which demand a continued control of the trypanosomes in culture which is not possible by cultivation in test tubes, I have cultivated Trypanosoma brucei in rat plasma on slides, using the tissue-culture methods of Harrison, Burrows, and Walton.[8] The plasma was either inoculated with the trypanosomes themselves or used as a medium for the growth in vitro of various infected tissues of the host. In the plasma of the rat I have been able to keep the trypanosomes in a normal condition for an indefinite period, whereas by the use of Ringer's fluid or blood bouillon the organisms die after a few days.[9]

For one strain of trypanosomes, herein designated as 'M,' with which the experiments were made I am indebted to Prof. F. G. Novy of the University of Michigan. It is the same Nagana material which was used by Kanthack, Durham and Blandford, and Laveran and Mesnil for so many years. A second strain, 'R', was secured from Dr. W. Brown of the Rockefeller Institute. However, since both strains showed the same changes in the plasma medium, I shall confine myself chiefly to a description of strain M. Also, in describing the morphology of the forms studied, I shall refer, in this paper, to the larger nucleus merely as 'nucleus,' and the smaller nucleus as blepharoplast, thus avoiding any discussion as to the fundamental origin or theoretical significance of these structures.

It will be necessary to keep in mind some general observations concerning the growth of this well known strain (M) *in the rat*, in order to interpret some of the phenomena observed in the plasma. The period in which the strain kills the rat after infection is generally three or four days in length, when 2 cc. of Ringer's solution, with six drops of blood taken from an animal shortly before its death, is used. When *one* trypanosome is used for inoculation, the lethal period lasts five days.[10] At the end of this period, shortly before death, there appear generally only those forms which have a well marked nucleus and are known as 'Schmalformen.' Schilling[11] mentions them at this period, and Oehler[12] considers them as 'Wucherformen.' Observed with dark-field illumination, 72 hours after infection, these animals appear as shown in figure 1, f and g. Text figure 1, d and e, represents trypanosomes 62 hours after inoculation. These animals are larger because they are probably not in such a rapid period of multiplication as animals f and g. Forty-eight hours after inoculation animals of the type figured in figure 1, b and c, can be discovered. These forms are the so-called 'Breitformen,' which are distinguished by their wealth of granules. Since Kühn and von Schuckmann[13] rarely observed forms filled with granules it would seem that they must have taken the material for their excellent cytological studies shortly before the death of the infected animal.[13] In strain M, after 48 hours infection, broad forms with granules prevailed, but forms without granules could also be detected. Trypanosomes which are about to die often show a granular appearance, but those observed in plasma, and figured in figure 1, a, b and c, were living and moving in the manner characteristic of Trypanosoma brucei. Thirty-six hours after subcutaneous inoculation the trypanosomes in the peripheral blood are very scarce. An example of an animal from the blood at this period is shown in text figure 1 a. The characteristic shape of the trypano-

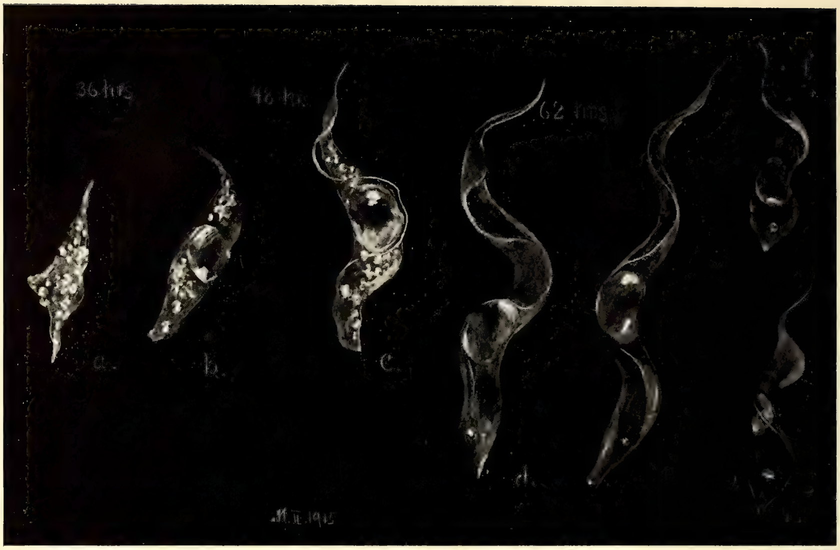
36 hrs
48 hrs
62 hrs
a.
b.
c.
d.
e.
f.
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Fig. 1

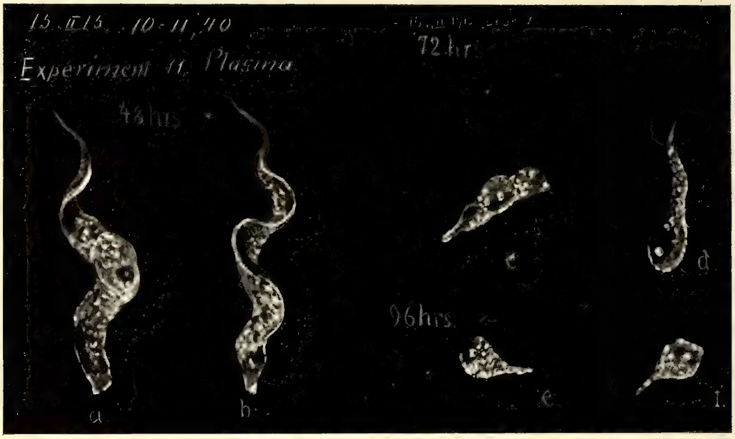
15.II.15. 10-11,40
Experiment 11. Plasma
72 hrs
48 hrs
a
b
c
d
96 hrs
e
f

Fig. 3

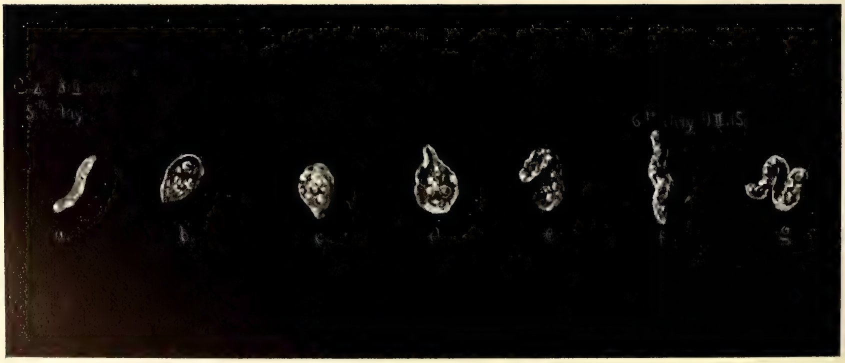
5th day
6th day 9.II.15
a
b
c
d
e
f
g

Fig. 6

somes is not yet assumed and the nucleus is not yet distinguishable. There remains, therefore, an interval of 36 hours after the new inoculation of strain M in which no typical trypanosomes could be discovered in the peripheral circulation. Crawley[14] shows that long continued use of dark-field illumination is inimical to protoplasm. Consequently great care must be taken in interpreting the different elements of the blood. The use of plasma instead of blood obviously eliminates many difficulties, because only a few elements of the blood are transferred into this culture medium.

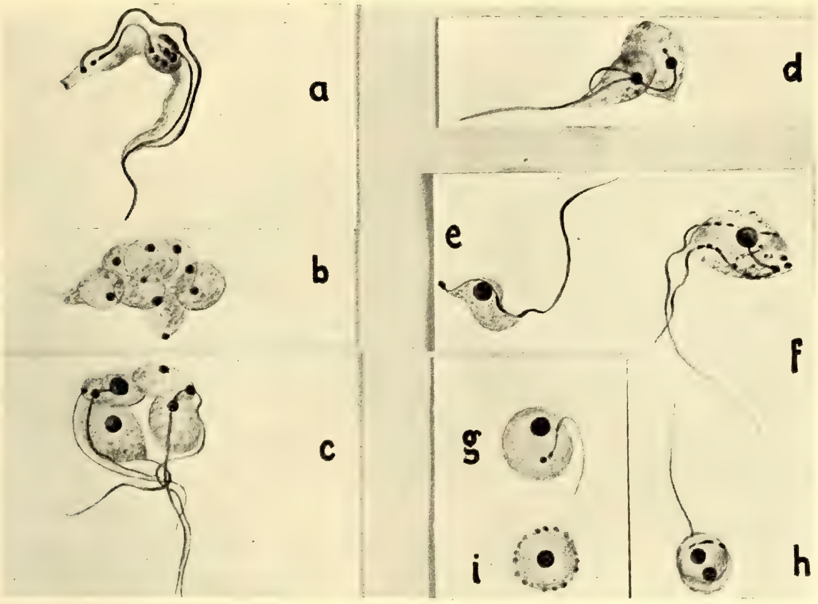

Fig. 2

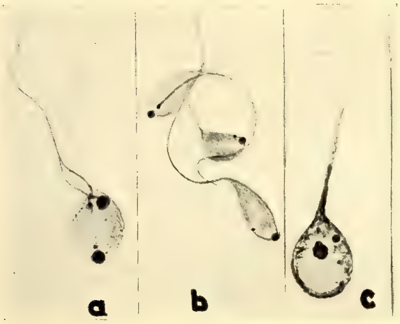

Fig. 4

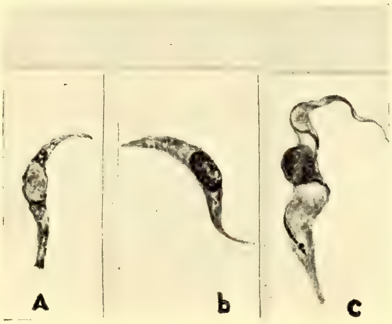

Fig. 5

DESCRIPTION OF EXPERIMENTS

The following short description of three typical experiments from a series of thirteen which were performed from September 25, 1914 to April 27, 1915, indicates that Trypanosoma brucei in slide plasma cultures undergoes different changes when maintained at a temperature identical with that of the mammalian blood (37°C.) from those which they undergo when maintained at a temperature (16°–18°C.) comparable with that of an invertebrate host.

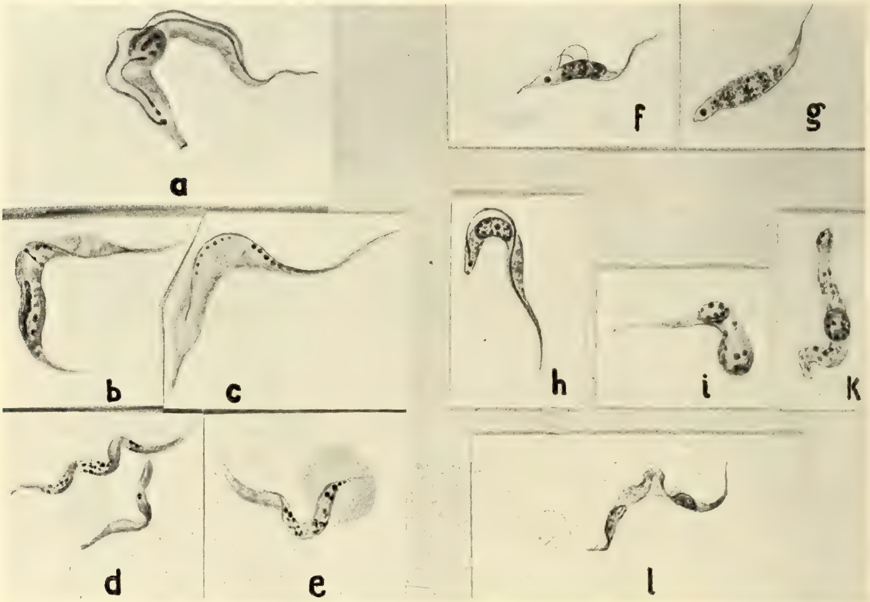

Fig. 7

Strain M, October 12 to 19, 1914. Culture kept at 37°C.—For this experiment the trypanosomes used resembled those of figure 1, animals f and g. The rat had been inoculated 84 hours, and died 2 hours after the experiment began. In figure 2, a trypanosome is pictured which has been six hours in the plasma medium. It was stained, after fixation with Schaudinn's fluid, with Giemsa feucht. This animal has the typical aspect of a trypanosome and proves that trypanosomes can live and divide in plasma at least six hours without changing their typical forms. Incidentally it has been observed in dividing forms just mentioned that the *nuclear* division lasts 2½ hours.

After having been subjected to the plasma for 24 hours the trypanosomes divide more rapidly. One organism frequently produces four or eight. Figure 2 b shows such a multiple division. In these specimens only nuclei are present, but several hours later almost every animal has a nucleus and a blepharoplast. The latter individuals grow and divide (fig. 2 e). The nucleus has become smaller, the blepharoplast is still lying in the posterior end of the animal, but the distance between them has been diminished. Also the undulating membrane has disappeared. Division into two animals is most frequent on the third and fourth day (fig. 2 d and f), and generally on the fifth day small individuals (fig. 2 e) with somewhat trypanoform appearance are numerous. On the fifth day round forms appear (fig. 2 g). That they are capable of division is shown in figure 2 h. These forms on the fifth and sixth day become motionless, and the flagellum encircles the animal. Often the staining capacity of the flagellum partially disappears, as is shown in figure 2 i. It is not necessary that the formation of these 'involution stages' last six days since sometimes three days after the trypanosome is taken from the rat, the stage as figured in figure 2 e is attained.

These small forms are, by *dark-field illumination*, actively motile, and finally after the flagellum encircles the cell, gradually come to rest. The nucleus appears yellowish, the blepharoplast has a reddish yellow tinge. These characteristic stages are the last forms visible in plasma culture at 37°C.

It seems probable that these forms found at the end of the cycle in the plasma similarly occur in the *mammalian* host, and in developing there give rise to the recurrent cycles of trypanosome form. This would be a general confirmation of the views of Moore and Breinl, that 'latent bodies' observed by them in blood are the cause of the cyclical appearance of those trypanosome forms which they studied, i.e., Trypanosoma gambiense, Trypanosoma equiperdum, and Trypanosoma lewisi. This view affords plausible explanation of the reappearance of trypanosomes in the blood, and would eliminate the necessity of accepting a theory which implies the occurrence of parthenogenesis.[15]

Strain M, February 13 to March 13, 1915. Culture kept at 16° to 18°C.—After being 62 hours in the rat, the trypanosomes were transferred to the plasma medium, and kept at a temperature of 16° to 18°C. At this temperature the changes in the plasma medium do not proceed in as rapid a manner as at 37°C, the temperature of the mammalian blood. These were chiefly forms *without* granules (fig. 1, d and e). Figure 3, a and b, represents animals which have been forty-eight hours in the *plasma*. In these forty-eight hours no form changes are revealed by dark-field illumination, but frequently forms without a discernible nucleus are present (fig. 3 b). The wealth of granules is remarkable, and divisions were noticed. The following day small actively motile forms appear (fig. 3, c and d). These are all without discernible nucleus when observed by dark-field illumination. In subsequent days these forms become smaller and do not change for an unlimited period, provided the organisms are transferred into new plasma under sterile conditions. Often they seem motionless, then again they have a corkscrew-like motion in the plasma. Frequently they flatten out, like a disk, or their edges fold over.

The preparations of preserved specimens show divisions into four and eight individuals as well as single individuals and, very rarely, the round forms, such as have been already described from the higher temperature (figure 2, b, c, d, e, f, g and h). Besides these forms similar to those described in plasma at 37°C., *other* types were found at 16° to 18°C. (fig. 4, a, b and c). Pear-shaped individuals are observed lying in clusters of from four to eight. The relative position of nucleus and blepharoplast has become changed in some of the animals observed at this period. The basal granule could not be distinguished, but the flagellum seems to arise from the blepharoplast. Figure 4, a shows such an animal, which perhaps might be termed crithidia-like because of the nucleus and the blepharoplast and the absence of an undulating membrane. This is the most frequent form in the plasma, and may be found even 30 days after the plasma is inoculated. These forms divide but undergo no further morphological changes.

Rounded forms of the above mentioned type (fig. 2, g and h), and of a slightly different type can also be observed on the fifth, sixth and seventh day after inoculation into the plasma (fig. 4, c). In these the flagellum has disappeared and only a tail of protoplasm is to be seen. They have a slight resemblance to the Dauerstadien in the frog trypanosome described by Doflein[16] except that in the latter forms the protoplasmic 'tail' has disappeared. In brief then, the full grown trypanosomes, in the plasma, at a temperature of 16° to 18°C., undergo changes which frequently lead to the formation of crithidia-like forms, and also sometimes, but rarely, to rounded forms.

After discovering that the crithidia-like and the rounded forms do not undergo further changes in the plasma (they were in the plasma from February 13 to 21), I inoculated them again into the rat. Five days after this inoculation small forms were observed (fig. 5, a, b). These were full grown at the seventh day (fig. 5, c) and caused the death of the rat at the ninth day of infection. Figure 6, which presents animals from another but exactly similar experiment, gives a good idea of these forms, which are present in the rat's blood on the fifth, sixth and seventh days after the crithidia-like forms are reinoculated into the rat. The individuals shown in figure 6, f and g, are probably identical with the forms illustrated in figure 5, a and b. It is evident from these figures that animals with typical trypanosome form are not present in the blood at this period. However, that the forms which are present develop in the blood into Trypanosoma brucei has been proved by four experiments in which the inoculation of crithidia-like forms was carried out (fig. 5 c). The typical trypanosome forms did not always appear after the first inoculation of infected plasma into the rat. Sometimes two or three passages were necessary to effect this phenomenon—but sooner or later Trypanosoma brucei with its characteristic form was present. Every precaution was taken to exclude possible sources of error by contamination, etc. The blood of all rats employed was examined before inoculation for Trypanosoma brucei. The animals were kept in a room where animals with Trypanosoma brucei had never been before, and further, the rats were etherized to eliminate the highly improbable source of infection, the rat flea. This new strain M proved highly virulent and caused the death of the rat in three days.

An attempt was made to determine in what organ of the rat the crithidia-like forms and the rounded forms underwent transformation into the typical trypanosome form. Pieces of spleen, liver, heart and lungs of rats, infected with these forms, were implanted into plasma and kept alive and growing for a considerable time. Study of these tissue cultures failed to reveal any full grown trypanosomes. Only twice were even crithidia-like forms found in a culture of lung tissue. It is necessary for these experiments to be repeated on a larger scale for it is certain that rats infected with crithidia-like forms carry them, or young trypanosomes, for a considerable time. It is quite probable that this phenomenon goes on in nature and that some wild animals are the carriers of the infection by means of the form under discussion, even where full grown trypanosomes cannot be detected.

Finally, I shall briefly describe some changes in Trypanosoma brucei in plasma which I have succeeded in obtaining but once. The experiment was started in the same way and under the same conditions as outlined above, the plasma being kept at 16° to 18°C., etc. After twenty-four hours numerous trypanosomes without definite nucleus appeared (figure 7, b, c, d, e). They resemble the forms of Trypanosoma brucei which Bradford and Plimmer[17] observed in the lungs. These animals I found dividing rapidly until at the third day two types were present as shown in figure 7, f and g. The following day also two different types could be distinguished: animals with a clearly defined nucleus, but with no trace of a blepharoplast or flagellum (fig. 7, i and k), and animals with nucleus, blepharoplast and flagellum (fig. 7, h). They were much smaller and narrower than the typical Trypanosoma brucei. The significance of this temporary disappearance of the nucleus is not clear. Such animals might be compared with von Prowazek's so-called 'indiferrente Formen.' However, the two different types which arise from these animals are not degeneration forms because they give rise to small trypanosomes (fig. 7, l) about eleven days after inoculation into the plasma. The appearance of the above mentioned types gives the only suggestion of dimorphism, which some might interpret as sexual in nature, which has appeared in my experiments.

Summary.—It is believed that the method employed in these experiments—which is a modification of tissue culture methods and excludes the possibility of the presence of confusing flagellate organisms—affords the means of following, outside the body of the invertebrate host, the sequence of changes in the life of trypanosomes.

Thus far its use has resulted in the discovery of dimorphic forms, latent or round forms, and crithidia-like forms in Trypanosoma brucei outside of the invertebrate host. The crithidia-like forms, when reinoculated into a rat, give rise to typical Trypanosoma brucei from which they had taken their origin.

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THE EFFECT OF PRESSURE ON POLYMORPHIC TRANSITIONS OF SOLIDS

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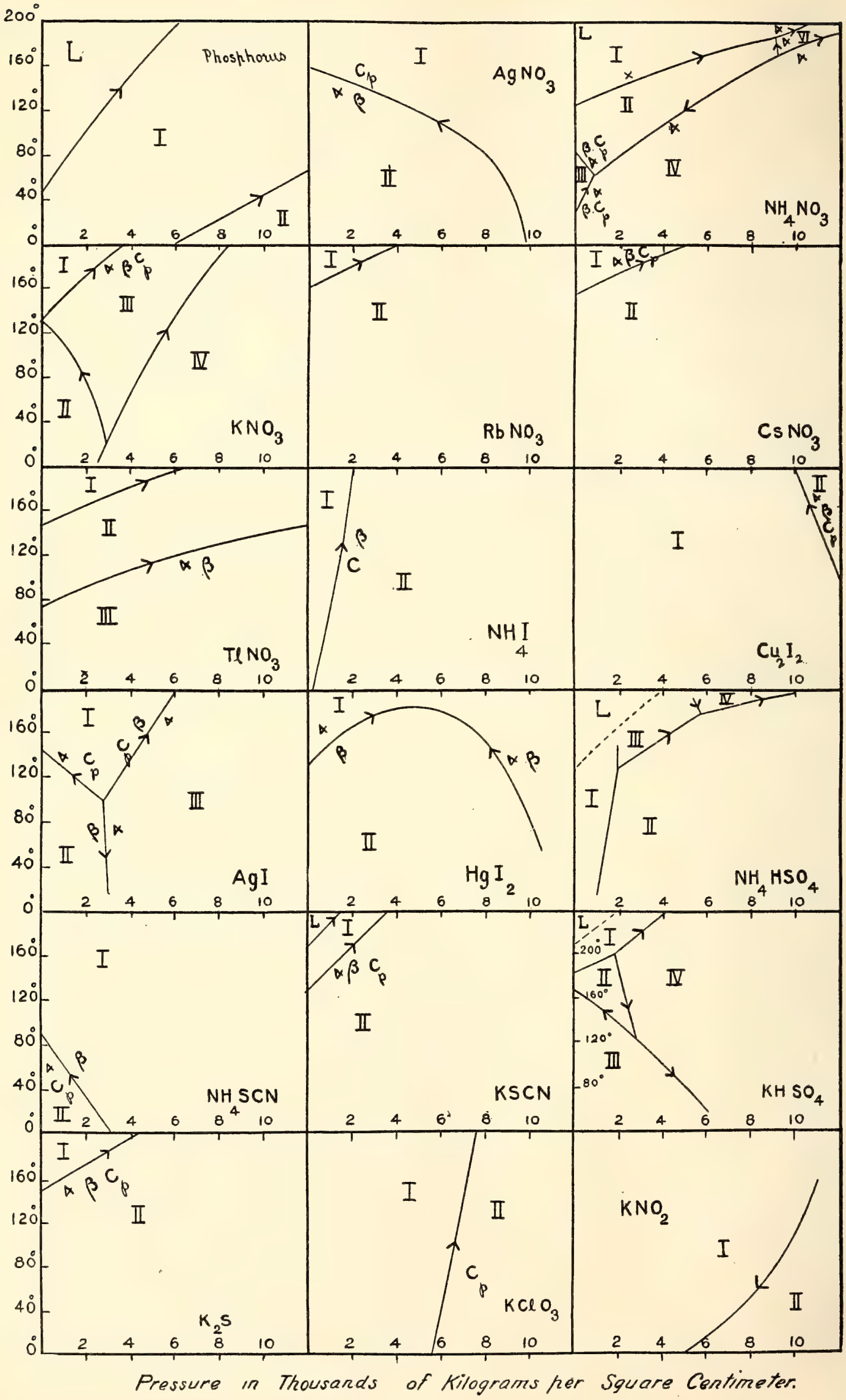
Presented to the Academy, August 7, 1915

This note presents in a compact form by means of diagrams many of the essential facts concerning the effect of high hydrostatic pressure on the polymorphic transitions of 30 substances. Five of these diagrams have been previously published,[1] eleven are to be published with greater detail in a forthcoming number of the *Proceedings of the American Academy*, and the rest await detailed publication. The experimental methods have been fully described previously. The investigation has been assisted in great measure by generous grants from the Bache Fund of the National Academy of Sciences and from the Rumford Fund of the American Academy of Arts and Sciences.

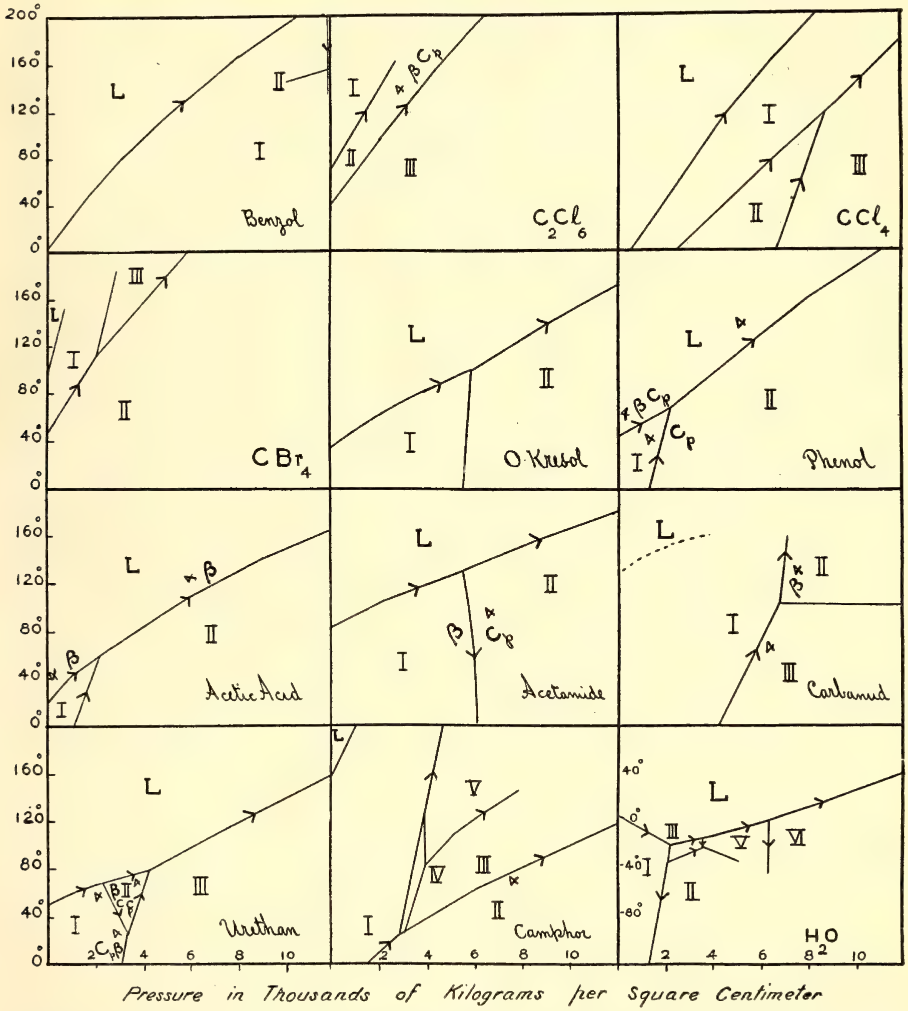
The diagrams show the transition curves on the pressure-temperature plane of the several solid phases, which are indicated by Roman numerals. The liquid phase, where it occurs, is denoted by an *L*. Notice that the temperature scale is changed in the diagrams for $KHSO_4$ and H_2O. NH_4NO_3 has one transition line not shown, of the ice type, beginning at −16°, and $RbNO_3$ has one at 219°, probably of normal type. The arrows on the transition lines indicate the directions in which the difference of volume of the two phases decreases numerically. An α, β or C_p placed on one side of a curve indicates that the phase on that side of the curve has the larger compressibility, thermal expansion, or specific heat. In the detailed presentation of data, the numerical values of all these factors, as well as of the latent heat of transition and change of energy are given.

The number of substances is perhaps sufficient to justify an enumeration of the relative frequency of different types of behavior. The first impression is one of bewildering complexity, it is obvious that the phenomena of polymorphism, even under high pressures, do not tend to any simple type. The apparent complexity is rather increased when one considers the diagrams of chemically related substances, such as the six nitrates, the four iodides, the two sulfocyanides, and acid sulfates. It is possible, however, to detect traces of regularity among the nitrates and sulfocyanides by putting into correspondence the phases which belong to the same crystalline system.

The variety of shapes possible for the individual curves is in striking contrast with the case for melting curves. Every rising melting curve


200°
160°
120°
80°
40°
0°
2
4
6
8
10
Phosphorus
L
I
II
AgNO$_3$
C_p
α β
NH$_4$NO$_3$
III
IV
V
VI
KNO$_3$
RbNO$_3$
CsNO$_3$
TlNO$_3$
NH$_4$I
C
Cu$_2$I$_2$
AgI
HgI$_2$
NH$_4$HSO$_4$
NH$_4$SCN
KSCN
KHSO$_4$
200°
160°
120°
80°
K$_2$S
KClO$_3$
KNO$_2$
Pressure in Thousands of Kilograms per Square Centimeter.

is concave toward the pressure axis, and rises continuously, and the two known falling melting curves are also concave downward. We have here examples of rising and falling curves with curvature in either direction, curves with a maximum temperature (HgI_2), and curves with a maximum or minimum pressure (H_2O and Benzol). No cases have



been found of a critical point, however. In the diagrams several curves are indicated as coming to an end; this simply means that for one reason or another further measurements were impossible. The relatively high frequency of falling transition curves is in contrast to the case for melting. A falling curve means that the phase stable at the higher temperature has the smaller volume. There are only two known cases for melting, whereas more than one quarter of the cases above are of this type. $KHSO_4$ affords a notable example where three curves of this type meet in a triple point. For a liquid, Δv always decreases with increasing temperature on either a rising or a falling curve. On the rising transition curves there are 37 cases of normal variation of Δv and 5 of abnormal variation; on the falling curves 8 normal and 8 abnormal cases.

The relative compressibility, thermal expansion, and specific heat of neighboring phases is significant. It is natural to expect that the phase of smaller volume will have the smaller compressibility and thermal expansion, and that the phase stable at the higher temperature will have the higher specific heat. If we call this behavior 'normal,' then on rising curves we find 9 cases of normal and 11 of abnormal compressibility, and on falling curves 1 normal and 7 abnormal. The expansion shows 5 normal and 7 abnormal cases on rising curves and 2 normal and 4 abnormal on falling curves. C_p is normal in 5 cases and abnormal in 7 cases on rising curves, and normal in 6 cases and abnormal in 1 on falling curves. The fact of abnormal C_p is of considerable significance from the point of view of the quantum hypothesis. It means (if we may apply the same considerations to C_p as to C_v, which is usually done) that the specific heat curves of the two modifications cannot be of the same character, but that somewhere between the transition point and absolute zero the one which is lower at the transition point must cross and lie above the other.

In addition to the substances enumerated above, about 100 others have been examined without finding other forms.

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ON ISOTHERMALLY CONJUGATE NETS OF SPACE CURVES

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Presented to the Academy, August 10, 1915

Bianchi[1] has called a parametric net of curves on a surface isothermally conjugate if, when the surface is referred to these curves, the second fundamental form, $D\,du^2 + 2D'\,du\,dv + D''\,dv^2$, may by a transformation $\bar{u} = U\,(u), \bar{v} = V\,(v)$ be made to take on the same shape as does the first fundamental form when the parametric net is isothermal; i.e., the parametric net is isothermally conjugate if $D' = 0$, $D = D''$. These nets have lately attained increased importance, so that Wilczynski's recent geometric interpretation[2] of Bianchi's condition is of great interest. In the present note, we propose to give a new and simple geometric characterization of isothermally conjugate nets which is entirely different from Wilczynski's.

Let $y^{(1)}, y^{(2)}, y^{(3)}, y^{(4)}$, be the homogeneous coordinates of a point in space, and let the four functions

$$y^{(k)}=f^{(k)}(u,v) \qquad (k=1,2,3,4) \tag{1}$$

define a surface S_y on which the curves u = const., v = const. form a conjugate net. Then the $y^{(k)}$'s satisfy a completely integrable system of two partial differential equations of the form[3]

$$\begin{aligned} y_{uu}&=a y_{vv}+b y_u+c y_v+d y,\\ y_{uv}&=b' y_u+c' y_v+d' y. \end{aligned} \tag{2}$$

The second of these is of the familiar Laplace type, characteristic of conjugate nets; the first shows that the conjugate net defined by equations (1) is isothermally conjugate if and only if

$$\frac{\partial^2}{\partial u \partial v}\log a=0. \tag{3}$$

The coefficients in equations (2) are not arbitrary, but are subjected to certain integrability conditions. One of the relations yielded by these conditions is that[4]

$$\frac{\partial}{\partial v}(b+2c')=\frac{\partial}{\partial u}\left(2b'-\frac{c}{a}-\frac{\partial}{\partial v}\log a\right),$$

or

$$b_v+2c'_v=2b'_u-\left(\frac{c}{a}\right)_u-\frac{\partial^2}{\partial u \partial v}\log a. \tag{4}$$

The minus first and first Laplace transforms of the point y are respectively

$$\rho=y_u-c'y, \qquad \sigma=y_v-b'y,$$

which represent covariant points on the tangents at y to the curves of the net passing through y. The surface S_ρ is the second focal sheet of the congruence of tangents to the curves v = const. on S_y, and S_σ is the second focal sheet of the congruence of tangents to the curves u = const. on S_y. Let us, with Wilczynski,[2] call the line $\rho\sigma$ corresponding to the point y the *ray* of the point y, and the totality of rays, which form a congruence, the *ray congruence*.

The osculating planes of the two curves u = const. and v = const. at a point y meet in a line which passes through y and which Wilczynski calls the *axis* of the point y. The totality of axes, which correspond to all the points y of the surface S_y, constitute a congruence, the *axis congruence.*

We may write the first of equations (2) in the form

$$y_{uu} - by_u - dy = ay_{vv} + cy_v.$$

The left-hand member represents a point in the osculating plane to the curve v = const., and the right-hand member a point in the osculating plane to the curve u = const., at y. Therefore, since the coordinates are homogeneous, the point

$$z = y_{vv} + \frac{c}{a} y_v$$

lies on the line of intersection of the two osculating planes, and the line yz is the axis of the point y.

We may determine the developables of the axis congruence as follows. If the point y moves to the point $y + dy$, the point z moves to $z + dz$, where $dy = y_u\, du + y_v\, dv$ and $dz = z_u\, du + z_v\, dv$. We wish the line yz to generate a developable. This will happen if and only if the four points y, z, $y + dy$, $z + dz$ lie in a plane, or what is the same thing, if the points y, z, $y_u\, du + y_v\, dv$, $z_u\, du + z_v\, dv$ are coplanar. We have on differentiation of equations (2)

$$\begin{aligned} y_{uvv} &= \alpha^{(12)} y_{vv} + \beta^{(12)} y_u + \gamma^{(12)} y_v + \delta^{(12)} y, \\ y_{vvv} &= \alpha^{(03)} y_{vv} + \beta^{(03)} y_u + \gamma^{(03)} y_v + \delta^{(03)} y, \end{aligned}$$

where in particular

$$\begin{aligned} &\alpha^{(12)} = c', \quad \beta^{(12)} = b'^2 + b'_v, \quad \gamma^{(12)} = b'c' + c'_v + d_,', \\ &\alpha^{(03)} = b' - \frac{c}{a} - \frac{\partial}{\partial v} \log a, \quad \beta^{(03)} = \frac{1}{a}(b'c' + b'_u - b_v + d'), \\ &\qquad \gamma^{(03)} = \frac{1}{a}[\, b'c + c'(c' - b) + c'_u - c_v - d\,], \end{aligned} \tag{5}$$

so that on using these and equations (2) we find

$$\begin{aligned} z_u &= y_{uvv} + \frac{c}{a} y_{uv} + \left(\frac{c}{a}\right)_u y_v \\ &= c' y_{vv} + \left(\beta^{(12)} + \frac{b'c}{a}\right) y_u + \left[\gamma^{(12)} + \frac{cc'}{a} + \left(\frac{c}{a}\right)_u\right] y_v + (\,)y, \\ z_v &= y_{vvv} + \frac{c}{a} y_{vv} + \left(\frac{c}{a}\right)_v y_v \\ &= \left(b' - \frac{\partial}{\partial v} \log a\right) y_{vv} + \beta^{(03)} y_u + \left[\gamma^{(03)} + \left(\frac{c}{a}\right)_v\right] y_v + (\,)y, \end{aligned}$$

in which the coefficients of y do not concern us. Consequently,

$$z_u du + z_v dv = \left[c' du + \left(b' - \frac{\partial}{\partial v} \log a\right) dv\right] y_{vv} + \left[\left(\beta^{(12)} + \frac{b'c}{a}\right) du + \beta^{(03)} dv\right] y_u$$
$$+ \left[\left(\gamma^{(12)} + \frac{cc'}{a} + \left(\frac{c}{a}\right)_u\right) du + \left(\gamma^{(03)} + \left(\frac{c}{a}\right)_v\right) dv\right] y_v + (\ ) y.$$

Now, if the points y, z, dy, dz are to be coplanar, the determinant of the coefficients of y_{vv}, y_u, y_v in the expressions for z, dy, dz must vanish; on expansion this determinant yields the quadratic in $du : dv$,

$$a\left[\gamma^{(12)} + \left(\frac{c}{a}\right)_u\right] du^2 - \mathfrak{D}\, du\, dv - a\, \beta^{(03)} dv^2 = 0, \tag{6}$$

where, on using (4), we find

$$\mathfrak{D} = d + ab'^2 - c'^2 + b'c + bc' + ab'_v - c'_u. \tag{7}$$

The quadratic (6) determines the direction in which y must move, in order that the axis yz may trace out a developable; there are two such directions at each point of S_y. We may regard (6) as a differential equation defining a net of curves on S_y having the property that if the point y traces out a curve of this net, the corresponding axis generates a developable surface. We call the two curves of the net which pass through the point y the *axis curves* of the point y.

In like manner, we may determine the developables of the ray congruence, i.e., the net of curves on S_y having the property, that if the point y traces out a curve of the net, the corresponding ray traces out a developable of the ray congruence. The differential equation defining this net of curves, which we call the *ray curves*, is without difficulty found to be

$$a\, \mathrm{H}\, du^2 - \mathfrak{D}\, du\, dv - \mathrm{K}\, dv^2 = 0, \tag{8}$$

where $\mathfrak{D}$ is given by (7), and

$$\mathrm{H} = d' + b'c' - b'_u, \qquad \mathrm{K} = d' + b'c' - c'_v \tag{9}$$

are the Laplace-Darboux invariants of the given conjugate net.

If we use (9), we find from (5) that

$$a\, \beta^{(03)} = \mathrm{H} + 2b'_u - b_v, \qquad \gamma^{(12)} + \left(\frac{c}{a}\right)_u = \mathrm{K} + 2c'_v + \left(\frac{c}{a}\right)_u,$$

the latter of which becomes, on use of (4),

$$\gamma^{(12)} + \left(\frac{c}{a}\right)_u = \mathrm{K} + 2b'_u - b_v - \frac{\partial^2}{\partial u\, \partial v} \log a.$$

The differential equation (6) of the axis curves may therefore be written

$$a\left[\mathrm{K}+2b'_u-b_v-\frac{\partial^2}{\partial u\,\partial v}\log a\right]du^2-\mathfrak{D}\,du\,dv-(\mathrm{H}+2b'_u-b_v)\,dv^2=0. \quad (10)$$

The differential equation of the asymptotic curves is

$$adu^2+dv^2=0 \qquad (11)$$

The pair of asymptotic tangents at y is of course harmonically separated by the tangents to the curves of our conjugate net. The differential equation

$$adu^2-dv^2=0 \qquad (12)$$

defines a new net of curves. It evidently has the property, that the tangents to the two curves of the net at the point y separate harmonically both the pair of asymptotic tangents and the tangents to the two curves of our conjugate net. It is moreover the only net which has this property; since it also is a conjugate net, we call it the *associate conjugate net.*

We shall define another net of curves which will be of importance in our geometric interpretation. The quadratic

$$a\,\mathrm{H}\,du^2+\mathfrak{D}\,du\,dv-\mathrm{K}\,dv^2=0 \qquad (13)$$

has for its roots the negatives of the roots of (8). It therefore defines a net such that the tangents to the two curves thereof at the point y are the harmonic conjugates of the two ray tangents (the tangents to the ray curves) with respect to the original conjugate tangents (the tangents to the curves of the original conjugate net). For convenience, let us call the curves defined by (13) the *anti-ray curves,* and the two tangents to the anti-ray curves at the point y the anti-ray tangents of the point y.

Let us now fix our attention upon a point y of the surface S_y, and let us regard equations (10), (12), and (13) as binary quadratics whose roots give respectively the pairs of axis tangents, associated conjugate tangents, and anti-ray tangents of the point y. The Jacobian of the forms (10) and (12) is

$$a\,\mathfrak{D}\,du^2+2a\left(\mathrm{H}-\mathrm{K}+\frac{\partial^2}{\partial u\,\partial v}\log a\right)du\,dv+\mathfrak{D}\,dv^2=0, \qquad (14)$$

and its roots give the pair of lines through y which separate harmonically both the pair of axis tangents and the pair of associated conjugate tangents of y. The Jacobian of the forms (12) and (13) is

$$a\mathfrak{D}\,du^2 + 2a\,(\mathrm{H} - \mathrm{K})\,du\,dv + \mathfrak{D}\,dv^2 = 0 \tag{15}$$

and defines the pair of lines through y which separate harmonically both the pair of anti-ray tangents and the pair of associate conjugate tangents of the point y.

The two Jacobians (14) and (15) coincide if and only if

$$\frac{\partial^2}{\partial u\,\partial v}\log a = 0, \tag{3 \textit{bis}}$$

i.e., if and only if the original conjugate net is isothermally conjugate. We may state our result as follows:

A necessary and sufficient condition that a conjugate net of curves on a surface be isothermally conjugate is that at each point of the surface the pair of axis tangents, the pair of associate conjugate tangents, and the pair of anti-ray tangents be pairs of the same involution.

By means of the various nets of curves defined in the course of the above interpretation, we have been enabled to deduce a number of properties of isothermally conjugate nets. We have included this more extended discussion in a longer paper, which is a sequel to the one on conjugate nets to which reference has already been made.

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[1] L. Bianchi, *Vorlesungen über Differentialgeometrie,* tr. M. Lukat, 2te Aufl., pp. 135 et seq.

[2] E. J. Wilczynski, *Trans. Amer. Math. Soc.,* **16,** 311–327 (1915).

[3] G. M. Green, *Amer. J. Math.,* **37,** 215–246 (1915). Cf. §1.

[4] Ibid., end of §3.

THE RÔLE OF THE LIVER IN ACUTE POLYCYTHAEMIA: THE MECHANISM CONTROLLING THE RED CORPUSCLE CONTENT OF THE BLOOD

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It is very generally considered by all except those who have paid special attention to the subject, that the number of red corpuscles per unit volume of blood is, in the normal individual, a fairly fixed quantity subject to gradual change only. A more careful study shows however that this number is subject to very rapid and great changes, and instead of being constant, that it is continually changing under physiological conditions.

Questions naturally arise as to what factors will cause a change in the number of red cells;[1] where these changes take place; whether the increase in number is relative or absolute; and finally if there is a mechanism controlling the red corpuscle content of the blood, and if so what is its function.

It has been found that asphyxia in any form causes the number of red cells to increase. Reduced atmospheric pressure, reduced partial tension of oxygen, obstruction of the air passages, reduced oxygen capacity of the blood, as in carbon monoxide poisoning, or increased oxygen consumption as in exercise, will cause an increase in the number of red cells. Furthermore, obstruction to the circulation in the lungs, and consequent interference with the oxygenation of the blood, as in congenital heart conditions, will cause polycythaemia. I have been able to produce this form of polycythaemia experimentally by the injection of either corpuscles hardened with formaldehyde, or an inert powder as lycopodium, or oil, thus inducing embolism of the lungs and obtaining as great an increase in the number of red corpuscles as 3,000,000 in fifteen minutes.

Besides, it is known that various substances occuring physiologically in the body as epinephrin, pituitrin, carbon dioxide, and substances entirely foreign to the body as nicotine, radium chloride, etc., may produce very marked changes in the red count.

And finally I have been able to show that the number of red cells is under nervous control. In cats frightened by a dog for a minute or so, the red count rose one or two millions in five minutes or less, and from excitement alone reached the extreme value of 16,776,000 in one instance and 14,464,000 and 14,920,000 in two other cases.

There has been a great deal of work done concerning the magnitude of the changes in number of red cells and the haemoglobin content of the blood at high altitudes, and in exercise, but up to this time there has been no experimental evidence as to the seat of the changes which cause this increase. On this account the following experiments were systematically undertaken to find out if possible where the changes take place which cause an increase in the number of red cells.

For this purpose epinephrin was chosen as a means of producing polycythaemia, as it can be obtained in a pure state, accurately graduated in amount, and applied as a stimulus with certainty when intravenously injected. Furthermore it occurs normally in the body, and ether does not interfere with its action. When injected in doses of 0.9 mg. per kilo, intravenously, it produces with great regularity an increase of one or two millions in the red count in fifteen or twenty minutes.

To obtain uniform results the animals were etherized, and blood taken from either the jugular or femoral veins by means of a hypodermic needle, which method has been shown to give a true index of the red count in the generally circulating blood.

The entire intestine, its mesentery, the omentum, spleen and pancreas were removed singly, and in other cases altogether, after which epinephrin was injected. Epinephrin caused the same increase in number of red cells in these animals in which the above organs had been removed as in normal animals. It is apparent then that these organs are not the chief seat of the changes by which the red count is increased.

As the liver could not be removed except by the introduction of mechanical complications, its part in the production of polycythaemia was sought by other means.

In the first place animals were completely tied in two above the diaphragm. This is accomplished by opening the chest on both sides between two of the lower ribs, cutting the tissues to the sternum and back bone, and tying the great vessels, the sternum, back bone and adjoining tissues securely with strong ligatures. Except for what blood could pass up and down inside the vertebrae, the animal was completely tied in two above the diaphragm.

In such an animal, the injection of epinephrin caused no increase in the number of red cells.

An operation in which the animal was again tied in two, but this time placing the ligatures about the aorta just below the mesenteric artery and above the renal arteries, and completely tying the rest of the vessels and tissues at this level, gives an animal similar to the one above, except that the blood supply to the liver, spleen, intestine, mesentery, pancreas and omentum has been added. As we know that removal of all these organs except the liver, has no effect on the production of polycythaemia after the injection of epinephrin, we have for our purposes, in this case merely added the liver to the animal which was divided above the diaphragm.

Epinephrin injected into such an animal increases the number of red cells as in a normal animal.

From these experiments it appears that the liver is the seat of the changes which cause an increase in number of red cells. Further proof of this was obtained by quite different means.

On account of the irregularity with which the hepatic artery branches it was considered a more certain means of shutting off the arterial blood supply to the liver to ligate the coeliac axis artery, than to attempt ligation of the hepatic artery. The coeliac axis artery supplies the liver, spleen and pancreas, and part of the stomach, but we have seen that removal of all of these except the liver has no effect on the production of polycythaemia after injection of epinephrin. If then the coeliac axis artery is ligated, and epinephrin injected, no change in the number of red cells is observed, but if this ligature is removed, even one-half hour after the injection of epinephrin, the red count will immediately increase, as if the usual dose of epinephrin had been injected into a normal animal.

A control experiment was done in which the coeliac axis artery was ligated, and after half an hour the ligature removed. This had no effect on the red count, which shows that mere interruption of the arterial circulation to the liver is not the only factor necessary to change the number of red cells.

I have also shown that ligation of the portal vein, as in the operation of removing the intestine, has no effect on the production of polycythaemia following the injection of epinephrin. Also in those cases in which the hepatic artery was ligated, epinephrin was carried to the liver by the uninterrupted portal circulation, and yet no change in the number of red cells took place.

Furthermore, in cases where the hepatic artery was tied, injection of epinephrin caused no increase in the number of red cells, although there was no interference with the blood supply to the kidneys, bone marrow, muscles, lungs or skin. These organs may then be considered as playing no part in the production of polycythaemia due to epinephrin injection.

It appears then that the liver is the organ capable of increasing the red corpuscle content of the blood after injection of epinephrin, and that the arterial blood supply to the liver must be intact to allow these changes to take place.

As to the changes which occur in the liver certain facts were observed in these experiments. There is a decrease in plasma volume, not however sufficient to account entirely for the increase in number of red cells. There are cells present in the circulation during polycythaemia which were not there before its production, as shown by a decrease in the average size of the red corpuscles, and a decrease in their percentage haemoglobin content. These cells show none of the usual signs of young cells. That is, they are not nucleated, they show no change in fragility, and they have no increased metabolism such as would result in an increased rate of reduction.

Concerning the mechanism which normally controls the red count, a few points of interest were observed.

It has been found that the red count is under nervous control, as shown by the increase in the number of red corpuscles after stimulation of the vago-sympathetic trunks, after asphyxia of the brain alone, and after emotional excitement as fear, rage, etc. It was also shown that the adrenals play a part in this mechanism, as epinephrin has been proved to be one of the most powerful means of increasing the number of red cells, and as the chief physiological stimuli which produce polycythaemia, namely asphyxia, exercise, fear, rage, etc., all cause a reflex stimulation of the adrenals, and an increased epinephrin output. Removal of the adrenals was found to exclude the production of polycythaemia in asphyxia, fear and rage, and after the injection of pituitrin, although these same animals responded to the injection of epinephrin by the usual increase in number of red cells. Furthermore, removal of the adrenals is followed after several hours by a complete loss of control of the red count.

When the number of red cells is increased in acute polycythaemia, we have seen that there is also an increase in haemoglobin, but not in proportion to the increase in number of red cells. This being the case, the blood is better equipped not only to carry more oxygen, but on account of the increased surface of the haemoglobin, to take up more oxygen per unit of time. The body is then better prepared to maintain a constant oxygen content of the blood under various physiological and pathological conditions.

We may therefore conclude that there is in the body a mechanism for regulating the red corpuscle content of the blood; that this mechanism is under nervous control, responding to nervous, chemical and emotional stimuli; that the adrenal glands play a part in this mechanism, and that the liver is the seat of the changes which increase the number of red cells, partly by a reduction in plasma volume, and partly by bringing cells into the circulation which are not normally present.[2]

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[1] The terms red count, number of red cells, etc., will be used for the sake of brevity instead of the more exact phrase "Number of red corpuscles per unit volume of blood."

[2] This article is given in full in the *Journal of Pharmacology and Experimental Therapeutics,* Vol. 7, Nos. 1 and 2 (1915).

THE POTENTIALS AT THE JUNCTIONS OF SALT SOLUTIONS

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A concentration-cell consisting of two identical reversible electrodes in contact with two differently concentrated solutions of the same salt gives an electromotive force which is the algebraic sum of three potentials—those at the two electrodes and that at the junction of the solutions. Since for most theoretical purposes the potential at the electrodes alone is desired, many attempts have been made to evaluate the potential at the liquid junction. The effect of interposing strong salt solutions on the electromotive force of the cell has also been studied with the view of eliminating the effect of the liquid junction.

It has been usual to test the correctness of the value assigned to the potential at the junction, or the efficacy of the artifice employed in eliminating it, by seeing whether the resulting electromotive force E at the electrodes can be calculated by the equation $E = (RT/F)\ln(C_1/C_2)$ where C_1 and C_2 are the concentrations of the ions in the two solutions as calculated from conductance measurements, and R, T, and F are the gas-constant, the absolute temperature, and the faraday respectively. Recent work[1] has indicated, however, that the voltages calculated by this formula are too high. It therefore seems desirable to investigate the subject of the liquid junction connecting two solutions of the same salt with a view of finding an expression, involving the fewest possible assumptions, for its electromotice force.

If the cell Ag, AgCl (solid) + KCl (0.1n), AgCl (solid) + KCl (0.01n), Ag operates reversibly until one faraday passes through it, one equivalent of chloride-ion enters the dilute solution and the same amount of chloride-ion is electrolyzed out of the more concentrated solution. The current is carried across the liquid junction by the movement of n_c equivalents of potassium-ion in the direction of the current and by the migration of $(1 - n_c)$ equivalents of chloride-ion in the reverse direction, n_c being the transference-number of the cation. The total effect of the passage of the faraday of electricity is the transference of n_c equivalents of salt, or $2n_c$ equivalents of ion, from the 0.1 normal KCl solution to the 0.01 normal KCl solution. The accompanying electrical work is the product $E\,F$ where E is the total E. M. F. of the cell and F is the value of the faraday. Now the electrical work and consequently the electromotive force at the liquid junction will be determined by the algebraic sum of the number of equivalents of ion that are carried across it from the concentrated to the dilute solution, in this case equal to $n_c - (1 - n_c) = 2n_c - 1$. If we now make the assumption that the osmotic work involved in the transferring of a gram equivalent of positive ion from a dilute to a concentrated solution is the same as the work necessary for the transfer of a corresponding amount of negative ion, we can obtain the electromotive force at the liquid junction by the simple proportion:

$$(E_L F : EF = (2n_c - 1) : 2n_c \quad \text{or} \quad E_L = E\,(1 - 1/2n_c). \qquad (1$$

This equation contains only the directly measurable quantities E and n_c, and contains no assumption concerning the concentrations of the ions in the two solutions.

A direct test of this simple equation is not possible; but an indirect one is afforded by the following considerations. The electrode-potentials of cells of the type Ag, AgCl (solid) + MCl (C_1), AgCl (solid) +MCl (C_2), Ag would be expected to be the same whether hydrogen or any one of the alkali metals is chosen for the radical M, if the concentrations of C_1 and C_2 are the same in each case and below about 0.05 normal. The sum of the electrode-potentials is determined by the difference of the osmotic pressures of the chloride ions in the two solutions; and this difference of osmotic pressure is, probably, nearly the same for dilute solutions of chlorides of univalent cations at corresponding concentrations, since the degrees of dissociation in dilute solution as determined by the conductivity method have been found to be nearly the same for these substances. If this is true, and if the assumption involved in the above-given expression for the potential at the liquid junction is correct, the sum of the electrode-potentials should be independent of the nature of the cation. This amounts to the assumption that the free energy of dilution of the chloride ion is the same whether hydrogen, potassium, or sodium is the cation, since the process at the electrodes during the operation of a cell consists in the formation of one equivalent of chloride ion in the dilute solution and the removal of the same amount of chloride ion from the concentrated solution per faraday passed through the cell. Jahn's accurate work (loc. cit.) on concentration cells with hydrochloric acid, potassium chloride, and sodium chloride is, fortunately, well adapted to test these conclusions.

Table I, which is self explanatory, gives the result of my calculations, based on Jahn's electromotive-force data, which are here given in millivolts. The transference-numbers are from Noyes and Falk's[2] compilation.

TABLE I

Calculation of the Liquid Junction Potential and Electrode Potentials (in Millivolts) of Chloride Concentration Cells With Silver—Silver Chloride Electrodes

SUBSTANCE	MOLS PER LITER	TRANSFERENCE NUMBER	E. M. F. OF CELL E	LIQUID JUNCTION POTENTIAL, E_L	ELECTRODE POTENNTIAL $(E - E_L)$	AVERAGE DEVIATION FROM MEAN
HCl.........	0.01665 0.001665	0.833	−92.35	−36.94	−55.41	0.24
NaCl........	0.01673 0.001674	0.396	−43.60	+11.46	−55.06	
KCl.........	0.01670 0.001674	0.496	−54.24	+00.44	−54.69	
HCl.........	0.03330 0.003329	0.833	−91.62	−36.64	−54.98	0.25
KCl.........	0.03347 0.003347	0.496	−54.03	+0.43	−54.47	
HCl.........	0.008315 0.001665	0.833	−64.87	−25.95	−38.92	0.04
NaCl........	0.008364 0.001674	0.396	−30.73	+ 8.08	−38.81	
KCl.........	0.008329 0.001670	0.495	−38.44	+ 0.39	−38.83	
HCl.........	0.006686 0.001665	0.833	−56.14	−22.45	−33.79	0.08
NaCl........	0.006686 0.001674	0.396	−26.53	+6.97	−33.49	
KCl.........	0.006700 0.001670	0.495	−33.30	+0.34	−33.64	

It will be observed that in each group of cells in which the solutions of electrolytes have the same concentrations, the calculated sum of the electrode-potentials has the same value within a few tenths of a millivolt. This is true even though the liquid-junction potential in the case of the hydrochloric acid cells is 40% of the total electromotive force, and of opposite sign to that in the case of the sodium chloride solutions. The agreement is as close as can be expected from our knowledge of the transference-numbers. Calculations based on other work by the same author show a similar agreement.

This result indicates, at any rate in the case of solutions of concentrations up to 0.03 normal, the correctness of the two assumptions involved in the calculations, namely, that the osmotic work of transferring equivalent quantities of the ions is nearly the same for positive ion and negative ions, and that the osmotic work of transferring chloride-ion from one salt concentration to another is nearly the same whether the cation is potassium, sodium, or hydrogen. It is probable that the first of these assumptions is substantially correct up to much higher concentrations than it can be tested by the method here employed, since this method involves also the second assumption just stated, which doubtless becomes inexact at higher concentrations. Equation (1) here considered must in any case give a much more exact value of the liquid potential than does the Nernst equation

$$E_L = \frac{RT\ (1-2n_c)}{F} \ln \frac{C_1}{C_2}.$$

How much the values calculated by the latter differ from those calculated by the former equation in the case of the more concentrated of these dilute solutions is shown in the following table:

SUBSTANCE	MOLS PER LITER	LIQUID POTENTIAL IN MILLIVOLTS		DIFFERENCE
		By equation (1)	By Nernst equation	
HCl................	0.01665 0.001665	−36.94	−38.03	1.09
NaCl..............	0.01673 0.001673	11.46	11.72	0.26
KCl................	0.01670 0.001674	00.44	00.45	0.01
HCl................	0.03330 0.003329	−36.64	−38.23	1.59
KCl................	0.03347 0.003347	0.43	0.45	0.02

The difference between the values for the liquid junction potential as calculated by the two methods is small for salts having ions with nearly the same mobility, but the difference increases rapidly with increased concentrations of the solutions, since the Nernst equation contains the assumption that the ions are 'perfect' solutes, an assumption which is not even approximately true at higher concentrations if the ion concentrations are calculated in the usual manner from conductance measurements.

In conclusion it may be noted that the value of E/n_c in equation (1) when multiplied by F, the value of the faraday, represents the maximum work or free-energy change attending the transfer of one equivalent of salt from one concentration to the other, and that therefore E_L may be calculated from any of the properties related to osmotic pressure, such as freezing-point. This free energy-change may also be obtained directly from electromotive force measurements of cells without transference of the type;Ag, AgCl (solid) + KCl (0.1n), $K(Hg)_x$ – $K(Hg)_x$, AgCl (solid) + KCl (0.01n), Ag, investigated by MacInnes and Parker.

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[1] See for instance, Jahn, *Zs. physik. Chem.* **33**, 545 (1900); Tolman and Ferguson, *J. Amer. Chem. Soc.* **34**, 232 (1912); MacInnes and Parker, *J. Amer. Chem. Soc.* **37**, 1445 (1915).

[2] *J. Amer. Chem. Soc.* **33**, 1454 (1911).

A STATISTICAL STUDY OF THE VISUAL DOUBLE STARS IN THE NORTHERN SKY

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The double star survey, initiated in April, 1899,[1] has now been completed to – 14° Declination for the winter sky and to – 22° Declination for the summer sky. This is about as far south as atmospheric conditions make it desirable to extend the survey at Mount Hamilton.

The original plan contemplated the examination with one of the refracting telescopes of the Lick Observatory of all stars as bright as 9.0 magnitude in the *Bonn Durchmusterung*, the identification of all previously known double stars, and the cataloguing of any new pairs, not exceeding $5''.00$ in angular separation. Limits must be imposed in work of this character, and those chosen, while necessarily arbitrary, were selected after careful study as being liberal enough for the purpose in mind, which was the accumulation of data for a statistical study of the number and distribution of the visual double stars.

The survey has yielded 4300 new double stars, 1328 discovered by Prof. W. J. Hussey, the others by the writer. Twenty-eight of these have angular separation slightly exceeding $5''.00$; 3199, or 74.4% of the whole number, fall within the limit $2''.00$, and 1280 (29.8%), within the limit $0''.50$. The distance limit adopted has thus been carefully observed, but for various reasons many stars fainter than the limit 9.0 were examined with the result that several hundred of our discoveries are fainter than this magnitude. It may develop that the most important contribution made by the survey, possibly the only one of permanent value, is this great addition to the number of known close visual binaries. Nevertheless, the statistical study has been the object which I have held definitely in mind from the beginning and I have regarded discovery as incidental.

The present paper gives the results from a partial statistical study for the northern half of the sky, the data including all visual double stars (as distinguished from spectroscopic binaries) as bright as 9.0 B. D. magnitude which fall within the limits set by my 'working definition' of a double star.[2] In all there are 5400 such pairs north of the celestial equator, 2823 of which were discovered in the course of the Lick Observatory survey. It is not claimed that this number is exhaustive, but it may fairly be said that it represents the capacity of the combination of telescope and observer under *average good observing conditions at Mount Hamilton.* Unfortunately the data are not strictly homogeneous, for part of the survey was carried out with the 12-inch refractor. A study of the relative efficiency of the 12-inch and 36-inch refractors in double star discovery leads to the conclusion that, if the entire work had been done with the latter telescope, about 250 additional pairs would have been discovered in the northern hemisphere.

According to Seeliger's count, the *Bonn Durchmusterung* contains 100,979 stars as bright as 9.0 magnitude north of the celestial equator; 5400 of these, or one in 18.7 on the average, have actually been found to be double within the chosen limits. If we add only 200 more pairs the ratio is increased to 1 in 18.03. We may therefore conclude that *at least one in every eighteen, on the average, of the stars as bright as 9.0 magnitude in the northern half of the sky is a double star visible with the 36-inch telescope.*

Table I shows the distribution of these double stars in galactic latitude by magnitude classes, the arguments, for convenience of comparison, being those adopted by Seeliger[3] in his discussion of the B. D. stars. The zones are 20 wide, the first extending from the north galactic pole to +70°, the fifth from +10 to −10 galactic latitude. Zone IX, touching the south galactic pole, lies entirely south of the celestial equator. The progressive increase in numbers in every magnitude class as we approach Zone V from the north or from the south was to be expected, for it is well known that the stars as a whole show a similar distribution. When, however, we compare the results with Seeliger's tabulation for all the B. D. stars to 9.0, we find that the frequency curves of double stars of every magnitude class rise to sharper maxima in the Milky Way than do the corresponding curves of the stars in general.

This is most strikingly exhibited in Table II which gives the percentage of double stars in the Milky Way zone, the 20° zone on either side of it, the area north of +30° and that south of −30° galactic latitude.

TABLE I

THE DISTRIBUTION OF DOUBLE STARS BY MAGNITUDE CLASSES AND ZONES OF GALACTIC LATITUDE

ZONE	MAG. TO 6.5	MAG. 6.6–7.0	MAG. 7.1–7.5	MAG. 7.6–8.0	MAG. 8.1–8.5	MAG. 8.6–9.0	TOTAL
I.............	19	13	14	29	40	84	199
II.............	43	28	50	68	114	193	496
III............	60	43	56	79	148	254	640
IV............	96	54	81	132	232	401	996
V.............	121	88	133	249	376	653	1620
VI............	84	51	81	134	221	395	966
VII...........	28	23	18	54	90	154	367
VIII..........	7	6	5	12	31	55	116
Total........	458	306	438	757	1252	2189	5400

Areas of all galactic latitudes were examined in winter, others are as in summer; the larger part of the work done with the 12-inch telescope lies between +27° and −3° galactic latitude; nearly the entire area south of −3°, and also the high northern galactic latitudes, were surveyed with the 36-inch refractor. The apparent increase in percentage in the Milky Way zone is therefore not due to the use of different telescopes nor to any other peculiarity in the collection of the data, and we must conclude that *close visual double stars are relatively more numerous in the Milky Way than elsewhere in the sky.*

TABLE II

PERCENTAGES OF DOUBLE STARS

GALACTIC LATITUDE	B. D. STARS TO 9.0	DOUBLE STARS	PERCENTAGE OF DOUBLE STARS
+90° to +30°	26948	1335	4.95
+30 to +10	19355	996	5.15
+10 to −10	26477	1620	6.13
−10 to −30	17831	966	5.13
−30 to −70	10368	483	4.66

I have tabulated the double stars in distance classes by steps of 0″.5 and find that the increased density toward the Milky Way is exhibited by stars of every class, though the percentages in Table II are most strongly affected by the close pairs because of their greater number.

Table III shows the relation between magnitude and angular separation. Since the progressive increase in the number of double stars as the angular distance diminishes is shown by stars of every magnitude class, it cannot be explained as merely a perspective effect, the more distant pairs appearing at smaller angles, for according to our present conceptions, stars of a given magnitude (e.g., from 8.6 to 9.0) are approximately at the same average distance from us. The relative angular separations of the double stars of this magnitude therefore reflect the relative linear distances between the components and it follows that *visual double stars as a rule revolve in relatively small orbits.*

The spectral classification of the majority of the 5400 stars under discussion is at present unknown. I have compared the list with the Draper Catalogue in volume 50 of the *Harvard Annals*, thus securing the spectral data for the brighter stars, and Prof. E. C. Pickering and Miss Annie J. Cannon have been kind enough to compare the list of double stars north of +60° and that of the double stars in the first two hours of Right Ascension with the new Draper Catalogue now in course of preparation at Harvard, which is designed to extend to 9.0 magnitude. Combining the data from these three sources and excluding duplicates we have Table IV.

TABLE III

THE DISTRIBUTION OF DOUBLE STARS IN THE NORTHERN HEMISPHERE BY ANGULAR DISTANCE AND MAGNITUDE

DIST.	0″.00 TO 0.50	0″.51 TO 1.00	(0″.00) TO (1.00)	1″.01 TO 2.00	2″.01 TO 3.00	3″.01 TO 4.00	4.01 TO 5.00	5″.01 AND OVER
Mag. to 6.5	75	63	(138)	83	62	41	31	99
6.6 to 7.0	82	52	(134)	59	42	40	21	14
7.1 to 7.5	103	67	(170)	99	64	48	31	29
7.6 to 8.0	178	132	(310)	164	107	85	63	26
8.1 to 8.5	310	223	(533)	285	173	128	111	21
8.6 to 9.0	508	413	(931)	532	317	217	191	11
Total.......	1256	954	(2206)	1222	765	559	448	200

The distribution of these stars with respect to the galactic plane closely resembles that of the stars in general; Classes B and A crowd toward that plane, classes F, G and K show a more uniform distribution. The totals show how marked is the preference of the visual double stars for Classes A and F. It may be added that 46 of the 83 Class B stars belong to the subclasses B8 and B9 and that 104 of the 124 Class K stars are marked 'K' or 'K0.' Clearly, *close visual double stars are rare among stars of either very early or very late spectral class.* This confirms my conclusion, in 1910,[4] based on an analysis of the spectral classification of 164 of the more rapidly moving binary stars on my observing program.

The ten stars of Classes M and Oe5 are obviously exceptional, and the number is too small for any discussion of value. The following points however may be noted: The stars are all bright; the proper motions are very small; orbital motion has been observed in only one of them (η *Geminorum*), and in this system it is very slow; the companion in each instance is much fainter than its primary, and the angular separation between the components is neither very small nor very large. Probably they are all very distant.

TABLE IV

SPECTRAL CLASSIFICATION OF DOUBLE STARS

ZONE	SPECTRAL CLASS						
	B	A	F	G	K	M	Oe 5
I	0	8	4	2	3		
II	2	19	14	5	12		
III	3	37	59	28	22		
IV	18	81	68	45	26	1	
V	47	101	54	31	24	5	2
VI	9	84	47	31	15	1	1
VII	3	19	33	16	14		
VIII	1	8	29	16	8		
Total	83	357	308	174	124	7	3

The percentage of very close double stars—under 0″.50—is considerably greater in Classes B, A and F than in Classes G and K, but the percentage of pairs over 2″.0 is about the same in all classes except K, in which it is larger. To discuss the correlation between angular separation and spectral class we need further data. I hope to take up this question and others in a later paper.

The conclusions here presented of course apply only to the data upon which they rest, and, in view of the many warnings we have had against basing 'laws of nature' upon incomplete statistics, I refrain from any comment upon the bearing they may have upon current theories of the evolution of double star systems. It is my opinion that we have not yet sufficient knowledge of the facts to establish any theory with reasonable certainty.

[1] "On Double Stars," R. G. Aitken, *San Francisco, Pub. Astr. Soc. Pac.*, **16**, 235 (1904).

[2] "The Definition of the Term Double Star," R. G. Aitken, *Ast. Nachr.*, **188**, 281 (1911).

[3] "Ueber die Vertheilung der Sterne auf der nördlichen Halb-kugel . . . ," H. Seeliger, *München, Sitz. Ber. Ak. Wiss.*, 1884, Heft 4.

[4] "Second Catalogue of Spectroscopic Binary Stars," W. W. Campbell, *Lick Obs. Bull.*, **6**, 41 (1910).

WALNUT MUTANT INVESTIGATIONS

By Ernest B. Babcock

DEPARTMENT OF AGRICULTURE, UNIVERSITY OF CALIFORNIA

Presented to the Academy, August 20, 1915

About fifteen years ago there appeared in a nursery of Southern California Black Walnut seedlings a new and very distinct form of *Juglans.* Two of the original trees still stand where first transplanted from the seed bed. These are the type specimens which I described[1] and named *Juglans californica* var. *quercina* because of their general resemblance to a small-leaved oak. Similar seedlings have been secured from two other trees of *Juglans californica* Wats. In 1914 seedlings of this same oak-like type appeared in a nursery of the Northern California Black Walnut, *J. californica* var. *hindsii* Jepson. This is a very interesting parallel mutation because *hindsii* was originally described as a species[2] and is separated from *californica* by several distinct features. However, this occurrence of parallel mutations does not in itself necessitate any change in nomenclature other than to designate *quercina* as mutation rather than variety. Therefore I propose to designate *quercina* individuals produced from seeds of *J. californica* as *J. californica* mut. *quercina* and to similar individuals produced from *hindsii* seeds as *J. californica* var. *hindsii* mut. *quercina.*

In 1910 I discovered that many otherwise normal trees of Southern California Black Walnuts produce a second crop of flowers, from one to two months later than the first crop and that these nuts are invariably teratological.[3] However, up to 1913 all the plants produced by the teratological nuts collected resembled *J. californica,* i.e., there was no evidence that teratology was the cause of the origin of *quercina.*

In 1912 I was able to locate an apparently normal tree of *Juglans californica* Wats., which produces a small percentage of *quercina* seedlings each year and to learn that these grew from normal nuts. This tree is designated as Garden Grove No. 16. It is probably the only perennial mutating individual accessible for experimentation. That the mutation always occurs in the pistillate flowers is shown by the fact that Garden Grove No. 16 is the only tree in a row of 21 (all of which were tested in 1912) that produced *quercina* seedlings. An objection has been raised to the designation of Garden Grove No. 16 as a mutating individual. The argument advanced is based on the assumption that all new types of organisms are due to recombinations of genetic factors and hence that if Garden Grove No. 16 produces two types of seedlings it must be a hybrid. To this objection I can only reply that thus far I have been unable to discover any morphological evidence that Garden Grove No. 16 is a hybrid.

In 1913 I gathered about 350 clusters of nuts from this tree and grew each lot separately. Among them were several clusters of teratological nuts. A single *quercina* seedling appeared in each of 42 of the normal clusters while three of the teratological clusters produced two or more *quercina* seedlings in each cluster. These results indicate that, while teratology *per se* is not the cause of the mutation, yet in a tree capable of producing *quercina* seedlings the mutation occurs more frequently among teratological than among normal flowers. The fact that in all the normal clusters producing *quercina* seedlings only one mutant came from each cluster, at once raised the question as to whether a certain location of the flower in the pistillate catkin might be associated with the occurrence of the mutation. This question is not yet fully answered but it is known that at least two different locations in the catkin are associated with the mutation and hence it can be stated definitely that no certain location of the pistillate flower is exclusively associated with the occurrence of the mutation.

On account of the interesting differences in chromosome numbers found among some of the evening primroses, we have determined the number in the somatic cells of both types of seedlings produced by Garden Grove No. 16. By planting one seed in a small pot excellent root-tips for cytological study are thrust out through the hole in the pot. The chromosomes are very small and cells that show satisfactory figures for counting chromosomes are comparatively rare. However, I am satisfied that the number of chromosomes in both types of seedlings is thirty-four. If further study reveals the same number of chromosomes in both *quercina* and *californica* seedlings secured from other sources, we must conclude that the mutation producing *quercina* is not due to a change in the number of chromosomes. The chromosome numbers for *hindsii* and its *quercina* mutants have not yet been determined.

Meanwhile, efforts have been made to test the genetic relation of *quercina* to *californica* by crossing. I have eight F_1 trees of the cross *J. californica* ♀ × *J. californica* mut. *quercina* ♂, all of which resemble *J. californica*. They produced some open pollinated nuts in 1914 and among the progeny of one of these eight (08 H_1F_1P8c) there has appeared one *quercina* seedling. This indicates that at least one of the eight is a veritable hybrid and not a parthenogen as might be the case judging from the results of certain crosses with oak pollen also made in 1908. Hence it is possible that the ratios produced in future tests of this individual will throw some light on the nature of the mutation which produces *quercina*. Efforts are being made to secure self-pollinated nuts from each of these trees and in 1916 I hope to secure sesqui-hybrids from the cross *J. californica* mut. *quercina* ♀ × 08 H_1F_1P8c ♂. The completion of these breeding tests is especially desirable in view of the diverse behavior of certain of the original *quercina* trees. One of the type individuals breeds true, while a cotype individual produces both *quercina* and *californica* seedlings and another cotype individual produces nothing but *californica* seedlings.

The facts reported in this paper may be interpreted in part as meaning (1) that the mutation takes place in female flowers only and appears in the first generation after the mutation occurs but upon crossing with the species type it is completely recessive in the F_1 generation; (2) that the nature of the mutation is such that only certain genetic factors are affected without having the chromosome number disturbed.

Another walnut mutant has appeared in the form of a laciniate-leaved type of *Juglans regia*. Open pollinated nuts occasionally reproduce the new type. It is expected that self- and cross-pollinations will be made in 1916 and possibly cytological investigations.

---

[1] Babcock, E. B., Studies in Juglans I. Study of a New Form of *Juglans californica* Wats., *Univ. Cal. Pub. Agr. Sci.*, 2, no. 1 (1913).

[2] Jepson, W. L., in *Bull. So. Cal. Acad. Sci.*, 7, 23 (1908).

[3] Babcock, E. B., Studies in Juglans II. Further observations on a New Variety of *Juglans californica* Wats., and on Certain Supposed Walnut-Oak Hybrids, *Univ. Cal. Pub. Agr. Sci.*, 2, no. 2 (1914).

HEREDITARY FRAGILITY OF BONE

By C. B. Davenport and H. S. Conard
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GRINNELL COLLEGE, IOWA

Presented to the Academy, August 30, 1915

While a weakness or brittleness of the long bones may arise from a number of pathological conditions affecting the bones, yet there has been recognized for 80 years a constitutional tendency to brittle bone that runs through families in such fashion as to demonstrate the presence of an hereditary factor. To it has been given the name fragilitas ossium or osteopsathyrosis.

In a classical case of osteopsathyrosis the femur is frequently found fractured at birth, but it mends quickly and smoothly. Repeatedly in life a slight knock causes a bone of the leg or arm to break; and sometimes before puberty the individual has suffered a score or more of fractures. The tendency is often outgrown after puberty. Lovett and Nichols[1] have shown that in osteopsathyrosis the periosteal bone formation especially is imperfect, since the Haversian systems are not formed.

An examination of a large number of family histories shows clearly that heredity in osteopsathyrosis is typically direct—i.e., the factor that determines the irregular bone formation is a dominant one. The view that it is due to "some disturbance of metabolism of the mother" is disproved by the instance of twins, described by Schwarz and Bass,[2] one of whom showed osteopsathyrosis at birth while the other did not. The rare exceptions to the rule that a generation is not skipped are explained by the well known principle of imperfection of dominance or possibly to a lack of knowledge of an infantile condition of a parent.

An association of osteopsathyrosis with blue sclerotics has often been pointed out. The condition of blue sclerotics also appears to be a dominant trait. It is barely possible that the association is not a necessary one but on the other hand, there may be a linkage between these traits; pedigrees are not yet extensive and numerous enough to decide.

That osteopsathyrosis depends upon a single dominant factor is further indicated by the fact that the proportion of children in an affected fraternity that shows osteopsathyrosis is approximately half—actually 55%.

The presence of biotypes of osteopsathyrosis can be recognized. In some families the slightest pressure results in fracture; in other families the bones are fairly resistant. The commonest bone to break is the femur. But in some families the humerus seems especially weak and in still others the clavicle is frequently broken. Thus, associated with the imperfect bone development are special family traits that modify the result.

It appears, to summarize, that of a parent who was in early life osteopsathyrotic at least half of the children will be similarly affected. But if neither parent, though of affected stock, has shown the tendency then expectation is that none of the children will have brittle bones. Moreover, when a parent is affected it is possible that his children will show the tendency at about the same time of life, in the same bones and to the same degree as he himself showed it.

The full paper with numerous pedigree charts will appear in the Bulletin of the Eugenics Record Office.

[1] Lovett, R. W. and Nichols, E. H., *Brit. Med. J., London,* 1906, II, 915.

[2] Schwarz and Bass, *Medical Record,* 1912, p. 317.

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EXPERIMENTS ON THE DEVELOPMENT OF THE LIMBS IN AMPHIBIA

By Ross G. Harrison
OSBORN ZOOLOGICAL LABORATORY, YALE UNIVERSITY

Presented to the Academy, October 2, 1915

Although previous experiments by Barfurth,[1] Byrnes,[2] Kammerer,[3] Braus,[4] Lewis[5] and others have yielded many interesting facts regarding the development and regeneration of the limbs in Amphibia, the bounds of the limb rudiment at very early stages have not been determined exactly in any species, and it is not known how wounds of a definite size and character in the limb region affect the subsequent power to give rise to a normal appendage. In order to control properly certain transplantation experiments upon the fore limbs in *Amblystoma*, relating chiefly to questions of position and laterality, it was, therefore, found necessary to determine first the regenerative power of the tissues of the limb region after the removal of the limb rudiment in whole or in part before differentiation had begun. Accordingly the following experiments were made:

1. Simple extirpation of the body wall of the fore limb region at a period before the limb bud becomes visible on the surface, the size of the wound being varied.
2. Extirpation of a definite portion of the limb region.
3. Removal of the limb rudiment and subsequent covering of wound with ectoderm taken from some other region of the embryo.
4. Removal of the mesoderm alone from the limb region, afterward replacing the overlying ectoderm.
5. Removal of the ectoderm alone leaving the underlying mesoderm intact.

6. Transplantation of small masses of mesoderm, not yet visibly differentiated, from the limb region to pockets under the skin on the side of the embryo.

The fore limb appears at a much earlier stage than the hind limb in the Urodeles and much earlier than either extremity in the Anurans. The operations were consequently done upon embryos about the time of appearance of the tail bud (fig. 1). In many cases, however, the embryos were slightly younger or slightly older than the one shown in the figure. The fore limb of *Amblystoma* develops as a thickening of the somatopleure which centers in the region of the fourth myotome and extends over into that of the third and fifth. In order to remove the tissues which form the appendage, a circular incision of about the diameter of these three segments was made with fine scissors, and the loosened piece including both ectoderm and mesoderm was then torn free and removed. The few mesoderm cells which are usually left sticking to the pronephros and around the edges of the wound were carefully removed in some cases and in others left in place. In some individuals the pronephros was left intact and in others it was removed. In nearly all cases the operation was done on one side, the other being left in its normal condition for comparison.

Table showing effect of removal of body wall of fore-limb region, the number of cases in each category being given

SIZE OF WOUND	WOUND NOT COVERED				WOUND COVERED			
	Not cleaned		Cleaned		Not cleaned		Cleaned	
	Regen.	Not regen.	Regen.	Not regen.	Regen.	Not regen.	Regen.	Not regen.
Not recorded	11	9			2	8		
3 somites	22	1	13	12			7	14
*3 somites							3	2
3¼ somites			1	0			1	0
*3½ somites							0	1
3½ somites	25	8	2	7			0	11
*3½ somites							0	2
4 somites	16	2	2	3			0	3
*4somites							0	2
4½ somites	2	0						
Total	76**	20	18	22	2	8	11	35
Percentage	79	21	45	55	20	80	24	76

* In the groups so marked the ectoderm of the limb region was healed back in place; in the other groups in which the wound was covered ectoderm from other regions of the body was used.

** Includes five cases, constituting the fifth group of experiments, in which a large part of the mesoderm was purposely left in.

Experiments. 1. The results of the simple removal of the body wall of the limb region are given in the left half of the Table. While the figures in the single classes are often not large enough to give significant percentages, it is clear that the cleaning of the wound of all scattered mesoderm cells reduces very materially the proportion of cases in which regeneration occurs, i.e., from 79 to 45 per cent. It is also apparent that when the size of the wound exceeds in diameter the length of three somites the proportion of cases in which limbs develop is considerably less, whether the wound is cleaned or not. When it is carefully cleaned and exceeds three somites in diameter, the healing is slow and there is a very high mortality, which accounts for the small number of experiments with wounds of this size appearing in the tabulation.

The age of the embryos, within the limits of the stages experimented upon, has no influence upon the regenerative capacity. The presence of the pronephros, which on account of its close proximity might possibly be thought to have some influence on the limb, seems not to affect its normal development, for a perfectly normal limb may be formed when the pronephros is removed. The removal of this organ facilitates, however, the cleaning away of the mesoderm cells and thus may affect indirectly the results of experiments, even though it exerts no formative stimulus.

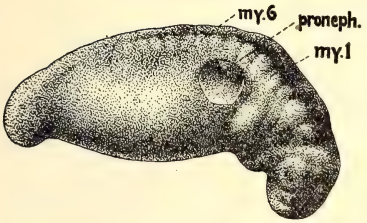

FIG. 1.

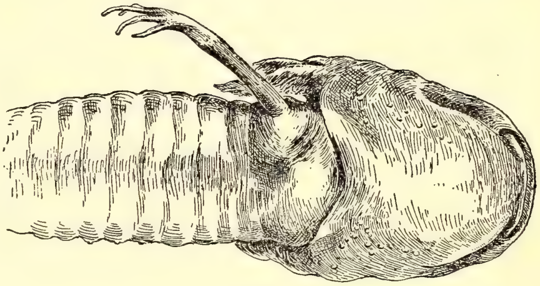

FIG. 2.

FIG. 1. EMBRYO OF AMBLYSTOMA PUNCTATUM SHOWING THE WOUND IMMEDIATELY AFTER OPERATION IN A TYPICAL EXPERIMENT. × 10.

FIG. 2. AMBLYSTOMA LARVA FROM WHICH THE LEFT FORE LIMB RUDIMENT WAS REMOVED, PRESERVED 87 DAYS AFTER OPERATION. × 5.

The limbs which develop after removal of the rudiment are retarded in development to a varying degree. In no case has a limb been found subsequently to arise, when the negative condition has persisted till the twelfth day, and in only three cases when it has remained till the tenth. Consequently, if development has not begun by the end of the second week it is safe to assume that it will not take place at all. A large number of cases observed till the end of the third or the fourth week, and in some instances much longer, show this to be true (fig. 2).

No extensive examination of the material has been made by means of sections, but five cases which were cut show, that even though the free appendage is lacking, a shoulder girdle of somewhat reduced dimensions is developed on the operated side, just as Braus[6] has found in the case of *Bombinator.*

2. The experiments of the second group, in which limited portions of the limb rudiment were removed—dorsal, ventral, caudal, or rostral half, or the central portion—all resulted in the formation of perfect limbs whose development was but slightly retarded.

3. A comparison of the right and left halves of the Table shows at a glance that the covering of the wound with indifferent skin is a considerable hindrance to the development of the limb. The cases, cited on the first line of the table, in which the wounds were not cleaned happen to give, however, a considerably exaggerated idea of the effect of skin transplantation, since the control experiments in which the wound was left uncovered, made at the same time and with the same material, showed also a very high percentage of cases of non-development. Some unknown factor must have affected the results here. The cases in which the wound was cleaned are gathered, however, from experiments made at various times, and these show very clearly the effect of covering the wound. In the experiments with wounds of but three somites in diameter the percentage of development is reduced from 52 to 33, while in the cases where the wound is three and a half segments or more in diameter, i.e., when the region of the anterior half of the sixth segment is included, the percentage of development is reduced to zero. The determination of this latter point is of prime importance for the main purpose of this work, since it shows how much it is necessary to remove, when transplanting other buds to the limb region, in order to be sure that the appendage which develops is not simply regenerated from the host.

4. The experiments in which the ectoderm of the limb region was healed back in place after removal of the mesoderm are only ten in number and are, therefore, insufficient to give significant percentages. They nevertheless confirm the result of the previous series. Five experiments with wounds of three segments in diameter yielded three cases in which development occurred and two in which it did not occur. Five other experiments with wounds four segments in diameter were all negative. The mesoderm was carefully cleaned off at the time of the operation in all experiments of this set.

5. The five cases in which the ectoderm alone was removed and the mesoderm, at least in great part, left in place all resulted in the develop-

ment of a normal limb. Since these differ only in degree from those experiments of the first group in which the wound was not carefully cleaned, they have been included in the tabulation under that class.

6. To transplant the mesoderm alone, the ectoderm is first torn free from the underlying tissues after the usual circular incision is made. The mesoderm of the region is then cut out in a single piece as large as possible, and transferred with a blunt needle to a small pocket under the ectoderm of another embryo previously prepared. The mass of cells is then pushed into the pocket and left, a procedure that is not always easily carried out because of the extreme stickiness of these cells.

Nineteen experiments were made, in four of which the embryo died before yielding results. In five cases the transplanted tissue was entirely resorbed; in three others it remained a small nodule; in one a long appendage without digits resulted; and in six there developed a limb of approximately full size which, however, in most cases showed some irregularities, usually in the form of reduplications.

The most perfect of these showed reduplication of only a single digit; another gave rise to a perfect limb with complete reduplication of fore-arm and manus. The one shown in figure 3 has the beginning of a supernumerary digit on the radial side of the hand. The other cases were more irregular. Sections show that in addition to the free appendage a small shoulder girdle is formed from the transplanted tissue. This is also the case when the entire limb rudiment is transplanted, as is shown both by Braus's[7] experiments upon *Bombinator* and my own upon *Amblystoma*.

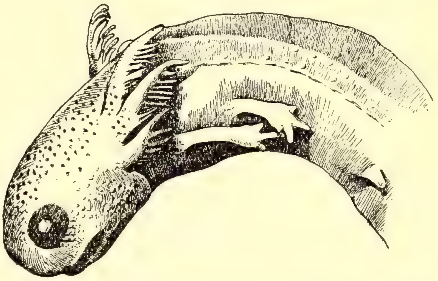

FIG. 3. AMBLYSTOMA LARVA WITH SUPERNUMERARY FORE LIMB, DEVELOPED FROM CELLS INOCULATED FROM THE MESODERM OF THE LIMB REGION OF AN EMBRYO IN THE STAGE SHOWN IN FIGURE 1. EXPERIMENT TR. MES. 1. PRESERVED 16 DAYS AFTER OPERATION. × 10.

Conclusions. The anterior limb of *Amblystoma*, according to the foregoing experiments, is already determined, at the time of appearance of the tail bud, in the mesoderm cells of that region of the body wall which lies close to the pronephros and ventral to the third, fourth, and fifth myotomes. The prospective significance of this group of cells as a whole thus is defined some time before differentiation becomes visible. Forming a narrow zone around this region there are cells which may vicariously give rise to a limb in case the rudiment proper is removed. These cells, responding to the stimulus of the wound, gradually move up toward the center, and covering the bare yolk, afterward in many cases develop into a normal limb. Miss Byrnes[8] some years ago called attention to this phenomenon in the case of the hind limb rudiment of *Rana* embryos, though the results of the present study, which has aimed to determine exactly the limits of the power to regenerate, show that it is much more restricted than she supposed to be the case. Perhaps, however, there is in *Amblystoma* the same difference in regenerative capacity between fore and hind limb regions that Braus,[9] found to be the case in the Anurans. Covering the wound with indifferent ectoderm, or healing back the original ectoderm of the limb region, hinders this movement of peripheral cells toward the nodal point from which the new limb may arise, and definitely prevents development, provided that the extent of the wound includes the region of the anterior half of the sixth somite in addition to that of the third, fourth and fifth. In case the wound is but three segments in diameter, development may or may not be prevented but the proportion of negative cases is considerably larger than when the wound of the same size is left uncovered.

Carried out upon a different form and on much younger embryos than the experiments of Braus, the present experiments afford additional evidence that the mesoderm cells of the limb region, while forming a definitely differentiated system as a whole, are nevertheless totipotent within that system as far as the skeletal and muscular elements of the limb are concerned. They can give rise to a perfect limb when placed in an unusual environment, even when the original arrangement of the cells is disturbed, as it necessarily is when the mesoderm alone is transplanted; and a small part can develop into a whole. Other experiments, not described here, show, however, that at the time of operation differentiation has already begun to some extent, though it has evidently not become irreversible. The ectoderm merely serves as a covering to the limb and no indication of any specific stimulus from this layer could be detected. Other structures in the region such as the myotomes, as shown by Byrnes (1898) and Lewis (*op. cit.*) or the pronephros, as found in the present investigation, do not affect the development or regeneration of the limb bud in any way.

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[1] D. Barfurth, *Arch. EntwMech., Leipzig,* **1** (1894).

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A MECHANISM OF PROTECTION AGAINST BACTERIAL INFECTION

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The means employed by the animal body to rid itself of bacteria have been conceived to be of two kinds: those of disintegration or lysis, and those of cellular inclusion or phagocytosis.

According to the former, the bacteria are acted upon by certain constituents of the blood serum—amboceptor and complement—which dissolve them; and according to the latter they are englobed by white blood corpuscles which digest them.

As a matter of fact, the first process has been inferred, rather than demonstrated. It is true that in shed blood the dissolution by lysis has been observed, but not in the living body. But even in shed blood or its serum constituents the solution occurs only with a part of the pathogenic bacteria, of which B. typhosus may be taken as an example. Such bacteria as pneumococcus, streptococcus, etc., are not subject directly to this form of dissolution. Phagocytosis, on the other hand, is a more general phenomenon and applies to a wide variety of bacteria.

It has long been known that when bacteria are introduced into and later disappear from the blood, they are not eliminated by the organs of excretion, but are destroyed in the organs themselves. The problem at issue relates to the manner of the destruction.

The question should be considered with reference to two states of the animal body, namely, the unprotected or normal, and the protected or immune state.

Taking certain forms of pathogenic or disease producing bacteria, a study was made as to the manner of their disappearance in protected rabbits. The pneumococcus and typhoid bacillus may serve as examples. Protection was secured by the employment of immune sera. In the case of the pneumococci, the type of pneumococcus and immune serum must coincide. In the experiments a type I pneumococcus and corresponding serum were employed.

Protection against pneumococcus. It has been shown that an active pneumococcus serum protects against a certain maximum quantity of pneumococcus culture, but that multiples of the serum do not protect equally against multiples of the culture. An effective culture of pneumococcus causes on inoculation fatal septicemia in the rabbit, followed by death in 24 to 48 hours or less. When an immune serum is employed, life may be saved or the surviving period merely prolonged.

The immediate effect of a serum injection is to cause the removal of the pneumococci from the circulating blood. This effect is produced in an incredibly short period of time—in a few minutes indeed. But the permanency of the removal depends in part on the quantity (or dose) of antiserum injected. Small doses of serum are more effective than large doses, and the former may be successful in saving life, while the latter are not.

The mechanism of the removal is as follows: when an immune serum is introduced into the blood of a rabbit suffering from pneumococcus septicemia, an almost immediate agglutination of the bacteria takes place. The larger the doses of the serum, within limits, the larger the size of the bacterial clumps that are found. The clumps are removed from the blood almost immediately by the organs—the spleen, liver, and bone marrow. What happens next is determined by the size of the clumps. If they are large, they cannot be ingested by phagocytes; hence they soon begin to multiply, and the bacteria reinvade the blood; if small, they are taken up by phagocytes and are digested. The animal succumbs on the one and survives on the other hand. Hence small doses of the serum causing smaller clumps may be more effective than large doses giving larger ones. No extra-phagocytic dissolution of the pneumococci seems to occur.

Protection against the typhoid bacillus. A similar mechanism operates in the rabbit inoculated with cultures of the typhoid bacillus. The typhoid bacilli, notwithstanding the fact that they are subject to serum-lysis, are taken out of the blood by the organs after clumping, and the clumps are ingested by phagocytes which digest them.

Pathogenic and non-pathogenic bacteria. Certain cultures of disease-producing bacteria are not, others are pathogenic for animals. The influenza bacillus appears in these two distinct varieties. When cultures by the non-pathogenic variety are injected into the circulation of rabbits, they are clumped and removed by the organs at once; when cultures of the pathogenic variety are inoculated, they are neither clumped nor removed. Hence a pathogenic effect may depend upon agglutinability of the bacteria—by the blood of normal or of immunized animals.

In other words, bacteria circulating in the blood are quickly removed when they are agglutinated or clumped, and the clumps deposited within the organs are taken up by phagocytes and digested. They appear not to be destroyed by solution or lysis through the operation of serum constitutents of the blood.

ON THE LIFE-HISTORY OF *GIARDIA*

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This paper is based upon the examination of 220 mammals, gray and white culture mice (98), culture rats (42), and wild *Peromyscus maniculatus gambeli* (Baird) (59), *Microtus californicus californicus* (Peale) (6), and coyotes [Canis ochropus ochropus (Peale)] (2) trapped near Berkeley. About 13% of the total were infected, 22 of the mice, and 4 of the *Peromyscus*, with *Giardia muris* and three of the *Microtus* with *G. microti* sp. nov. Several hundred slides prepared by the Schaudinn-iron haematoxylin wet method were examined and stages in binary and multiple fission and in encystment obtained in quantity.

In the last edition of Kolle and Wassermann's *Handbuch der pathogenen Mikroorganismen* Jollos (1913) denies the existence of multiplicative phases in *Lamblia* in the free stage in the intestine of the host. Rodenwaldt (1911) had also held this view in Prowazek's *Handbuch der pathogenen Protozoen* and Alexeieff (1914) and Prowazek and Werner (1914) as the result of recent investigations have confirmed it. Castellani and Chalmers in the last edition (1913) of their *Manual of Tropical Medicine* accept Noc's (1908) conclusion, seemingly made without study of comparative material, that there is only one species concerned in the intestinal infections alike of mice, rats, cats, and man, and make the inference that such vermin become sources of infection to man by the contamination of his food and water with their cyst-bearing faeces.

It comes thus to be of interest to human and comparative medicine to learn whether or not these intestinal flagellates which produce a chronic enteritis in young mice and are under suspicion of similar activity in man, may multiply in the free state in the intestine, or whether infection is limited to the number of ingested spores, and especially to learn whether there are separate species restricted to given host species or groups of species of rodents, and another species found in man. If the latter be the case, the probability of these mammals being sources of infection is reduced considerably, unless it be that the parasites transform from one 'species' to another with change of host. The biological problems of host specificity, and transformation by environment are thus involved in this question.

It is the purpose of this paper to demonstrate the occurrence of binary and multiple fission in *Giardia* (=*Lamblia*), in both the free (hitherto denied) and encysted stage, to describe mitosis in these minute organisms, to offer confirmatory evidence for the presence of several species instead of one only in the genus, to add to the skepticism regarding autogamy and the existence of an *Octomitus* stage in the life-history, advanced by Hartmann (1911), and to offer some inconclusive evidence of maturation in the so-called conjugation cysts. The paper will be published in full in the University of California Publications in Zoology.

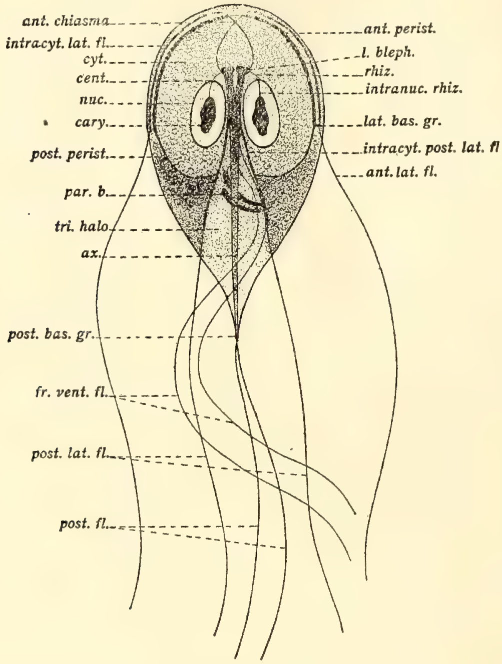

Fig. 1. Ventral view of *Giardia microti* sp. nov. in early prophase of mitosis, somewhat diagrammatic. × 3400. Abbreviations: *ant. chiasma*, anterior chiasma; *ant. lat. fl.*, anterior lateral flagellum; *ant. perist.*, anterior peristome; *ax.*, axostyle; *cary.*, karyosome; *cent.*, centrosome; *cyt.*, cytostome; *fr. vent. fl.*, free ventral flagellum; *intracyt. lat. fl.*, intracytoplasmic part of the antero-lateral flagellum; *intracyt. post. lat. fl.*, intra-cytoplasmic part of the postero-lateral flagellum; *intranuc. rhiz.*, intranuclear rhizoplast; *lat. bas. gr.*, lateral basal granule of antero-lateral flagellum; *l. bleph.*, left blepharoplast; *nuc.*, nucleus; *par. b.*, parabasal body; *post. bas. gr.*, posterior basal granule of posterior flagella; *post. fl.*, posterior flagella; *post. lat. fl.*, posterolateral flagellum; *post. perist.*, posterior peristome; *rhiz.*, rhizoplast joining nucleus to blepharoplast; *tri. halo:* triangular halo about posterior part of the axostyle.

Giardia microti sp. nov. (fig. 1) in common with the other species, is a diplozoic flagellate, a binucleate somatella with eight flagella in four pairs, antero-lateral, postero-lateral, free ventral, and posterior, the last emerging from the posterior tip of the axially located axostyle, the ventrals from the ventral surface, and the others from the lateral margins. The flagella form a part of an integrated neuromotor system of fibrils which unite the two component cells by a single chromatic system staining more or less deeply with iron haematoxylin. This system consists of a rhizoplast from the centrally located karyosome of each nucleus which passes through the centrosome on the nuclear membrane to the blepharoplast on its side at the head of the axostyle.

From the two blepharoplasts the single axostyle (not two as heretofore reported) passes posteriorly to emerge as the posterior pair of flagella. From the blepharoplast or near it, also arise the other flagella. The antero-lateral passes forward within the cytoplasm, crosses its mate of the opposite side uniting at the crossing in a node, the anterior chiasma, and thence curves posteriorly along the margin of the cytostome to emerge from the lateral basal granule as a free antero-lateral flagellum. The postero-lateral passes obliquely posteriorly within the cytoplasm to emerge near the margin, and the free ventrals emerge at once on the ventral surface. The blepharoplasts of the two sides of the axostyle are united across its head by a transverse bridge commissure. The antero-lateral flagella run in close conjunction with the thickened chromatic anterior peristome and the postero-laterals are in contact with the posterior arc of the peristome. This thickened chromatic margin lies in the rim of the deeply cupped ventral cytostome, the organ of attachment of the parasite.

Dorsal to the axostyle is a pair of chromatic bodies, the parabasals which contain stainable material whose volume fluctuates with the conditions of metabolism and which may function as reservoirs of cytoplasmic chromidial material. They appear to have a characteristic shape in each species. In *G. muris* they are stout ellipsoidal, and nearly fused. In *G. microti* (fig. 1) they are slender, curved, tapering, parallel structures. In *G. muris* each is connected with the axostyle near the blepharoplast by a slender chromatic fibre.

These organisms multiply in both the free and the encysted stages, by simple binary fission, and by multiple fission leading to the formation of a 16-nucleate, 8-zooid plasmodium or somatella, or a 16-nucleate, potentially 8-zooid cyst. Binary fission occurs by a well defined mitosis followed by a delayed plasmotomy.

Mitosis in *Giardia* is much like that described by Kofoid and Swezy (1915) for *Trichomonas*. It begins in the blepharoplast whose division is followed by that of the axostyle which splits distally to form two on the anterior ends of which the daughter blepharoplasts are attached. The centrosome at the upper pole of the nucleus divides and the daughters assume polar positions on the membrane of the elongating nucleus, spinning out a temporary paradesmose between them upon the outside of the persistent nuclear membrane. The nucleus, from which an intranuclear chromidial cloud had emerged, with subsequent increase of cytoplasmic chromidia, exhibits an enlargement of the karyosome, its breaking up into eight chromatic granules probably representing four precociously split chromosomes which form in parallel rows and merge into a split skein from which four chromosomes emerge in the equatorial plate at the metaphase. These constrict, the daughter chromosomes migrate toward the poles of the intranuclear spindle, and the nuclear membrane constricts the parent into two daughter nuclei.

Plasmotomy is long delayed. The separation is longitudinal, the two daughter nuclei of each side forming the nuclei of the daughter zooids, which hang together assuming a great variety of positions before the protoplasmic bridge finally parts.

Multiple fission is of three types. The first is in free individuals in which three mitoses follow in succession before plasmotomy occurs and each zooid during the process acquires its full complement of organelles before the next division ensues. The result is a 16-nucleate, free-moving plasmodium composed of 8 fully equipped zooids which they separate singly. These small individuals are not *Octomitus*-like and are not to be confused with *Hexamitus muris* Grassi (= *Octomitus*), which species occurs in the same host with *G. muris*. *Giardia*, according to our evidence, does not pass through an *Octomitus* stage as Hartmann (1911) proposed.

Multiple fission in free forms occurs also by nuclear division with which the extranuclear organelles do not keep pace in multiplication, so that free individuals with the normal single equipment of these of extranuclear organelles are crowded with 16 nuclei in a bilateral or ultimately an irregular arrangement. The later history of such individuals has not been traced.

Single encysted individuals may also undergo multiple fission. The extranuclear organelles tend to disappear, the axostyle and posterior peristome lingering longer than the other chromatic parts as the nuclei go through three succeeding divisions producing a 16-nucleate mass within the cyst. Such cysts seem not to undergo further development in the rectum of the host in which they are found.

We find no evidence that the so-called conjugation cysts contain two sister individuals, and that therefore autogamy occurs. Pairs of individuals in end-to-end back-to-back position as in cysts are found free without evidence of their origin by fission. This condition may precede encystment. The two individuals which may simulate maturation divisions in so far that the first division of each nucleus gives rise to two nuclei, one with abundant chromatin, and one (first polar nucleus?) with little chromatin. The second division occurs in both of these daughter nuclei. The smaller nucleus divides forming two still smaller nuclei and the larger one forms one nucleus rich in chromatin (the gamete nucleus?) and one with little chromatin (the second polar nucleus?). However, no evidence of reduction in number of chromosomes is demonstrable. It is obvious that, since these divisions occurred in both nuclei of the somatella, we might have a binucleate gamete, and should the two 'conjugants' in the cyst unite, a binucleate zygote, a condition highly improbable. The alternative explanation of this simulation of maturation is that we have here some form of degenerative multiple fission but the evidence of this is not conclusive.

We find evidence of groups of structural characteristics which are diagnostic of the several species in the genus. The same organelles with the same fundamental relations of position occur in all of the species at comparable stages. The specific differences consist in modifications in shape, size, proportions, relative size and shape of the cytostome and its relation to the margin of the body, in the size, shape, and position of the parabasal bodies, in general stainability, and in relations to hosts. It is possible thus to characterize *G. agilis* Alexeieff from the tadpole, *G. sanguinis* (Gonder) from *Elanus*, a South African falcon, *G. duodenalis* (Davaine) from the European rabbit, *G. muris* (Grassi), from *Mus*, *Epimys*, *Arvicola* (?) and *Peromyscus*, *G. microti*, sp. nov. from *Microtus*, and the species widely known as *Lamblia intestinalis* (Lambl) from man. Conclusions as to the identity of *Giardia* (= *Lamblia*) of man and rodents now generally accepted are of doubtful validity and require critical verification.

The generic name *Lamblia* Blanchard 1888 should give way to *Giardia* Kunstler 1882 on ground of priority since species of the two are morphologically so alike that no generic distinction can be made between them.

Our work has shown that *Giardia muris* and *G. microti* produce a readily recognizable enteritis in mice, and that both binary and multiple fission take place in the free non-encysted stage. It may be found abundantly in material well smeared out from the intestinal epithelium. Multiple fission results in a 16-nucleate, 8-zooid plasmodium which later undergoes disintegrative plasmotomy. There is no *Octomitus* stage in the life-history of *Giardia*. There is typically one axostyle in the trophozoite. Prophase stages in which this has divided to form two axostyles are often seen. The so-called 'Rätzelkörper' are homologues of the parabasals of the trichomonads. The extranuclear organelles are united with the karyosome, centrosome, and blepharoplast in an integrated neuromotor apparatus. Mitosis is intranuclear, with precocious splitting of the four chromosomes which subsequently fuse in four in the equatorial plate. Free pairs of individuals are found united in back-to-back position as in the so-called conjugation cysts. Nuclei in these cysts undergo two divisions simulating reduction divisions in which, however, chromosomes reduction has not been demonstrated. No evidence in support of autogamy and no proof of sexual reproduction has been discovered.

Morphological characters separate six species in *Giardia*. The parasite in mice appears to be distinct from that in man. The generic name *Giardia* Kunstler should supersede *Lamblia* Blanchard on grounds of priority.

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THE INORGANIC CONSTITUENTS OF ALCYONARIA

By F. W. Clarke and W. C. Wheeler

UNITED STATES GEOLOGICAL SURVEY, WASHINGTON

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The zoantharia, madreporaria, or stony corals have been repeatedly analyzed, and with generally concordant results. Thirty analyses, made in the course of the present investigation, of which this paper is a preliminary notice,[1] have confirmed the older data. These corals consist mainly of calcium carbonate, with one or two per cent of minor impurities, and a little organic matter. The same is true of the coralline hydrozoa, of which six analyses, representing the genera *Millepora* and *Distichopora*, have also been made. The alcyonaria, however, which include the red corals, the gorgonias, and other fan-like or branching forms, are quite different; and they are generally characterized by the presence in them of magnesium carbonate, and often of calcium phosphate also. Many of them, moreover, are rich in organic matter. To these statements there is one known exception. The blue coral, *Heliopora cerulea*, is an isolated organism, closely resembling the true corals, and like them in its chemical composition.

In the course of our research upon the inorganic constitutents of marine invertebrates, 22 analyses of alcyonarians have been made, representing the following species:

1. *Heliopora cerulea*, Pallas. Blue coral. Southern Philippines.
2. *Tubipora purpurea*, Lamarck. Singapore, Straits Settlements. Latitude 1° 20′ N.
3. *Corallium elatior*, Ridley. A red coral. Murotsu, Tosa, Japan. Latitude about 33° N.
4. *Primnoa reseda*, Verrill. East of Nova Scotia. Latitude 44° 16′ N.
5. *Lepidisis carophyllia*, Verrill. Off Nantucket shoals. Latitude 38° 53′ N. Depth of water 3,168 metres. Temperature 3°. 3′. C.
6. *Pennatula aculeata*, Dana. St. Peter's Bank. Latitude 44° 47′ N. Temperature 4°.5 C.
7. *Paramuricea borealis*, Verrill. Southwest edge of the Grand Bank.
8. *Paragorgia arborea*, Milne-Edwards and Haime. Le Have Ridge, off Nova Scotia.
9. *Alcyonium carneum*, L. Agassiz. Southwest of Stellwagen Bank, off Race Point Light, Cape Cod, Massachusetts. Temperature 3°.9 C.
10. *Gorgonia suffruticosa*, Dana. Fiji Islands.
11. *Gorgonia acerosa*, Pallas. Nassau, Bahamas. Latitude 25°, 5′ 6″ N.
12. *Gorgonia acerosa*, Caesar's Creek, Southern Florida. Latitude about 23° 30′ N.
13. *Muricea humilis*, Milne-Edwards. Parahyba do Norte, Brazil.
14. *Muricea echinata*, Valenciennes. Cape San Lucas, Lower California. Latitude 22° 52′ N.
15. *Plexaurella grandiflora*, Verrill. Mar Grande, Bahia, Brazil.
16. *Ctenocella pectinata*, Valenciennes. Torres Straits, Australia. Latitude about 10° S.
17. *Xiphogorgia anceps*, Pallas, Caesar's Creek, Florida.
18. *Rhipidogorgia flabellum*, Linné, Bermuda. Latitude about 32° N.
19. *Rhipidogorgia flabellum*, Andros Island, Bahamas. Latitude about 25° N.
20. *Leptogorgia pulchra*, Verrill. La Paz, Gulf of California. Latitude 24° 16′ N.
21. *Leptogorgia rigida*, Verrill. Cape San Lucas, Lower California.
22. *Phyllogorgia quercifolia*, Dana. Fernando de Noronha, Brazil. Latitude 3° 50′ S.

The complete analyses of these alcyonarians, which will be published in our final report, show very variable proportions of organic matter.

In Nos. 1, 2, 3, and 5, very little was found; while the others contained quantities ranging from 13 to 61%. Rejecting this variable constituent, and recalculating the inorganic portion to 100%, the reduced or rational analyses assume the following form:

	1	2	3	4	5	6	7	8
SiO_2	0.15	1.40	0.00	0.13	0.11	1.70	0.44	0.15
$(Al,Fe)_2O_3$	0.07	0.57	0.15	0.88	0.05	1.01	0.30	0.03
$MgCO_3$	0.35	12.23	11.56	6.18	6.92	7.71	8.03	9.05
$CaCO_3$	98.93	84.61	86.57	90.39	92.24	85.62	85.11	88.04
$CaSO_4$	0.50	1.19	1.32	1.59	0.68	0.84	4.69	2.17
$Ca_3P_2O_8$	trace	trace	0.40	0.83	trace	3.12	1.43	0.56
	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

	9	10	11	12	13	14	15
SiO_2	18.05	0.55	0.22	0.04	0.56	0.11	0.45
$(Al,Fe)_2O_3$	5.80	0.28	0.22	0.24	0.07	0.06	0.15
$MgCO_3$	9.21	13.43	12.52	13.29	12.64	12.28	13.79
$CaCO_3$	52.23	79.84	81.45	79.48	84.47	83.79	85.61
$CaSO_4$	1.36	5.43	1.95	4.08	1.73	2.93	trace
$Ca_3P_2O_8$	13.35	0.47	3.64	2.87	0.59	0.83	trace
	100.00	100.00	100.00	100.00	100.00	100.00	100.00

	16	17	18	19	20	21	22
SiO_2	0.21	0.14	0.21	0.24	0.09	0.28	0.34
$(Al,Fe)_2O_3$	0.13	0.07	0.28	0.07	0.03	0.21	0.26
$MgCO_3$	15.65	13.04	12.64	13.19	13.71	14.13	15.73
$CaCO_3$	81.44	80.96	63.38	80.75	74.99	75.36	72.99
$CaSO_4$	1.69	3.83	2.40	2.95	2.91	2.07	2.11
$Ca_3P_2O_8$	0.88	1.96	1.09	2.80	8.27	7.95	8.57
	100.00	100.00	100.00	100.00	100.00	100.00	100.00

Analysis No. 1, of *Heliopora*, might be that of an ordinary coral, being almost non-magnesian and therefore different from all the others. The material of No. 9, *Alcyonium*, was obviously impure, with much admixture of sand and mud. That the species is remarkably rich in phosphate, however, seems to be clear; but it needs further investigation upon a better sample. In two of the *Gorgonias* the black, wiry axis was separately examined, although there was not enough of it for satisfactory analysis. In *G. suffruticosa* the axis lost on ignition 94.39%, and in *G. acerosa* 96.35%, mostly organic matter. The inorganic residues were small in amount, but partial analysis showed that they differed in composition from the more abundant calcareous envelopes. In some genera the axes are mainly calcareous; but in others the organic centers are very large, and the species are difficult to deal with analytically. Further investigation upon the differences in composition between cortex and axis is needed, but on biological rather than geological grounds. As the purpose of this investigation is to determine what each group of organisms contributes to the formation of marine limestones, an extended study of their organic matter would be hardly relevant.

If, now, we arrange the alcyonaria in the order of ascending magnesium carbonate, a remarkable relation appears connecting composition with the temperature of the habitat. *Heliopora*, being anomalous, is not included in the table. *Alycyonium* is also excluded, on account of its impurities. The percentages of $MgCO_3$ and $Ca_3P_2O_8$ are both given.

Magnesium carbonate and calcium phosphate in alcyonaria.

SPECIES	LOCALITY	LATITUDE	$Ca_3P_2O_8$	$MgCO_3$
Primnoa reseda	Nova Scotia	42° 16′ N.	0.83	6.18
Lepidisis caryophyllia	Off Nantucket	48° 53′ N.	trace	6.92
Pennatula aculeata	Nova Scotia	44° 47′ N.	3.12	7.71
Paramuricea borealis	Grand Banks		1.43	8.03
Paragorgia arborea	Nova Scotia		0.56	9.05
Corallium elatior	Japan	33° N.	0.40	11.56
Tubipora purpurea	Singapore	1° 20′ N.	trace	12.23
Muricea echinata	C. San Lucas	22° 52′ N.	0.83	12.28
Gorgonia acerosa	Bahamas	25° 5′ N.	3.64	12.52
Muricea humilis	Brazil	7°–8° S.	0.59	12.64
Rhipidogorgia flabellum	Bermuda	32° N.	1.09	12.64
Xiphogorgia anceps	Florida	22° 30′ N.	1.96	13.04
Rhipidogorgia flabellum	Bahamaas	25° N.	2.80	13.19
Gorgonia acerosa	Florida	22° 30′ N.	2.87	13.29
Gorgonia suffruticosa	Fiji		0.47	13.43
Leptogorgia pulchra	L. California	24° 16′ N.	8.27	13.71
Plexaurella grandiflora	Brazil		trace	13.79
Leptogorgia rigida	C. San Lucas	22° 52′ N.	7.95	14.13
Ctenocella pectinata	Torres Straits	10° S.	0.88	15.65
Phyllogorgia quercifolia	Brazil	3° 50′ S.	8.57	15.73

Although records of temperature and depth of water are available for only a few of these alcyonaria, the suggested relation is clear. The organisms from cold, northern waters, or from very deep waters, are low in magnesia, while those from warm regions are much higher. The same relation appears in our analyses of echinoderms, and is unmistakable; even though it is as yet unexplained. It is not rigorously exact; but some apparent irregularities are due to the disturbing effect of impurities, such as sand or mud, which appear in the analyses as silica and sesquioxides. If these were rejected the percentage of magnesia would be raised. Variations are also to be expected because of cold or warm currents, and differing depths of water. Very deep water, even under the equator, is always cold; while shallow bays, even quite far north, may be relatively warm. Possibly also, the alcyonaria may form several distinct series, not perfectly comparable as regards chemical composition. *Corallium* and *Tubipora*, for example, are compact forms, with little organic matter; and they are lower in magnesia than the genera with horny, organic axes, such as appear at the end of the table. It is also noteworthy that the highest proportions of calcium phosphate are commonly found associated with high values for magnesia.

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AN EXPERIMENTAL ANALYSIS OF THE ORIGIN AND RELATIONSHIP OF BLOOD CORPUSCLES AND THE LINING CELLS OF VESSELS

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Studies on the origin and development of the cellular elements of the blood and the so-called endothelial cells which line the blood vessels in the normal embryo are peculiarly difficult on account of the important rôle that wandering mesenchyme cells play in these processes. The problem is also further confused by the perplexing mixture of cells of different origin brought about by the early established circulation of the body fluids. The development of no other embryonic tissue is so disturbed by mechanical and physical conditions.

A study of living fish embryos with the high power microscope has made it possible to observe the behavior of the wandering cells and to follow them in their development. The disadvantages due to the intermixture of cells in the blood current have been overcome by the investigation of embryos in which a circulation of the blood is prevented from taking place.

When the eggs of the fish, Fundulus heteroclitus, are treated during early developmental stages with weak solutions of alcohol, the resulting embryos in many cases never establish a blood circulation. In other respects these embryos may be very nearly normal and the development and differentiation of their tissues and organs often proceed in the usual manner, though at a somewhat slower rate. The heart and chief vessels are formed and the blood cells arise and develop in a vigorous fashion. The heart pulsates rhythmically but is unable to propel the body fluid since its venous end does not connect with the yolk vessels. And in many cases its lumen is partially or completely obliterated by periblastic material and nuclei which seem to be sucked into the heart cavity from the surface of the yolk.

In these embryos without a circulation of the blood, one is enabled to study the complete development of the different types of blood corpuscles in the particular regions in which they originate. There is no contamination of the products of a given region through the introduction of foreign cells normally carried in the blood stream.

The actual power to form blood cells possessed by the different organs and tissues may be determined in the experimental embryos having no blood circulation. And the true haematopoetic function is thus clearly contrasted with the ordinary reproduction or multiplication of blood cells which might take place within the tissue spaces of such organs in the normal embryo.

The debated question regarding the production of blood cells from those cells lining the blood vessel wall may be convincingly answered, at least for the species here studied.

The results and conclusions derived from these experiments may be summarized as follows:

1. The fish embryo is capable of living and developing in an almost normal fashion without a circulation of its blood; this fact was first recorded by J. Loeb in 1893, yet this is the initial study of blood and vessel formation in such embryos. Red blood cells may be seen to arise and differentiate in these living embryos in two definite localities; the one within the posterior body region, and the other the blood islands on the yolk.

The blood cells remain confined to their places of origin, yet they attain a typical red color and may persist in an apparently functional condition on the yolk-sac for as long as sixteen to twenty days. The normal embryo becomes free swimming at from twelve to fifteen days, but individuals without a circulation never hatch although they may often live for more than thirty days.

All recent investigators have claimed that there are no blood islands present on the bony fish yolk-sac. Yet the presence of such islands is readily demonstrated in living Fundulus embryos, in the normal as well as in those with no circulation.

2. The plasma or fluid in the embryos which fail to develop a circulation begins to collect at an early time in the body cavities. The pericardium becomes hugely distended with fluid, as well as the lateral coelomic spaces and the Kupffer's vesicle at the posterior end of the embryo. The great distension of the pericardium due to this fluid accumulation pushes the head end of the embryo unusually far away from the surface of the yolk. The heart is thus often stretched into a long straight tube or string leading from the ventral surface of the head through the great pericardial cavity to the anterior yolk surface.

No blood vessels, or very few if any, form on the extreme anterior portion of the yolk-sac, so that the venous end of the heart is never connected with veins. And the heart does not draw fluid into its cavity to be pumped away through the aorta. When the heart cavity does contain fluid it is unable to escape and small floating particles may often be observed rising and falling with the feeble pulsations.

3. The hearts in embryos without a circulation are lined with a definite endocardium, but the myocardium or muscle wall is poorly developed, sometimes consisting of only a single cell layer. Pigmented cells are not present in the wall of the normal heart, but in the experimental hearts these large chromatophores are invariably found. The cavity in many of the hearts is almost if not entirely obliterated by the presence of periblastic material and huge amorphous periblast nuclei.

The conus end of such hearts leads directly to a more or less closed ventral aorta. Portions of the aortic arches are seen as open spaces, and the dorsal aortae are almost invariably seen as typical spaces lined by characteristic embryonic endothelium.

A point of much importance is the fact, that *neither these hearts with their endothelial linings nor any portion of the aortae at any stage of development have ever been seen to contain any form of red blood corpuscle.* Cells of this type are completely absent from the anterior regions of the embryo.

4. Pigment cells normally occur on the Fundulus yolk-sac and arrange themselves along the vascular net so as to map out the yolk-sac vessels in a striking manner. Loeb has thought that this arrangement along the vessel walls was possibly due to the presence of oxygen carried by the corpuscles within the vessels. In the embryos without a yolk-sac circulation the pigment cells arise but rarely become fully expanded so that the usual long branched processes are represented only by short projections; the chromatophore consequently seems much smaller than usual.

The unexpanded pigment cells, however, wander over the yolk-sac and collect in numbers around the plasma filled spaces. The yolk surface of the pericardium and the periphery of the Kupffer's vesicle are often almost covered with pigment. The hearts are during early stages full of plasma and the pigment cells form a sheath around them, while such cells are never present on the normal hearts during the embryonic period.

These facts would seem to indicate that the plasma rather than the blood corpuscles contains the substance which attracts the chromatophores and initiates their arrangement along the normal vascular net of the yolk-sac.

5. A definite mass of cells characteristic of the fish embryo is located in the posterior half of the body between the notochord and the gut and extends well into the tail region. This so-called 'intermediate cell mass' is the intra-embryonic red blood cell anlage in many of the species.

The peripheral cells of the mass as claimed by Swaen and Brachet, or the mesenchyme about the mass, Sobotta, form a vascular endothelium which encloses the central early blood corpuscles. In individuals without a circulation the erythroblasts arise in a normal manner in this centrally located position and become erythrocytes or red blood corpuscles filled with haemoglobin. Typical vascular endothelium completely surrounds the erythrocytes which, instead of being swept away as usual by the circulating current, remain in their place of origin. All of the early blood forming cells of this intermediate mass give rise to red corpuscles and never to white blood cells.

6. Contrary to the opinion of most recent observers on blood development in Teleosts, the Fundulus embryos both with and without a circulation, possess blood islands on the posterior and ventral portions of the yolk-sac. These blood islands are formed by wandering mesenchymal cells which migrate out from the caudal region of the embryo. They represent all that remains on the yolk-sac of the peripheral mesoderm in the Teleosts and probably wander away from mesoderm related to that of the intermediate cell mass. The intermediate cell mass may possibly represent the bulk of the peripheral mesoderm which is here included within the embryonic body, while in other meroblastic eggs it is spread out posteriorly over the yolk. The only mesodermal elements of the yolk-sac in Fundulus are the independent wandering mesenchyme cells, some of which group themselves to form the blood islands, while others give rise to the yolk vessel endothelium, and still other wandering cells develop into the chromatophores.

7. The non-circulating red-blood corpuscles within the embryo remain in a fully developed condition for eight or ten days and then undergo degeneration. In an old embryo of sixteen days it is sometimes found that very few of the corpuscles in the intermediate mass are still present and these are degenerate. The vascular endothelium has been lost and numerous mesenchyme cells have wandered in to lie among the corpuscles.

On the yolk-sac the corpuscles no doubt have a better oxygen supply and here they maintain their color longer, but finally also present a degenerate appearance with small densely staining nuclei and cell bodies much reduced in size.

8. Vascular endothelium arises *in loco* in many parts of the embryonic body other than those localities in which blood cells form. The endothelium is in all cases utterly incapable of giving rise to any type of blood cell. This incapacity cannot be attributed to the abnormal condition of the embryo, since true blood cell anlagen in the same specimen produce blood corpuscles in abundance.

The lining of the vessel walls in the fish embryo has no blood forming function.

9. Neither lymphocytes nor other types of white blood corpuscles, have been found to arise in the yolk-sac blood islands, nor within the intermediate cell mass. The embryonic white blood cells are found most abundantly in the anterior body and head regions, and there occupy extra-vascular positions usually lying among the mesenchymal cells.

The sources of origin of the white and red blood corpuscles in Fundulus embryos are distinct, and these two different types of cells cannot be considered to have a monophyletic origin except in so far as both arise from mesenchymal cells.

The adult blood of Fundulus contains lymphocytes and several varieties of granular leucocytes.

10. There is evidence to indicate that definite environmental conditions are necessary for blood cell proliferation or multiplication. Blood cells do not normally divide when completely enclosed by vascular endothelium. This is the key to the shifting series of so-called haematopoetic organs found during embryonic development.

Erythroblasts, embryonic red blood corpuscles, lying about spaces unenclosed by vascular endothelium, proliferate steadily and give off their products into the space from which they find their way into the embryonic vessels. Should such an erythroblast be carried by the circulation to another unlined space, it may become arrested there and again undergo a series of divisions, giving rise to other erythroblasts. When, however, these spaces become lined by endothelium, the blood cell reproduction stops.

In most embryos the earliest blood cell formation occurs in the yolk-sac blood islands. The cells in these islands continue to divide until they become surrounded by endothelium, then the yolk-sac blood islands lose their haematopoetic function and become a vascular net through which the blood circulates. The liver now takes up the rôle of harboring dividing blood cells within its tissue spaces. When these spaces become vascularized by endothelium, here again the blood cells no longer multiply but merely circulate.

Finally, in the mammalian embryo, one organ after another ceases to offer the necessary harbor for dividing blood cells, until the red bone marrow is the only tissue presenting the proper relationship of spaces and vessels, and here alone the erythropoetic function exists to supply the red blood cells for the entire body circulation. The red blood corpuscles are always produced so as to be delivered into the vessels and thus very soon occupy an intra-vascular position and cease to divide, while the white blood cells arise and remain for some time among the mesenchymal tissue cells in an extra-vascular position.

11. Lymphocytes and leucocytes, so-called white blood cells, along with the invertebrate amoebocytes, are all generalized more or less primitive wandering cells, and are almost universally distributed throughout the metazoa.

Erythrocytes, red blood corpuscles, are very highly specialized cells with a peculiar oxygen carrying function, due to their haemoglobin content. In contrast to the universal distribution of the leucocytes, the erythrocytes, the red corpuscles, are only found in the vertebrate phylum and in a few of the higher invertebrate groups. Yet in these invertebrates the oxygen carrying blood cell never presents the typically uniform appearance of the vertebrate erythrocyte. The oxygen carrying function in many invertebrates is confined to the liquid plasma.

The typical vascular endothelial cell is widely distributed in the animal kingdom and appears to be a simple slightly modified mesenchymal cell.

These three very different types of cells all seem to arise from mesoderm—the mesenchyme. Yet the present investigation would indicate that each arises from a distinctly separate mesenchymal anlage. The erythrocyte, red cell, anlage is localized and perfectly consistent in the quality of its production. The lymphocyte and leucocyte, white cell, anlage is more diffusely arranged and not definitely localized in any particular cells group. The vascular endothelium appears to be formed *in loco* in almost all parts of the embryonic body, and its formation is absolutely independent of a circulating fluid or the presence of blood cells.

On the yolk-sac of Fundulus embryos one finds four distinctly different products, red blood corpuscles, endothelium, and two varieties of chromatophores, differentiating from the apparently similar wandering mesenchymal cells. The environment in which the four types differentiate is identical as far as is possible to determine, and the only explanation of their various modes of differentiation is that the original mesenchymal cells that wandered out were already of four potentially different classes. These differences in potentiality within the cells produced the four different types of structure in one and the same environment. The four types of cells are then in an embryological sense polyphyletic in origin.

Therefore, *vascular endothelium, erythrocytes and leucocytes, although all arise from mesenchyme, are really polyphyletic in origin; that is, each has a different mesenchymal anlage.* To make the meaning absolutely clear, we may consider the origin of the liver and pancreas cells a parallel case, both arise from endoderm but each is formed by a distinctly different endodermal anlage, and if one of these two anlagen is destroyed the other is powerless to replace its product.

A more complete account of this work, with a discussion of the literature bearing on the subject, appears in the September and November numbers of the *American Journal of Anatomy.*

NOTICES OF BIOGRAPHICAL MEMOIRS

The following biographical memoirs have been published by the Academy since the last notices of such memoirs appeared in the June number of the Proceedings.

HENRY MORTON (1836–1902). By EDWARD L. NICHOLS. *Biographical Memoirs of the National Academy*, vol. **8**, pp. 143–151.

This Memoir consists in a description of the life-work of Henry Morton. It includes an estimate of his personality by Mr. Thomas A. Janvier quoted from the Morton Memorial Volume published by the Alumni Association of the Stevens Institute of Technology of which Mr. Morton was the first president.

PETER LESLEY (1819–1903). By W. W. DAVIS. *Biographical Memoirs of the National Academy*, vol. **8**, pp. 155–240.

This Memoir discusses the life-work of Peter Lesley, under the headings: Ancestry; Boyhood; First Geological Work; Early Appalachian Study; Theology and Geology at Princeton; A Year Abroad; Preaching in Pennsylvania; Geological Work in Boston; Preaching in Milton; Engagement and Marriage; The Lyman Family; Peter and Susan Lesley; Married Life in Milton; Lesley's Hobby, "Arkism;" Return to Pennsylvania and Geology; Anti-Slavery Opinions; Professional Work, its Pleasures and Pains; "Coal and its Topography;" Catastrophic Views; Soil-Creep; Lesley as a Draftsman; Personal Opinions; The American Philosophical Society; The Iron Manufacturers' Guide; A Later Estimate of Rogers; Personal Items; Lowell Institute Lectures; Man's Origin and Destiny; Lesley as a Philologist; "Five Types of Earth Surface;" Two Years Abroad; More Personal Items; A Characteristic Incident; Theoretical Views in Professional Reports; The University of Pennsylvania; Proofreading; The "Second Survey;" The Drudgery of Survey Work; Distractions and Entertainments; The American Association; Trips to Europe; The Dictionary of Fossils; Summary of Pennsylvania Geology; Recognition; Lesley in his Old Age.

At the close the author of the Memoir presents a personal appreciation of Mr. Lesley.

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SALTS, SOIL-COLLOIDS, AND SOILS

By L. T. Sharp
COLLEGE OF AGRICULTURE, UNIVERSITY OF CALIFORNIA

Presented to the Academy, November 1, 1915

To reclaim permanently and manage alkali lands successfully, or to use fertilizer salts with intelligence and profit, requires a consideration of the effects of these salts on the physical condition of soils. Not only must we bear in mind the effects of the salts while present in the soil, but also the condition that may result when these same salts are removed partially or wholly by natural or artificial drainage. In our experience in California this latter phase has proved to be a more difficult and perplexing problem than the former. The importance of this subject justifies the publication of this brief discussion, prior to the appearance of the detailed statement with reference to three years of work which we have carried out upon it.

We know that, if a salt is added to a soil suspension, certain obvious effects are produced. Thus the common acids and their salts with the alkalies, the alkaline earths, and the heavy metals, as well as many other salts, flocculate suspensions in water of the soil mass. We associate this flocculated condition with an improvement in the working qualities of the soil. Another class of salts, those which give rise to an alkaline reaction, such as the alkaline hydrates, and as assumed, the alkali carbonates, possess within certain concentrations, the opposite power of deflocculating soil suspensions. Field soils so affected reflect this condition by their imperviousness to water and their inferior cultivating qualities.

Undoubtedly these phenomena are to be associated with the soil particles of such a fine state of division as to make their relatively large surface a factor of special importance. Possibly such particles may properly be designated colloids. Hence in attempting to explain salt effects on soils, we must look to the laws formulated from studies on colloidal materials. Foremost among these may be mentioned those concerning the migration and electrical charges of colloids by Picton and Linde[1] and the subsequent contribution by Hardy,[2] setting forth the important rule that ions which carry charges opposite to those associated with particular colloids will act as precipitants for those colloids. This rule could be anticipated from the previous discovery of Schulze[3] that the flocculating power of an ion is in some measure dependent upon its valence. More recently Tolman[4] has gone over the field anew, and, by assuming that stable colloidal solutions exist only when the surface tension between the particles and the liquid is equal to zero, has developed a very interesting and highly plausible theoretical explanation for the phenomena occurring in dispersoid systems. By means of these facts we are able to predict with some assurance the effect accompanying the constant presence of a particular salt on the physical condition of a soil. It is of particular significance in this connection to remember, however, that these conceptions regarding the behavior of colloids are based on systems wherein the salt remains in contact with the particles. Such is rarely the case in normal field soils; for various agencies are constantly affecting the position of the salts, and as we shall see the leaching out of certain salts from a soil seems to produce an entirely new system, to which the laws referred to may not be strictly applicable. At least the conditions are sufficiently puzzling to merit the following discussion.

If a soil to which sodium chloride or sulphate has been added is subjected to leaching processes, it assumes an entirely new set of physical properties, characterized by a more or less complete deflocculation.[5] Evidently a new system, differing from that composed of the soil in contact with its own solution, or in contact with the salt solution, has resulted from the treatment. This phenomenon was first observed by us in a field experiment consisting of cylinders containing a clay loam soil at Davis, Cal., to which solutions of sodium chloride, sulphate, and carbonate had been added. It will suffice here to say that the salt-treated soils became very impervious to water during the winter season, and when dry were extremely intractable. The untreated control soils were easily managed at all times.

Results similar to those observed in the field were secured in the laboratory when sodium lactate, acetate, nitrate, chloride, sulphate, carbonate, and hydrate, potassium chloride, and sulphate, and ammonium sulphate were washed from the Davis soil. Magnesium chloride is also slightly effective in the same direction, while calcium chloride and barium chloride, when similarly leached from it, seem to leave the soil in its original condition. Not only were various salts tried, but also various soils with the same salt. Thus the washing out of sodium chloride from the Davis clay loam, the Berkeley adobe, the Anaheim sandy and the Oakley blow sand brought about a marked increase in the diffusion of the soil colloids. Apparently, then, we are dealing with a phenomenon of general application.

Turning to the literature we find reports of some interesting observations which concern the phenomenon under consideration, Adolph Mayer[6] noted a decrease in percolation when certain salts are washed from soil; and according to Van Bemmelen[7] the same thing occurs when clays or the hydrated oxides of tin, silica, and manganese are treated in the same manner. He also calls attention to the turbidity of the filtrate which appears as the salts are washed out. A similar turbidity has been observed by us in the filtrates from soils being washed free of salts. Warington[8] also refers in a general way to the appearance of somewhat similar phenomena when soils previously treated with acids are washed with water. Some further observations pertaining to this subject have also been recorded by Warington,[9] Hall,[10] Krüger,[11] McGeorge,[12] and others. Their observations dealt in the main with fertilizer salts and particularly with the after effects of sodium nitrate. In general, the ill effects of long continued use of this salt, are attributed to the formation of sodium carbonate.

Finding that the salt and water treatment of soils referred to brought about such striking results, some experiments were undertaken with the view of studying the mechanism by which these effects are produced. These investigations followed certain theories which had been advanced to account for the soil conditions noted above.

If a soil which has been treated as outlined above is suspended in water it yields a suspension much richer in solid matter than a suspension similarly formed from the untreated soil. Boiling the normal soil in distilled water produces a somewhat similar effect. Likewise, the same general condition of diffusion appears if NaOH in certain concentrations is added to a suspension of the normal soil. We also found that it required more NaCl to flocculate the suspension of a soil from which this salt had been previously leached than to flocculate a suspension of the normal soil. Evidently the colloids of the salt-treated soil are in a high state of diffusion. Furthermore, there is some indication that colloids of a new type, or perhaps additional colloids of the same general type, have been formed.

In either case an increase in the internal surface of the soil would be expected. That such an increase had occurred seemed evident from the data secured by the use of Briggs'[13] centrifugal method for moisture-equivalents. However, a study of the hygroscopic coefficients of the variously treated soils failed to reveal any increase in surface. In all probability Mitscherlich's[14] method would be the most appropriate one for obtaining information regarding this point, but it has not yet been tried by us.

The interchange of ions when salt solutions are placed in contact with soils forms the working basis for the hypothesis that new colloids have been formed. The filtrates from soils which have received salt applications followed by leaching with water contain calcium and magnesium in amounts chemically equivalent[15] to the sodium taken up in exchange therefor by the silicates. It is to this new sodium silicate compound as the first possibility that we attribute the increased colloidal properties noted above. Potassium and ammonium salts seem to produce similar compounds, while calcium and barium salts under the same conditions do not produce the colloidal hydrated silicates. These new substances, together with the normal soil colloids, remain in a flocculated condition as long as free acid[16] or an excess of the soluble salts added or those formed by the interchange is present.

As flocculated colloids their effect on the physical condition of the soil is either beneficial or passes unobserved. Hence as long as salts are present in sufficient quantity no objectionable feature appears, but when these flocculating agents are removed by leaching with water, the colloids become diffused and offer great resistance to the further passage of water.

In explanation thereof, we might assume in accordance with Tolman's ideas that the normal lyophobic soil colloids when suspended in water have a positive surface-tension, hence greater than zero, but that the new hydrated sodium silicate when suspended in the same medium automatically provides by hydrolysis a sufficient quantity of OH ions to lower this surface-tension and to bring it nearer the zero value, so that the colloids, in this case, would have a far greater tendency to become and to remain dispersed. This assumption requires the consideration that both the normal colloids and those resulting from the interchange of ions carry negative charges, which brings us immediately to our next hypothesis which deals in the main, with the nature of the suspending medium, rather than with the changes occurring in the solid substances.

This hypothesis is largely based on the contention that the behavior of soils under the influence of salts agrees in some measure with the laws which are thought to govern the behavior of dispersed systems to which salts have been added. Obviously this would be the case when we are considering suspensions of the soil mass in various solutions. Some doubt, however, may be expressed as to the validity of this statement when a soil of normal moisture content is compared to the dispersoid of a disperse system, or to a soil suspended in a liquid. For in the normal soil such factors as the surface tension of the liquid-air surface may be of such magnitudes as to materially alter conditions so that the laws referred to would not be applicable to such systems. In order to see how far the analogy can be drawn let us consider, for example, clay. Clay has been shown to be a lyophobic colloid which assumes the negative charge of OH ions when suspended in water. The addition of more OH ions as by NaOH within certain limitations lowers the solid-liquid surface-tension so that a greater diffusion results. If a positive ion is added to the system, the OH ions are neutralized, the surface-tension increases, and the clay particles increase in size until they finally settle to the bottom of the vessel. So far the well known facts agree with the theory and, moreover, it is evident that the nature of the medium of suspension is an important factor. When a salt, such as sodium chloride, is added to a soil and subsequently leached out, it may be possible that the medium of suspension has been materially altered. The selective adsorption of the sodium by the soil may eventually give rise to OH ions, in which case the soil would become more or less diffused. By means of the hydrogen electrode we have been able to obtain specific data with respect to the quantity of OH ions present in the medium of suspension. Our results so far indicate that there is not a sufficient quantity of OH ions in the solution bathing the soil particles of the salt treated soil, to account for its unusual degree of diffusion. These data bear significantly on the general behavior of the soil suspension under various conditions.

If a soil which has been previously treated with salt and leached with water is subjected to drying at 130° C., it loses only a small portion of its diffusible colloidal material. Whatever other bearing this may have upon the subject, it at least indicates that the change is of a permanent character.

In the course of these investigations we have also noted that salt solutions of low concentration (less than $\frac{1}{1500}$ normal) have little or no effect on a soil suspension, so that the mere dilution of the salt solutions can not account for the effects observed. Moreover, it has been found that it is not necessary to wash the soil entirely free from salt in order to produce the deflocculated condition.

One other interesting observation with respect to the difference in behavior of sodium hydroxide and carbonate came to our notice. Sodium hydrate is a stabilizing agent in a fairly wide range of concentrations. Sodium carbonate, on the other hand, failed to stabilize at any concentration. Both substances are precipitating agents at concentrations greater than $\frac{1}{15}$ normal. Our studies with the hydrogen electrode include systems of soil suspended in various concentrations of these substances. In case sodium hydroxide or carbonate is added to a soil the bases combine directly with the silicates forming addition-compounds, as has been also pointed out by Sullivan (loc. cit.[15]). Water or carbonic acid is the end-product, hence the washing process so essential for the appearance of diffusion in soils after the use of neutral salts is not so necessary in the case of alkaline salts.

Briefly stated the experiments upon which the above summary is based have enabled us to throw upon the subject of salts in relation with soil colloids a light which has never before, particularly by soil chemists, been cast upon it. Not only is the way opened for an extensive experimental field in the physical chemistry of the soil, but there are indications that the principles involved, part of which are for the first time gathered together and correlated by us, will be of profound practical significance in the important subjects both of 'alkali' and of fertilizer salt applications to soils. Many of the perplexing aspects of the physical effects of continued nitrate of soda and other fertilizer applications to soils, as well as those following the leaching out of alkali, will, we hope, be largely cleared up by the investigations soon to be reported at length.

My thanks are due to Dr. Chas. B. Lipman for many helpful suggestions and for his critical reading of the manuscript.

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[1] *London, J. Chem. Soc.*, **71**, 568 (1897).

[2] *London, Proc. R. Soc.*, **66**, 115 (1899).

[3] *J. prakt. Chem., Leipzig*, Ser. 2, **25**, 445 (1882).

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[7] *J. prakt. Chem.*, Ser. 2, **23**, 388 (1881).

[8] *Physical Properties of Soil*, Cambridge Univ. Press, 1900, p. 30.

[9] Loc. cit.

[10] *The Soil*, 1910, p. 252; also *London, J. Chem. Soc.*, **85**, 964 (1904).

[11] Cited from *U. S. Dept. Agric. Off. Exp. Sta. Rec.*, **29**, No. 8, 719 (1908).

[12] *Hawaiian Agric. Exp. Sta., Bull.* No. 35, 1914.

[13] *U. S. Dept. Agric. Bur. Soils, Bull.* No. 45.

[14] *Bodenkunde für Land-und Forstwirte*, p. 51.

[15] After reviewing this subject Sullivan draws the following conclusion—"So far as the evidence goes, then, the action of silicates, clay, and other constituents of the earth's crust on solutions of such salts as do not dissolve in water with alkaline reaction consists in an equivalent exchange of bases." *U. S. Geol. Sur., Bull.* No. 312, p. 27 (1907).

[16] For a discussion of the presence of free acid in salt solutions in contact with soils the reader is referred to the work of Parker, *J. Agric. Res.*, **1**, 179 (1913).

THE CHILD AND THE TRIBE

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In a previous article, 'The Indian and Nature,' (These PROCEEDINGS, **1,** 467) I outlined the Siouan Indian's conception of Nature and his attempt to reflect that conception in his tribal rites and gentile organization. The new born child was regarded as 'a new life' that had come 'into the midst' of the various forms of nature. The babe had no personal place in the tribe until it became old enough to pass through the ceremony called by the Omaha 'Turning the Child,' by which it entered the tribe and received one of the names belonging to its father's village (gens). This change in the status of the child was effected through the presence during the ceremony of those dual forces by which Wakonda, it was believed, had brought all forms into being and made their perpetuity possible, namely, the masculine element resident in the Sky, and the feminine element dwelling in the Earth, the former was represented by the 'Four Winds' invoked by the priest in the opening ritual song, and the latter was present in the Stone upon which the child was 'turned' by the priest and made to face successively the four directions.

It is to be noted, that during this important ceremony the parents of the child were not present within the sacred tent, nor did they have any part in the rite. The dual forces alone became responsible for the birth of the child into the tribal organization. No stronger or more practical evidence could be given to show the vital character of the Indian's belief in what he conceived to be the established order of Nature, than this surrender of his child to that order as manifested in the rites and organization of the tribe.

Through this ceremony, not only did the child enter into its place in the tribe, but at that time it took its first authorized steps in 'the path of life.' The Omaha ritual song that accompanies these intial steps of the child, sets forth wherein, the Indian believed, lay the power through which man received the ability 'to stand' and to 'go forth' in life. (A free translation of the song is here given.)

Here unto you has been spoken the truth,
    Because of this truth, you shall stand.
Here, declared is the truth!
Here in this place has been shown you the truth;
    Therefore, Arise, go forth in its strength.

Studying this rite in connection with the one used to introduce the child to the cosmos, given in the previous article, together with the various tribal ceremonies of the Siouan group of tribes, it becomes clear that 'the truth' referred to in the ritual song, given above, is that man is an integral part of an order established and sustained by the invisible, undying power, called Wakonda. When the child, through this ceremony, entered the tribe it became a recognized member of the cosmic family as established by Wakonda and drew its strength to pursue the path of life from this truth. Henceforth help could be asked of Wakonda and be brought to the growing child through the performance of the duties laid upon it as a member of the tribe. The children thereafter grew up under the influence of the Indian's belief in an ever present though invisible Wakonda.

For the male child, there was a supplementary rite which followed that through which he was born into the tribe. This ceremony pertained to the life of the boy as a future warrior and protector of the tribe. The rite was called, by the Omaha, 'Wébashna,' to cut the hair. There is evidence indicating that this rite belongs to a later period in the evolution of the tribe than the preceding ceremony of 'Turning the Child.' The limitation of space forbids the presentation of evidence showing how, through the specializing of duties with their attendant ceremonies, the political growth of the tribe slowly developed. In the following rite we discern that the warrior has been raised above the mere fighter, he has become the man of power, one entitled to be placed in close relation to those natural manifestations of power seen in the fury of the storm and heard in the rolling of the thunder. In the efforts toward political unification of the tribe, such rites as those connected with Thunder would conduce to the welding of the people, by the inculcation of a common dependence upon a powerful god and the putting of a sign of consecration to this god upon the head of every male member of the tribe.

In the ceremony of Wébashna, the priest took the boy to the space within the sacred tent west of the fire, there, both facing the east, the priest gathered a tuft from the crown of the boy's head, cut it off and laid it away in a parflesh case, which was kept as a sacred repository, singing as he cut the hair the following ritual song, explanatory of the act. (A free translation is given.)

Grandfather! far above, on high,
The hair, like a shadow, passes before you;
Grandfather! far above on high.
Dark like a shadow, the hair sweeps before you into the midst of your realm.

Grandfather! far above, on high,
Dark like a shadow, the hair passes before you,
Grandfather! dwelling afar, on high,
Like a dark shadow, the hair sweeps before you into the midst of your realm.
Grandfather! far above, on high,
The hair, like a shadow, passes before you.

The hair laid away in the sacred case was sent symbolically to the Thunder dwelling 'far above, on high,' who was addressed as 'Grandfather,' a term of highest respect. The severing of the lock, implied the consecration of the life of the boy to Thunder a symbol of Wakonda the power that controlled the life and death of the warrior. Every man had to be a warrior and thus become a part of the only bulwark of defence to the home and to the tribe. Because of the belief in the continuity of life, a part could stand for the whole, therefore in this rite, by the cutting of a lock of the boy's hair and giving it to Thunder, the life of the child was ceremonially given into the keeping of Wakonda.

The hair of a person was popularly believed to have a vital connection with the life of the body, so that any one becoming possessed of a lock of hair might work his will on the individual from whom it came. In the light of customs that obtained among the people, the hair, under certain circumstances, might be said to typify life.

It is to be noted in this connection, that later, when the hair was allowed to grow on the lad's head, a lock on the crown was parted in a circle from the rest of the hair, kept constantly distinct and always neatly braided, no matter how frowzy and tangled the remainder of his hair might be. It was upon this lock that the war honors of the warrior were worn. It was this lock that was cut from the head of a slain enemy and formed the central object in the triumph ceremonies, for the reason, that it preeminently represented the life of the man who had been killed in battle.

The ritual song that followed the cutting of the lock of hair from the head of the boy makes clear the meaning of the act. (The following is a free translation.)

What time I will, then only then,
A man lies dead, a gruesome thing;
What time I will, then only then,
A man lies dead, a gruesome thing;
What time I will, then only then,
Like a shadow dark, the man shall lie;

What time I will, then suddenly,
A man lies dead, a gruesome thing;
What time I will, then only then,
Reddened and stark, a man lies dead;
What time I will, then only then,
A man lies dead, a gruesome thing.

The word meaning 'dark like a shadow' used in the preceding song to indicate the lock of hair cut from the boy's head as a symbol of his life and offered to Wakonda, is in this song applied to the man, who 'like a shadow dark' 'shall lie' when his life has been taken by the god. The use of the word bears out the meaning of the act that accompanied the singing of the previous song, that, by giving the lock of hair, the life of the giver was placed in the keeping of the Thunder as representing Wakonda. This song shows that the god intends to do as he wills with the life that has been offered him. There are other ceremonial songs which iterate the belief that a man dies only when Wakonda so decrees.

These two rites which connected the life of the child with that of the tribe also initiated the indirect religious teaching as given to Indian children. Of course the little ones who passed through these rites were too young to understand their meaning but the vague memory of the dramatic acts was augmented as year by year the ceremonies were repeated. The cover of the sacred tent where these rites took place was always turned up from the ground, thus exposing the frame work of poles, through which all that took place within the tent was easily seen and the songs heard by the crowd of old and young people that gathered to witness the ceremonies.

In a like indirect manner the children acquired familiarity with the composite character of the tribal organization, which was primarily religious in character and to which the political system was subsidiary. Mention was made in the previous article, already referred to, of the custom of the father cutting his son's hair in an established manner, one that was meant to typify the sacred symbol of his village (gens). This custom of so cutting a boy's hair was kept up from the time the child passed through the rites that made him a member of the tribe, until the period of his second dentition. As the children played together, because of the queerly cropped head of their companions, they came to know to which village (gens) each playmate belonged. They also became accustomed to the use of symbols and to know what the symbolic cut of a boy's hair represented, whether it indicated an animal, a bird, or a cosmic force, as the Wind. Thus unconsciously, the children drifted into the atmosphere of the Indian's belief, that he, as man, was included in the great unity of Nature's family.

The children also learned the place assigned to each village (gens) in the tribal order. While the people were on the great tribal buffalo hunt and during the elaborate ceremonies that took place at its close, the tribe camped in the ceremonial order. At this time the Omaha encampment was in a circular form with an opening toward the east. The north half of the circle, with its five villages (gentes), formed the Sky division; the south half of the circle, with its five villages (gentes), formed the Earth division. In this way, the dual forces, that were regarded as formative instruments in Nature, were represented. There is an interesting bit of evidence as to the use of native logic observable in the following arrangement whereby all the leaders in the ceremonies connected with the material welfare of the people, as, the securing of food by cultivating the maize, hunting, etc., and those, relating to the secular government and to war, were men who belonged to the Earth division; while those rites that were concerned with the securing of help from the supernatural and the maintenance of the tribal order, such as those cited above that pertain to the child, were in charge of leaders who belonged to the Sky division.

The vital existence of the abstract dual forces represented in the tribal organization was made real to the boys and fixed in their minds by the following custom. Fighting was ordinarily not encouraged among the lads, but there was one occasion when it was not interfered with by the elders, and that was when boys crossed the invisible line that divided the Earth from the Sky division while the tribe was camped in the ceremonial order. If a boy should be sent by his father on an errand that required the lad to cross this line, he did not dare to go alone, he would gather his friends, belonging to his own division, to go with him and help him in the battling that was sure to take place. It may truly be said that it was by hard knocks that this fundamental division of the tribe was beaten into the heads of the boys.

The children learned the rudiments of tribal beliefs through object lessons, such as those described above, rather than through oral teaching. A child was always obliged to observe certain forms of respect toward the symbol sacred to its village (gens), it must not touch or taste that which was associated with the symbol, for the symbol belonged to the tribal rites through which the people appealed to Wakonda, therefore, whatever was connected with the symbol was set apart from any familiar use. Beyond the insistence upon this form of respect to the symbol sacred to his village (gens), there was no attempt to coerce the child to observe any rite or to direct its mind to think upon tribal beliefs.

A child gifted with an inquiring mind would be apt to ask questions concerning the symbols and the customs attending them of his father or mother and if his parents belonged to the thinking class, they might put the child's thoughts on the trail that in time would lead him to a knowledge and an understanding of the religious beliefs expressed in the tribal rites, otherwise, the boy or girl would grow up in unquestioning ignorance of the truths hidden within the dramatic rites of the tribe.

The sacred legend of the Omaha lays stress upon 'thought.' 'And the people thought,' was always the prelude to any change for the betterment of the tribe. The Indian thinker became the Indian seer, but, the man must seek for himself the path to that height.

THE CORRELATION OF POTASSIUM AND MAGNESIUM, SODIUM AND IRON, IN IGNEOUS ROCKS

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In its modern development the science of petrology is devoted, in great part, to the physico-chemical study of igneous rocks, regarded as congealed solutions, and of the conditions of separation of their constituent minerals. Distinct from, and in some respects more general than, this, is the study of the distribution of the chemical elements of which they are composed.

This distribution may be spatial; that is, according to the occurrence of groups of rocks, or rock series, over different areas of the earth, each with its dominant chemical characters. Such districts are known as petrographic provinces or comagmatic regions. Their study involves the consideration of the original homogeneity or heterogeneity of the globe, as well as, possibly, certain tectonic factors, as suggested by Harker, Becke, and others, and does not concern us here.

Or the distribution may be according to a chemical correlation, expressed by the presence of certain pairs or groups of elements, which vary together, however they may be spatially distributed. In other words, in rock magmas, solidified as igneous rocks, as well as in their constituent minerals, certain of the elements are found to be generally and persistently associated together, the one being abundantly present when the amount of its correlate is high, and the converse, but not usually so when the rock magma is dominantly characterized by the abundance of another, non-correlated element, in which case another element, correlate with it, will appear as the co-dominant one.

This phase of petrology, which is of a complexity probably much greater than is superficially apparent both as to character and cause, has, as yet, received comparatively little attention. J. H. L. Vogt,[1] J. F. Kemp,[2] L. de Launay,[3] and W. F. Hillebrand[4] have discussed the subject, but have confined their observations almost wholly to the distribution of the elements according to the relative amount of silica (silicity) shown by the igneous rocks.

A few years ago I showed[5] that these relations are far more complex than the simple one based on the silicity. Thus it is clearly obvious from a study of rocks and rock analyses that, among other relations, lithium, zirconium, the elements of the rare earths, glucinum and others, are most abundant in dominantly sodic magmas, irrespective of their silicity; barium apparently in those dominantly potassic; titanium, vanadium, nickel and manganese in those high in iron; and chromium and platinum in those high in magnesium. It was also stated that "soda not uncommonly tends to vary with the iron oxides, while potash shows similar relations to magnesia."

The recent compilation of a second edition of *A Collection of the Chemical Analyses of Igneous Rocks*,[6] embracing all those published between 1883 and 1914 (numbering nearly 10,000), and soon to be published, has added greatly to the data available. This cumulative evidence now greatly strengthens, if it does not quite establish, the view that in igneous magmas potassium and magnesium on the one hand, and sodium and iron on the other, tend to vary together.

The scope of this paper does not permit the presentation in full or in detail of the data in regard to this or other correlations which have been noted, but a short statement may be of interest, in advance of a more extended publication, as calling attention to a phase of chemical petrology which has been somewhat neglected, but which would seem to be of fundamental importance.

The evidence is of two kinds; mineralogical and petrographical.

Among the important mineral groups, found in igneous rocks, which contain the elements in question, the law enunciated above holds good almost without exception. The sodic pyroxenes and amphiboles, such as aegirite, acmite, riebeckite, arfvedsonite, barkevikite, kaersutite, and basaltic hornblende, all show, along with practical absence of potassium, a strong preponderance of iron over magnesium, the latter usually being present in these only in traces or at least in very small amounts. The few reliable analyses of augites from highly potassic rocks, on the other hand, show little soda, but a marked preponderance of magnesium over iron. It may be noted, incidentally, that there are no natural potassic pyroxenes or amphiboles corresponding to the sodic ones.

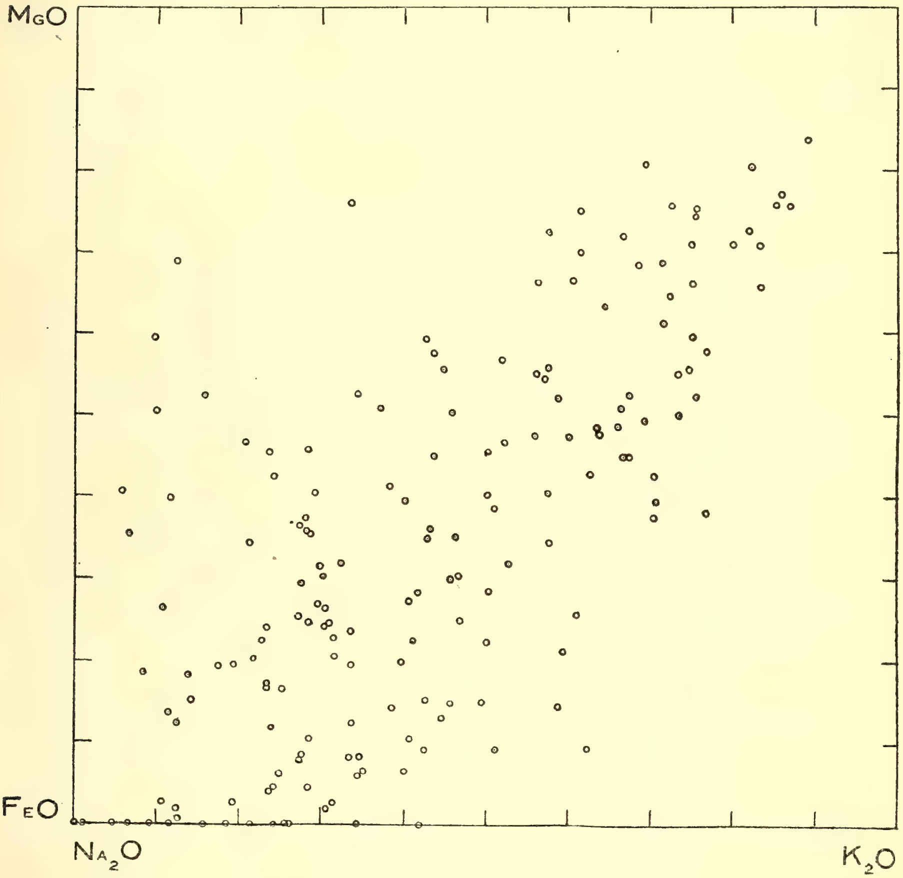
On the other hand in the ferromagnesian micas, biotite and phlogopite, the alkali metal is entirely or almost entirely potassium, the amount of sodium being generally negligible. In these there is usually a very marked predominance of magnesium over iron. Only in the potash mica, lepidomelane, which is characterized by high ferric iron, does iron dominate over magnesium, but it is to be noted that this variety seems to be confined to the sodic rocks. It is also noteworthy that there are no true soda micas, except paragonite, which is only known in metamorphic rocks. It may be added that the potassium-lithium mica, zinnwaldite or cryophyllite, carries much more iron than magnesium, and attention was called above to the fact that lithium is usually associated with sodic rocks.

Of the multitudinous examples of the general law furnished by igneous rocks only a few prominent ones can be given here.

Highly potassic rocks are very rare, but the few known regions are characterized by high magnesium relative to iron. Such are the wyomingite, orendite and madupite of the Leucite Hills, Wyoming; the jumillite and verite of Murcia, Spain; the leucitic rocks of Celebes and of the Gaussberg, in the Antarctic; many of the leucitic rocks of the volcanoes of Western Italy (through here the dominance of magnesium over iron is not so pronounced). There are also many other scattering examples of syenites, shonkinites, minettes, leucitites, leucite tephrites, and other dopotassic rocks. Indeed the same general relation holds good even in such rocks as peridotites, pyroxenites and hornblendites, in which the amounts of magnesium and iron are very high and those of the alkalies very low.

Of the converse case, high sodium with high iron relative to magnesium, very striking examples are the highly sodic rocks of the Julianehaab District in Southern Greenland, in which magnesium is usually present only in traces and often wholly absent; the ijolites and other nephelite rocks of the Kola Peninsula in Finland; the nephelite syenites and related rocks of Ontario, Norway, the Transvaal, Portugal, Brazil, and other regions; and the sodic lavas of Pantelleria, Madagascar, and the Great Rift Valley in East Africa. Indeed illustrations of the law are so abundant among the highly sodic rocks as to make selection difficult.

In the figure is given the result of plotting nearly two hundred superior analyses of rocks from all over the globe. To avoid complications due to the presence of considerable lime, a subject for further investigation, only peralkalic and some domalkalic rocks were selected, chiefly among the perpotassic, dopotassic, and persodic subrangs. The analyses belonging to the sodipotassic and dosodic subrangs are so numerous that only a very few of the many available were used. These analyses represent rocks from Greenland, Canada, Montana, Colorado, Wyoming, Scotland, Norway, Finland, Bohemia, Italy, Madagascar, Celebes, Africa, Australia, Antartica and other localities, so that, as a whole, variations due to regional peculiarities are eliminated.



It may be added that the silica percentages of the rocks represented by these analyses vary from 39.24 to 73.68, thus covering nearly the entire range.

The clustering in a narrow band, extending diagonally from high soda and iron in the lower left quadrant, to high potash and magnesia in the upper right, with the practical absence of analyses in the upper left and lower right quadrants, is very evident.

The chemical composition of igneous rocks is so complex and the number of their chemical constituents is so great; the conditions of differentiation and solidification are so varied and so complicated; and the knowledge of the application of the laws of physical chemistry to them is as yet so meager by reason of the lack of data, that some irregularities and apparent exceptions are to be expected. Furthermore there are other correlative relations which have not been dealt with here, and it is conceivable or indeed probable that one or more of these may under certain conditions, whether of composition, differentiation or solidification, supersede the one which is the subject of the present discussion without invalidating the truth of this.

It would be premature to enter here into a discussion of the causes of the correlation of these four elements. Any such would necessitate the consideration of the other correlations which have been detected. It may be said, however, that even though such concomitant variations may be due to similar solubility relations, or some such physico-chemical factor, yet that such similarities are themselves presumably due to certain intimate relationships between the elements which are commonly and somewhat loosely called affinities.

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[1] J. H. L. Vogt., *Zs. prakt. Geol.*, 1898, 326.

[2] J. F. Kemp., *Ore Deposits*, 34–37 (1900).

[3] L. de Launay, *La Science Géologique*, 637, 1905; *Gîtes Mineraux*, 1, 46–51 (1913).

[4] W. F. Hillebrand., *Bull. U. S. Geol. Surv.*, 305, 21 (1907).

[5] H. S. Washington., *Trans. Amer. Inst. Min. Eng.*, 1908, 749–767.

[6] H. S. Washington., *Prof. Paper U. S. Geol. Surv.*, 14, 1903.

THEOREM CONCERNING THE SINGULAR POINTS OF ORDINARY LINEAR DIFFERENTIAL EQUATIONS

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In earlier papers I have considered the effect of a linear transformation of dependent variables upon the solutions of ordinary linear differential equations in the vicinity of a singular point.

This type of transformation led me to the notion of *equivalence* which is fundamentally important for the classification of singular points.

Ordinary linear differential equations also preserve their form under an arbitrary transformation of the independent variable. I shall prove here that this second type has no additional significance for the purposes of classification since the solutions are carried over into the solutions of *equivalent* differential equations by such transformations.[1] The precise statement of the result is contained in the following

THEOREM. *Let* $Y(x)$ *be a matrix*[2] *of functions* $y_{ij}(x)$ $(i, j = 1, 2, \cdots, n)$, *each analytic for* $|x| \geqq r$, *with determinant* $Y(x)$ *not zero for* $|x| \geqq r$. *A necessary and sufficient condition that* $Y(x)$ *forms the matrix solution of a linear differential system*

$$\frac{dY}{dx} = A(x)\,Y,$$

where the elements $a_{ij}(x)$ *of* $A(x)$ *are analytic* $(q = 0)$ *or have a pole of at most the qth order* $(q > 0)$ *at* $x = \infty$, *is that, for every function* $\varphi(x)$ *of the form*

$$\varphi(x) = x + l\,x^{-q} + m\,x^{-q-1} + \cdots,$$

the matrix $M(x)$ *defined by the matrix equation*

$$Y(\varphi(x)) = M(x)\,Y(x)$$

is composed of elements $m_{ij}(x)$ *analytic at* $x = \infty$.

Let us prove first that the condition is *necessary*.

The function $M(x)$ defined for any choice of $\varphi(x)$ is single-valued as well as analytic for $|x|$ sufficiently large. In fact, when x makes a circuit of $x = \infty$, $Y(x)$ is altered to $Y(x)C$ where C is a matrix of constants of determinant not zero. But since $\varphi(x) - x$ remains finite as x becomes infinite, $\varphi(x)$ will make essentially the same circuit if $|x|$ be sufficiently great. Thus $Y(\varphi(x))$ changes to $Y(\varphi(x))C$, and $M(x)$ is unaltered in value.

Now the expression for $M(x)$

$$Y(x_0)\,Y^{-1}(x_m) \qquad (x_0 = \varphi(x);\ x_m = x)$$

may also be written as a matrix product

$$[Y(x_0)\,Y^{-1}(x_1)]\ [Y(x_1)\,Y^{-1}(x_2)] \cdots [Y(x_{m-1})\,Y^{-1}(x_m)].$$

Here we will assume that the values $x_0, x_1, x_2, \ldots, x_m$ of x are equally spaced, each after the first differing from its predecessor by $\Delta x = (x_1 - x_m)/m$.

From the differential equation itself, it is clear that

$$Y(x_i) - Y(x_{i+1}) = \Delta x[A(x_i) + E_i]\,Y(x_{i+1}) \qquad (i = 0 \text{ zero}, 1, \cdots, m-1)$$

where E_i will have arbitrarily small elements if Δx is small enough.

For $|x|$ taken large enough, it is also clear that we will have an equality of the form

$$|a_{ij}(x)| < a|x^q| \qquad (i, j, = 1, 2, \cdots, n)$$

for the elements of $A(x)$. Hence if I denotes the matrix with elements $\delta_{ij}(\delta_{ij} = 0,\ i \mp j;\ \delta_{ii} = 1)$, and if A denotes the matrix whose elements are all equal to the positive constant a, we find that the elements of the matrix

$$Y(x_i)\, Y^{-1}(x_{i+1}) \quad \text{or} \quad I + \Delta x(A(x_i) + E_i)$$

are less in absolute value than the corresponding elements of the matrix

$$I + 2|\Delta x|A|x_i|^q.$$

Also on account of the form of $\varphi(x)$ we have for $|x|$ large enough

$$|x_0 - x_m| \leqq k|x_i|^{-q}, \ (k \text{ independent of } i \text{ or } x).$$

and thence

$$|\Delta x| \leqq \frac{k}{m}|x_i|^{-q}.$$

Substituting this value of Δx above, we conclude that the elements of $Y(x_i)Y^{-1}(x_{i+1})$ are less in absolute value than the corresponding elements of

$$I + \frac{2k}{m}A.$$

Thus, by the product formula, $M(x)$ will have elements whose absolute value is less than that of the corresponding elements of

$$\left(I + \frac{2k}{m}A\right)^m$$

whose elements are obviously limited in absolute value for all values of m.

Consequently the elements of $M(x)$ are limited in absolute value for $|x|$ large enough, and are necessarily analytic at infinity.

Let us show now that the stated condition is *sufficient*.

To this end we will consider the family of functions

$$\varphi(x) = x + \rho x^{-q}$$

where ρ is a parameter.

Defining $M(x,\rho)$ by the equation

$$Y(x + \rho x^{-q}) = M(x, \rho)\, Y(x),$$

we observe that $M(x,\rho)$ is made up of elements single-valued and analytic in x and ρ if

$$|x| \geqq {}^{1} + r, \qquad |\rho| \leqq 1,$$

inasmuch as in this event

$$|x + \rho x^{-q}| \geqq |x| - |x^{-q}| \geqq |x| - 1 \geqq r$$

Moreover, in view of the stated condition, the elements $m_{ij}(x,\rho)$ of $M(x,\rho)$ are analytic in x at $x = \infty$.

Consider now the integral formula

$$\frac{1}{x} m_{ij}(x,\rho) = \frac{-1}{2\pi\sqrt{-1}} \int \frac{m_{ij}(\xi,\rho)\, d\xi}{\xi(\xi - x)} \qquad (|x| > 1 + r)$$

where the integration is performed around the fixed circle C: $|\xi| = 1 + r$ in a positive sense. This reduces to the Cauchy integral formula by the substitution $x = 1/x'$, $\xi = 1/\xi'$, which holds since $m(x,\rho)$ is analytic for $x \geqq 1 + r$ and at $x = \infty$.

We now perceive from the form of the integrand that $m_{ij}(x,\rho)$ must be analytic in x *and* ρ for $x = \infty$.

Differentiating the equation of definition for $M(x,\rho)$ with respect to ρ we obtain

$$x^{-q} \frac{d}{dx} Y(x + \rho x^{-q}) = \frac{d}{d\rho} M(x,\rho)\, Y(x).$$

The elements of $dM(x,\rho)/d\rho$ are also analytic in x and ρ at $x = \infty$. Letting ρ approach zero we obtain, for $|x| > 1 + r$,

$$x^{-q} \frac{dY(x)}{dx} = \frac{d}{d\rho} M(x,0) Y(x)$$

which establishes at once that $Y(x)$ is the solution of a differential system of the desired form.

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[1] This result was partly known to me in 1908. For a special case, see *Trans. Amer. Math. Soc.*, **14** 462–476 (1913), in particular pp. 475–476.

[2] For the elements of the theory of matrices here used see Schlesinger, *Vorlesungen über linearen Differentialgleichungen*, pp. 18–19.

A QUANTITATIVE STUDY OF CUTANEOUS ANALGESIA PRODUCED BY VARIOUS OPIUM ALKALOIDS

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The pharmacological and physiological literature contains but few quantitative and scientifically accurate comparisons of the narcotic or analgesic properties of the various opium alkaloids. This is chiefly due to a lack of an adequate method for studying the subject. The observations so far recorded are of a clinical character, and the conclusions drawn, even in the cases where alkaloids of reliable purity were used, show the greatest diversity of opinion. Thus, Claude Bernard[1] regarded narcein, an inert substance according to the majority of investigators, as a powerful narcotic; again Fronmüller[2] ranked narcotin in activity as next to morphin; while Baxt[3] extolled the wonderful pain-relieving virtues of papaverin.

In the present investigations we have made use of a large Baltzer inductorium for producing quickly and conveniently finely graded pain stimuli. Having standardized our apparatus with the help of Dr. C. W. Hewlett of the Physical Laboratory of this University, we were able to express the values of these stimuli, quantitatively, in Henrys or C. G. S. units, and in this way compare the strengths of the stimuli required to produce pain under various conditions.

The first step in the investigation was to determine whether a sufficiently constant pain threshold could be established. Martin[4] and his coworkers have studied the threshold of electrocutaneous sensation with the induction current, by dipping a finger in a liquid electrode, and have found that a definite sensory threshold can be established, subject to physiological diurnal, nocturnal, and fatigue variations. Furthermore, Martin, Grace and McGuire[5] in the only pharmacological study by this method, have found a definite lowering of the sensory threshold after administration of acetphenetidin by mouth.

In our work we have made use of fine platinum electrodes, studying the effect of the induced current on individual pain points or groups of pain points in four different regions of the body. In this way our chances of error were lowered fourfold. The points most convenient for study employed by us were the skin on the back of the hand between thumb and forefinger, the tip of the nose, the tip of the tongue, and the lips.

In complete agreement with the localization of pain points on the surface of the body, as described by v. Frey,[6] and others, we have found that at any given spot of the body a definite pain sensation can be elicited by changes in mutual inductance of sufficient intensity, and that for every group of pain points a pain sensation of exactly the same quality and intensity can be elicited by exactly the same intensity of electrical stimulus. By practice we were able to distinguish changes in pain sensation produced by moving the secondary towards or away from the primary coil a distance of not more than 0.1 cm.

Through numerous observations it was established that the normal pain threshold remains surprisingly constant for many hours in succession, and, through a series of other experiments, lasting over 25 hours each, the diurnal and nocturnal variations were found to be very slight.

Having determined the normal pain threshold in any given experiment, a drug was administered by subcutaneous or intramuscular injection, and, after its absorption, repeated readings were made. In this way a rise or fall in the pain threshold, or an analgesic or hyperalgesic effect respectively, was detected and measured. The experiments were performed on Dr. Macht and two medical students, Messrs. N. B. Herman and C. S. Levy.

Being fully aware of the fact that certain subjective elements, inherent in the character of the investigation, enter into our experiments, we have taken all possible precautions to eliminate errors arising from this source, and have made numerous control experiments.

Each experiment was carried out in the same room, and under perfectly constant conditions. Readings were always taken with the subject in the same position, and the subject was never allowed to look at the apparatus, but sat either with eyes closed or fixed on some distant point. The electrodes employed were of course the same in any one experiment; the distance, between the electrodes was kept fixed; the pressure with which they were applied to the surfaces was kept constant; the direction of their application was the same; and the wetness of the surfaces stimulated was maintained the same as nearly as could be judged.

When a drug was administered, the subject was ignorant of its nature. Furthermore, as controls, normal saline and other inactive substances were often substituted in place of the drug without the subject's knowledge. It may be remarked in passing, that owing to the conflicting experiences of previous observers, we could not know the true pharmacological action of most of the alkaloids studied; thus further eliminating any subjective bias.

Action of Opium Alkaloids Individually.—By the above methods the six principal opium alkaloids, morphin, papaverin, codein, narcotin, narcein and thebain were studied. Administered in moderate therapeutic doses, it was found that in respect to their analgesic power beginning with the strongest and ending with the weakest they ranged themselves in the following order: Morphin (10 mgs.)→Papaverin (40 mgs.)→Codein (20 mgs.)→Narcotin (30 mgs.)→Narcein (10 mgs.)→ Thebain (10 mgs.).

In respect to morphin in one of us an idiocyncrasy was noted. The subject (N. B. H.) was rendered hypersensitive to pain, and this effect could be measured quantitatively. This was an interesting confirmation of the existence of undoubted cases of persons, who are not relieved by morphin, but are rendered even more sensitive by it.

Table Showing the Maximal Effects of the Various Drugs Used for D. I. M.

Time of reading	Morphin sulphate 10 mgs.	Morphin sulphate 5 mgs.	Papav. sulph. 40 mgs.	Codein phosph. 20 mgs.	Narcotin HCl 20 mgs.	Narcotin HCl 8 mgs.	Narcein HCl 10 mgs.	Thebain HCl 10 mgs.	Narcophin 20 mgs.	Narcophin 10 mgs.	Narcophin 5 mgs.	Pantopon 10 mgs.
	C.G.S. units	C.G.S. units	G.G.S. units	C.G.S. units	C.G.S. units	C.G.S. units	C.G.S. units	C.G.S. units	C.G.S. units	C.G.S. units	C.G.S. units	C.G.S. units
Hand												
Before injection	2553	4554	2829	2208	2484	5520	6624	7038	5451	3174	3588	4899
After injection	4209	4554	6210	2967	3036	5106	6210	7038	17664	5106	4209	6624
Tongue												
Before injection	1588	1897	1173	2208	2277	3036	2760	2622	1622	1725	1656	2070
After injection	3174	1794	2277	3036	2553	2760	2622	2553	3626	2829	2208	3588
Lip												
Before injection	1002	1312	639	966	1104	2070	1312	2139	829	932	897	1156
After injection	2208	1380	1656	1346	1588	1794	1312	2070	4345	1139	1151	1725
Nose												
Before injection	1156	1002	518	1588	1244	1244	1622	1794	1156	1156	1036	1622
After injection	2277	1070	2484	1897	1656	1104	1520	1794	3450	2070	1380	2622

Action of Combination of Alkaloids.—After a study of the individual alkaloids, the action of combinations of morphin and narcotin meconates (narcophin) and other salts were studied. It was found that a given dose of narcophin has greater analgesic power than is represented by the arithmetical sum of the effect of its constituents, morphin and narcotin. Thus 5 mgs. of narcophin produces distinct analgesia, while 5 mgs. of morphin alone, or 10 mgs. of narcotin alone, produces no such effect. This observation is in complete agreement with Straub's[7] views on synergism of these two substances. A similar action was observed with a mixture of the total opium alkaloids (Pantopon).

The accompanying table illustrates the maximal effects of the various drugs used on one of us, as expressed by the quantity of stimulus in C. G. S. units required to produce the threshold sensation of pain.

Although by the above described method only cutaneous sensations could be studied, we think that our observations are of some value in the study of the very important subject of analgesia. This research has been endowed in part by a grant from the Council on Pharmacy and Chemistry of the American Medical Association; the complete data with many tables will appear in the *Journal of Pharmacology and Experimental Therapeutics*, Vol. 7, No. 5, January, 1916.

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[5] Martin Grace and McGuire, *J. Pharmacol. Exp. Therap.*, 6, 527 (1915).

[6] von Frey, *Leipzig, Abh. Ges. Wiss.*, 66, 186, 293 (1894); 67, 166 (1895), and *J. Amer. Med. Ass.*, 47, 695 (1906).

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THE SURFACE-TENSION AT THE INTERFACE BETWEEN TWO LIQUIDS

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While working with Haber upon a theory of muscular motion it was found by Harkins that the capillary-tube method for the determination of surface-tension is very inaccurate whenever a basic solution is used. This method is also extremely sensitive to the action of dust particles and to the presence of certain impurities, since the surface involved in the measurement is very small. Of the other available methods the two best seem to be the measurement of surface waves and the determination of the weight of a falling drop. Of these two the former requires a very elaborate and expensive apparatus if the determinations are to be made with considerable accuracy, while on the other hand the drop-weight method makes use of comparatively simple apparatus and gives results which are reproducible with considerable accuracy.

The most complete treatment of the mathematical theory of the relation between the forms of drops and surface-tension is given in a book published in 1883 by Bashforth and Adams.[1] Much later than this, in 1906, Lohnstein[2] applied the general theory to the special case of the hanging drop just before its fall and the residue left after its fall, and from the difference he obtained the magnitude of the falling drop itself. The equation which he used for the relation between the weight (W) of the falling drop and the surface-tension α is

$$W = 2\pi r \alpha \, \mathrm{f}\left(\frac{r}{a}\right), \tag{1}$$

where r is the radius of the tip and a is the square root of the capillary-constant, and $\mathrm{f}(r/a)$ is a function of r/a. Since $\mathrm{f}(r/a)$ varies from 1.0 to 0.6, it is evident that neglect of this correction may cause errors as great as 40% in the surface-tension. It seems somewhat remarkable that in work with this method the correction has been neglected almost as often as it has been applied. Lohnstein determined the values of the function of r/a for different values of r/a, and his calculation of these corrections from a theoretical standpoint would seem to make it possible to use the drop-weight method as an independent method for the determination of surface-tension.

Unfortunately, however, when it becomes desirable to determine surface-tension, it is found that Lohnstein did not carry his calculations to a sufficient degree of accuracy to make this method available for determinations where an accuracy greater than 4% is desired. It is therefore important that this correction, which is a very fundamental one in work on capillarity, should be determined with a greater degree of exactness. Forms of apparatus devised by the writers seemed to make it possible to determine this correction experimentally under conditions which are more ideal than is possible than when usual methods are used. Thus the substitution of experiments on the liquid-liquid interface for the ordinary method in which a liquid-air interface is used makes it possible to compare the drop-weight results with those obtained in a capillary tube of large bore (1.5 mm.) when the capillary rise is great enough (150 mm.) to give accurate measurements. Not only can the diameter of the large capillary be determined more accurately than that of the smaller tube used for measurements on a single liquid, but in addition it is much easier in the large tube to keep the surface of the meniscus in a pure condition. Then in the drop-weight determination itself the drop falls very much more slowly when it falls into a liquid than when it falls into a gas, so that there is less disturbance in the drop at the time of fall than when it breaks away at a relatively high speed. The corrections determined in this way are given in Table 1, and are shown in the form of a curve in figure 1. In the figure the ordinates represent the values of the function of r/a, and the abscissae give the values of r/a. The complete circles represent Lohnstein's theoretically determined values, while the circles given in outline represent the new values determined experimentally. The figure shows that the experimental values give with Lohnstein's first two points for small values of r/a a much smoother curve than is given by his own theoretical values. The experimental values were determined by the use of a number of different liquids, and measurements were made both upon liquid interfaces and at the surface of a single liquid.

Morgan, who has carried out an extensive series of investigations[3] on the drop-weight method, does not use the general Lohnstein relation, but considers that the law of Tate holds for drops which have a 'normal' form. This law he expresses in the form: 'Surface-tension = Constant × Drop Weight.' The use of this equation is equivalent to the assumption that in figure 1 the curve at the bottom is coincident with its horizontal tangent. From the form of the curve it may be seen that this is not strictly true at any point, but that no serious error is involved in this rule if the determinations of surface-tension are always made with a tip which gives a value of r/a very nearly that at which the tangent touches the curve. Usually the tube used does not meet this very specific requirement, and the result is therefore different from the true result by the distance between this tangent and the curve.

TABLE I.

Experimental determination of the values of the function f (r/a)

Point No.	*r/a*	f(r/a)	*Interface*	*Temperature degrees*
1	0.281	0.709	Water: Ethyl Carbonate	25
2	0.366	0.685	Water: Benzene	25
3	0.441	0.672	Water: Dimethylaniline	25
4	0.484	0.654	Water: Ethyl Carbonate	25
5	0.592	0.639	Water: Benzene	10
6	0.621	0.636	Water: Benzene	20
7	0.633	0.632	Water: Xylene	25
8	0.636	0.634	Water: Benzene	25
9	0.648	0.634	Water: Benzene	30
10	0.649	0.632	Water: Toluene	25
11	0.709	0.620	Water: Air	25
12	0.837	0.615	Water: Hexane	25
13	0.845	0.616	Aqueous solution of Sodium Chloride: Benzene	25 25
14	1.071	0.612	Benzene: Air	25
15	1.387	0.620	Aqueous Solution of Strontium Bromide: Hexane	25

The results of Morgan and McCann[4] upon 5 different liquids with 16 different tips have been used to calculate the values of the function

$f(r/a)$. Of these only the results obtained on benzene have been plotted in figure 1. This curve is higher than that obtained by us, which means that their values for the surface-tension are less than ours. This, however, does not mean that the drop-weight results given by Morgan and McCann are incorrect, but that the capillary-constant for benzene which they use, and which they do not themselves determine, is lower than that found by us. They use an average constant taken from the results of Ramsay and Shields and other workers. Since the reading of this paper the supposition that these workers have in general obtained too low a capillary-constant has been confirmed by Richards and Coombs,[5] who considers that the results of Ramsay and Shields, Renard and Guye, and Walden and Swinne, are too low, and that the discrepancy is of the order of 3%.

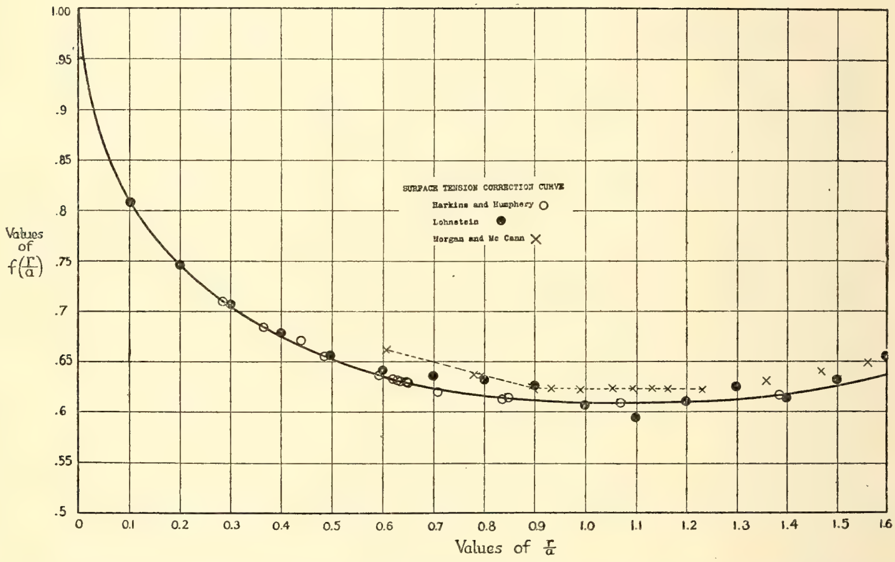

FIG. 1.—(NOTE) THE CURVE HAS BEEN DETERMINED ACCURATELY FOR VALUES OF $\frac{r}{a}$ UP TO 0.7. FOR VALUES GREATER THAN THIS ONLY THE GENERAL FORM OF THE CURVE HAS BEEN DETERMINED.

For values of r/a between 0.9 and 1.2 Morgan and McCann get a straight line for the values of the function, which would mean that Tate's law holds. Our determinations in this region have not yet been made with any great accuracy, since most of the work up to the present time has been on values of r/a between 0.3 and 0.7; but at least it can be easily seen that the general form of their curve is very different from ours.

This improved drop-weight method has been applied by us to the determination of the surface-tension at the interface between two liquids, and especially to the problem of the change of surface-tension at such an interface as that between benzene and water when the reaction of the aqueous phase is changed from acid to basic. This problem has been investigated to some extent by von Lerch,[6] who used the capillary-tube method and thus obtained very poor results.

The problem just referred to has an important bearing on the mechanism of muscular action. Two important facts have been established in regard to the motion of the muscles: first, that the active part of the muscle is always electrically negative to the part at rest, and second, that the active muscle shows an acid reaction. It has been shown by Hill that the amount of energy set free during contraction is directly proportional to the length of the fibrilles, and therefore to the area of their surfaces. According to Bernstein the force of contraction produced by a stimulus has a negative temperature-coefficient. All of these facts seem to suggest that the origin of muscular motion should be sought for in some form of surface energy. This was suggested as early as 1878 by Fitzgerald, who considered that changes of surface-tension are responsible for the phenomenon.

Haber and Klemensiewicz investigated the problem from a physico-chemical standpoint. It may be considered that the fibrilles of the muscles form one phase and the sarcoplasma another phase of a two-phase system. A similar two-phase system was constructed by the use of the interface water-benzene, and it was found that the change of reaction of the aqueous phase from basic to acidic caused a very rapid variation of electromotive force close to the neutral point. The change was found to be of the order of 0.5 of a volt. In similar systems von Lerch had found very rapid changes of surface-tension at the neutral point, so that if his results were reliable there would be a very good physico-chemical basis for the surface-tension theory of muscular action. Such a theory would be that a chemical reaction in the active muscle, such as the production of lactic acid or possibly of carbonic acid, causes the sarcoplasma to becomes acid, and thus changes the electromotive force, which in turn changes the surface-tension and causes muscular motion.

Unfortunately for this explanation our work indicates that the magnitude of the change of surface-tension at the neutral point is not so great as that found by von Lerch. The data determined by the writers will be presented in later papers to be published in the January (1916) number of the *Journal of the American Chemical Society.*

This research has been supported by a grant of three hundred dollars from the Warren Fund of the American Academy of Arts and Sciences, and for this aid we wish to express our indebtedness.

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[1] Cambridge University Press.

[2] *Ann. Physik, Leipzig*, Ser. 4, 20, 237–68, 606–18 (1906); *Zs. physik. Chem., Leipzig*, 64, 686 (1908); 84, 410 (1913).

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[6] *Ann. Physik.*, Ser. 4, 9, 434 (1902).

OUTLINES OF A PROPOSED SYSTEM OF CLASSIFICATION OF THE NEBULAE BY MEANS OF THEIR SPECTRA*

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Presented to the Academy, October 22, 1915

This paper may properly be considered a continuation of one previously published in these PROCEEDINGS, 1, 266 (1915). In that article the opinion is expressed that the behavior of the line 4686A might serve as the basis of a classification of the nebulae. The notion will be discussed here in greater detail. About eleven of the planetary nebulae have been studied with a fair degree of completeness, and while this number is too low to afford the most secure basis for broad generalizations, the observations seem to point the way to a rational system of classification of these objects on the basis of their spectra.

In figure 1 are reproduced the spectra of nine planetary nebulae and the great nebula in Orion. As stated in the earlier paper the method adopted for observing the spectra of such nebulae consists in placing the slit of the spectrograph directly across the image. In this way the length of a spectral line is made to furnish a measure of the extent of the occurrence of the emitting material in the nebula.

The first spectrum shown is that of N. G. C. 7027. This nebula consists of two nuclei, of unequal brightness, surrounded by fainter nebulosity. In photographing the spectrum the slit was placed in the line of the two nuclei. The spectrum shows the nebula to be unusually homogeneous. Some of the fainter lines appear to be short but that is probably merely the result of their faintness, as they are no shorter than the strong lines appear with greatly reduced exposure. None of the lines exhibit any tendency to undue localization in the nuclei. The fourth strong line from the right is 4686A, whose origin has so engaged the attention of physicists in recent years. It is quite as long as lines of equal strength in other parts of the spectrum. Flanking this line on either side are groups of fainter lines which should be noted, as they play important parts in the discussion which is to follow.

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* This and the following paper contains, in abbreviated form, the substance of one read before the eighteenth (Pacific Coast) meeting of the American Astronomical Society, under the title: *The spectra of the gaseous nebulae and some points of correspondence between them and other celestial spectra.*

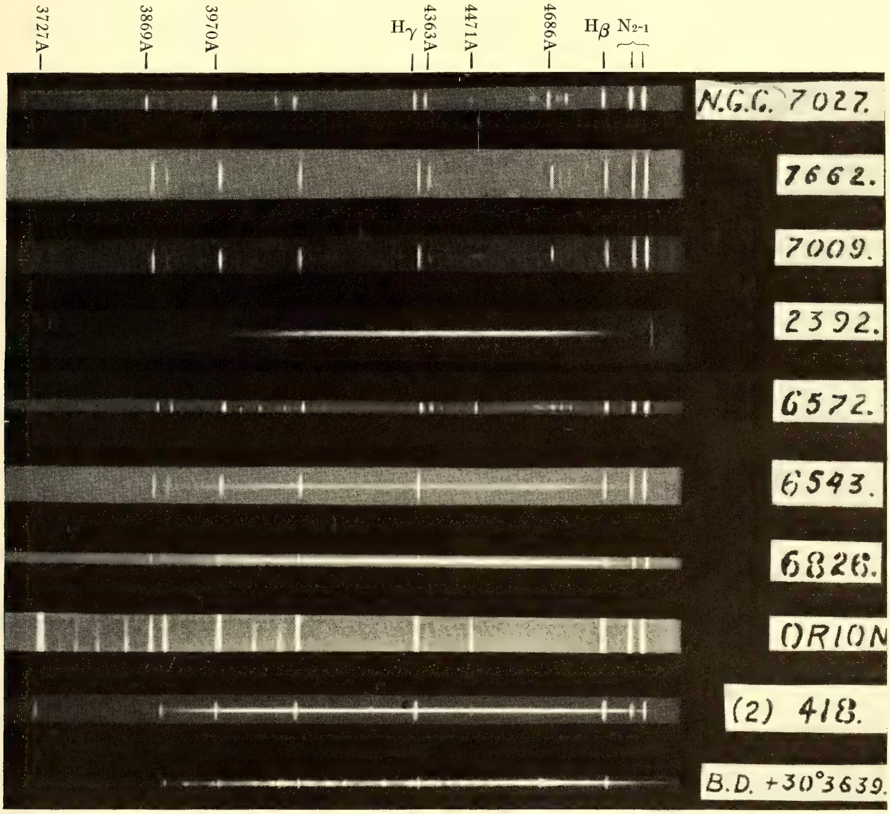

FIG. 1. SPECTRA OF NEBULAE.

The nebula N. G. C. 7662, whose spectrum follows, consists of a nucleus surrounded by an elliptical ring, the whole lying on a somewhat complicated background of fainter nebulosity. The increased brightness of most of the nebular lines where they cross the ring is plainly evident, particularly where the lines are not over-exposed. The line at wave-length 4686A and its neighbors are shorter than the others. They terminate sharply at the ring, and do not extend out into the faint outlying nebulosity and their brightness at contact with the ring itself is not greatly increased, which shows that they are relatively bright in the comparatively dark area within. 4363A is longer than the members of this group but shorter than the hydrogen ($H\beta$, $H\gamma$, $H\delta$, $H\epsilon$) and nebulium (N_{2-1}) lines. Although the exposure extended over 17 hours the continuous spectrum of the nucleus is only faintly recorded, a fact which may be due, in part, to poor observing conditions.

Both in form and in spectrum the object N. G. C. 7009 bears a striking resemblance to 7662, and the purpose of exhibiting the spectrum is partly to call attention to this fact. Both of these spectra appear to bear a strong resemblance to that of the ring nebula in *Lyra* as described by Max Wolf, and it may be that in general objects of this form have a characteristic spectrum. The spectrum of the nucleus of N. G. C. 7009 is continuous, but the record is not strong enough to prove the presence or absence of dark lines.

N. G. C. 2392 is the nebula in *Gemini* which consists of two concentric rings of nebulosity surrounding a very bright nucleus. The usual nebular lines, including those due to hydrogen and helium, are present in the outlying nebulosity, but in the nucleus the hydrogen and helium lines are reversed, that is, they appear as dark lines. 4686A on the contrary is brightened and probably broadened on crossing the spectrum of the nucleus. One might expect, from this apparent tendency of the line to concentrate in the nucleus, to find it comparatively short in the nebula, but as a matter of fact it appears to be quite as long, in this object, as any of the other lines. In fixing the position of the spectrum in an order, the controlling factor of whose arrangement is the degree of concentration of the line 4686A, there is therefore some doubt as to which of these phenomena should control. As a nebular line the one in question is relatively longer in this nebula than in N. G. C. 7662, which should place it before that object, while its increased breadth in the nucleus would seem to indicate a position intermediate between N. G. C. 7009 and 6572. In the dilemma I have tentatively assigned it to the latter position. In addition to the dark lines referred to above, the ζ *Puppis* series is also dark in the spectrum of the nucleus.

The nebula N. G. C. 6572, or Struve 6, is a small object which brightens gradually to a hazy nucleus near the center. It will be noticed that the line 4686A is confined to the nucleus, and that it has completely lost its monochromatic character. It has the appearance of a hazy ball. The lines flanking it on either side are very short, and some of them are broadened. The lines of these groups become very active in the *novae* and some of them play important rôles in the Wolf-Rayet (Class O) spectra. The well-known lines 4634 and 4650A are among them. The spectrum of this nucleus has several of the characteristic

Wolf-Rayet bands, in addition to that at 4686A. One of these shows very faintly at 4057A. While the nucleus is undoubtedly to be considered a Class O star, I am in some doubt as to its exact position in the Harvard scale.

The spectrum of N. G. C. 6543 very much resembles the one immediately above it, except that the nebulous envelope is comparatively much larger. Five of the critical lines in the vicinity of 4686A are very short, while two of them: 4641 and 4713 are of medium length. The nucleus band, 4057A, visible only with difficulty in N. G. C. 6572, is strongly developed here. Unlike the one at 4686A, this band does not appear to have its counterpart in a true nebular spectrum, that is, it has not been observed except as a broad band confined to nebular nuclei, or other Class O stars. As in most of the other spectra the lines are of various lengths, helium being intermediate in extent between 4686A and hydrogen. The other lines fall in their usual order of length.

The following subject is the spectrum of the central region of N. G. C. 6826. The spectrum of the nucleus resembles that of the preceding object, but has the additional feature of interest that the nucleus band at 4686A, and probably the bright hydrogen lines, are accompanied on their more refrangible edges by dark lines in the spectrum of the nucleus. Dark ζ *Puppis* lines are also present, though faint, and H and K (Calcium) are also dark. In addition to 4686A, 4658 and perhaps 4650 occur as bright bands in the nucleus.

As has already been remarked, the nebula N. G. C. Index 418, situated in the southern part of the constellation of Orion, was found by Campbell to have a hydrogen atmosphere of greater extent than that of its 'nebulium atmosphere. The photograph confirms this result, and shows in addition the restricted occurrence of helium. The details of the nucleus spectrum are lost on this plate, through overexposure, so that it is necessary to state that 4650 and 4686A are present as broadened bands confined to the nucleus, the first named line being much the stronger. In the downward progression this line has developed from a comparatively insignificant nebular line into the strongest band in the spectrum of the nucleus.

The spectrum immediately below is that of the well-known Wolf-Rayet star B.D. +30°3639 which Campbell found to be surrounded by a hydrogen atmosphere. As announced a year ago it is in reality a planetary nebula with a highly complicated nucleus. The line 4686A and most of the neighboring lines which occur as nebular lines in N. G. C. 7027 are here represented by broad bands in the nucleus. Helium is in the nucleus, while hydrogen is still outstanding in the nebula.

The nebula N. G. C. 40, the spectrum of which is shown in figure 2, is a large dim ring with a comparatively bright nucleus. The spectrum was photographed with a very short camera, so the scale is small. The nebular spectrum is shown to consist principally of the hydrogen lines and the ultra-violet line at 3727A. As is usual in such cases the lines show knots at their extremities corresponding to the bright edges of the ring. N_1 and N_2 are present, though they are exceedingly faint, and appear to be short. The spectrum of the nucleus is over-exposed on this plate and is shown to better advantage, with reduced exposure, in figure 3. The original negatives of this spectrum are very small and the necessary enlargement has been so great that the definition is not of the best. It will be seen, however, that the spectrum of the nucleus is made up of numerous hazy bright bands and measurement proves these to be well-known Wolf-Rayet radiations. There are seventeen of them in addition to the hydrogen bands. The brightest is 4650A, and the lesser one beside it is 4686A. If it is true that the nebulae are condensing into stars there are reasons for supposing that this one is further along in the process than any of the others we have studied.

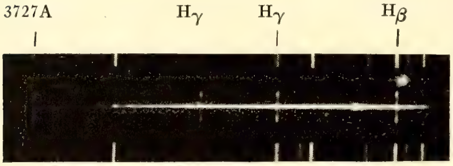

FIG. 2. SPECTRUM OF N.G.C. 40.

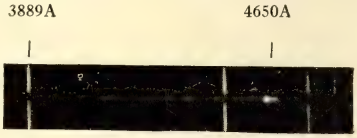

FIG. 3. SPECTRUM OF NUCLEUS N.G.C. 40.

In presenting these spectra an endeavor has been made to arrange them according to the degree of concentration of 4686A and some of the neighboring lines. It will be seen that the successive objects stand in very close relation to one another, yet at one end of the scale is a purely gaseous nebula, and at the other a banded star with only the shreds of nebulosity clinging to it.

Quite aside from any bearing which these observations may have on the subject of nebular evolution it appears to me that they offer the basis for a satisfactory classification of the nebulae. The observation of many of the nebulae for the purpose of investigating the degree of concentration of the lines which have been discussed would be impracticable but this, I believe, would hardly be necessary, for it will be observed that, having arranged the spectra in the order shown, there are certain progressions in the relative intensities of some of the comparatively bright lines which would serve to fix the positions of any of the spectra in the adopted scale. For instance, consider the doublet at 3970A. The component to the right is H_ϵ, the one to the left is of unknown origin. These lines are so close that they are only incompletely resolved on some of the plates, a rough estimate of their relative intensities may, however, readily be made, and it will be seen that in the upper spectrum the left hand component is much the stronger. As we go down the line it loses steadily in strength until in N. G. C. 6543 and 6826 the two components are equal. In the Orion nebula, which has been inserted here, the left hand member is much the weaker of the two, while in the last two objects this component is practically missing. The same may be said of 3869A; it is strong in the upper pictures and finally fades out in the descent. In what appears to be a lesser degree there is a corresponding variation of the lines N_{1-2}; that is, the so-called chief nebular lines gradually weaken with reference to those of hydrogen. The line 3727A, on the other hand, undoubtedly strengthens in the downward progression, though this does not show very clearly in these plates by reason of the fact that the 36-inch refractor is an unsuitable instrument for studying the ultra-violet, the region of the spectrum in which these lines lie. The spectrum of the Orion nebula, in which this line shows so prominently, was made with the 12-inch telescope; aside from the undue strength of 3727A, which is to be accounted for by this fact, this spectrum is found to correspond closely with those on either side of it, and appears to be intermediate between them. There is therefore no essential difference between it and the spectra of the planetary nebulae which may be said to belong to the same class.

In the course of this work the spectra of nine planetary nebular nuclei have been examined. In two cases the observations so far made are not competent to determine with certainty the nature of the spectra. The remaining seven are undoubtedly Class O stars. I think it a fair inference that the nebulae and the Class O stars in general stand next to each other in the course of stellar evolution. If, as some astronomers believe, certain of the red stars stand next to the nebulae, it is remarkable that we do not find them associated with these objects. All of the nebular nuclei stand high on the temperature scale, if we may judge from their great photographic, as compared with their visual, brightness. So far as the development of stars from gaseous nebulae may be concerned the evidence seems to favor the usually accepted theory, rather than the red to red progression favored by Lockyer and some others.

SOME PROBABLE IDENTITIES IN WAVE-LENGTH IN NEBULAR AND STELLAR SPECTRA*

By W. H. Wright

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Presented to the Academy, October 22, 1915

Coincidence between the bright nebular lines and lines in the spectra of the stars and terrestial sources are usually regarded as limited to the radiations of hydrogen, helium, the line at wave-length 4686A, and the related series of ζ *Puppis* lines. There are other correspondences to which attention should be drawn. There is in the nebulae a very faint line close to $H\delta$ which became pronounced enough in the spectrum of *Nova Geminorum* no. 2 to modify very perceptibly the form of the H_δ band. In fact it was first suspected in the *nova* from this circumstance, and afterward found, with prolonged exposure, in a number of nebulae. Its wave-length is uncertain on account of its faintness and the overpowering strength of the H_δ line which, with the low dispersion used, is very close. The mean of five measures gives 4097.6A. The wave-length of the dark line in ε *Orionis* is, according to R. H. Curtiss,[1] 4097.5A. This stellar line has been the subject of much discussion in studies of spectral classification. It has its maximum, according to Miss Cannon, in Class Oe stars. It has been assigned by various observers to as many elements, but the best case seems to have been made out by Lockyer, Baxendall and Butler who identify it with a line of abnormal behavior in the spectrum of nitrogen.[2] Whatever its chemical source its strength in Class O stars argues for its common origin in these and in the nebulae.

A line which plays an equally, or perhaps more, important part in stars a little further along in their development is 4267.30A. This is universally ascribed to carbon, and the wave-length quoted is Hartmann's laboratory determination made with a highly rarefied tube containing a gaseous hydrocarbon compound. A faint nebular line has been photographed in this vicinity by several observers, but only rough determinations of its position have been published. The mean of five of my measures places it at 4267.28A. The line is very faint and this value may be in error, say 0.2 of an angstrom. The South Kensington observers have found[3] the carbon line 4267.30 to be accompanied by a doublet in position 4647.53, 4650.92. If this doublet were in the nebulae it would not be resolved on these plates but would be represented by an apparently single line of slightly greater wave-length than the mean, as the red component of the double is the stronger of the two. In other words a nebular line of slightly greater wave-length than 4649.2A would meet the requirements. There is actually such a line at 4649.5A. Both this and the one at 4267A probably exist as bright bands in the spectra of the Wolf-Rayet stars.[4]

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* See note on previous title, p. 590.

The following is a summary of the wave-length comparisons which have been made:

	Star.	
Nebula	*Nitrogen*	*Carbon*
4097.6	4097.5	
4267.28±		4267.30
4649.5		4649.2 +

In considering the significance of these comparisons it should be borne in mind that the observations of the nebulae were made with a single prism spectrograph and that the lines are faint and difficult to measure. I think, however, that the evidence renders probable the presence in the nebulae of carbon and nitrogen, and fortifies the assumption of a close relationship between the nebulae and the early type stars.

There are other possible points of correspondence between nebular and stellar spectra which will be referred to in a more complete presentation of the subject.

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[1] *Pub. Ast. Obs. Univ. Mich.*, 1, 120 (1915).

[2] *London, Proc. R. Soc.*, A, 82, 534 (1909).

[3] *Ibid.*

[4] A paper is just at hand by Mr. T. R. Merton, entitled: On a spectrum associated with carbon, in relation to the Wolf-Rayet stars, *Lond., Proc. R. Soc.*, A, 91, 498 (1915). In this the author makes the suggestion that the two Wolf-Rayet bands in question, with others, are due to carbon.

ENERGY TRANSFORMATIONS DURING HORIZONTAL WALKING

By Francis G. Benedict and Hans Murschhauser

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Presented to the Academy, November 2, 1915

No form of muscular exercise enters so universally into the lives of all individuals as does horizontal walking, but most of the earlier researches on the energy transformations consider walking on a horizontal plane as incidental to or as a base-line for the work of ascent, particularly in connection with mountain climbing. From the fundamental contributions of Zuntz and Durig and their associates, it has been concluded that for an individual walking on a horizontal plane the energy required to move one kilogram, either of body-weight or of superimposed load, one meter in a horizontal direction is equivalent to 0.55 gram-calorie. These workers likewise noted the distinct influence of increased velocity upon the energy requirement for the same amount of work.

Prior to a direct calorimetric study of the influence of walking in a horizontal direction, as well as the work of ascent and descent, the present study was made to elaborate the earlier researches on horizontal walking. A modified form of the universal respiration apparatus was employed and a specially designed treadmill. The factors measured were the oxygen consumption, the carbon-dioxide production, the respiration-rate, the distance walked, the number of steps taken by the subject, and the height to which the body of the subject was raised in the up and down motion of walking. The values for the resting metabolism as determined for both the lying and the standing relaxed positions were taken as base lines for comparison with the values obtained with the subject while walking. A few experiments were made when the subject was walking at a high rate of speed and likewise when running.

TABLE I

SUBJECT NO.	AGE	NUDE WEIGHT	HEIGHT	CARBON DIOXIDE PER MINUTE	OXYGEN PER MINUTE	HEAT-PRODUCTION PER MINUTE
	yrs.	*kilos.*	*cm.*	*cc.*	*cc.*	*cals.*
I...........	29	69.7	180	223	280	1.34
II...........	31	68.3	177	214	258	1.25

TABLE II

SUBJECT NO.	NO. OF PERIODS	WEIGHT WITH CLOTHING	AVERAGE RATE OF WALKING PER MINUTE	INCREASE IN HEAT OUTPUT OVER STANDING	HEAT OUTPUT PER HORIZONTAL KILOGRAMMETER
		kilos.	*meters*	*cals.*	*gm.-cals.*
I.......	53	73.10	75.9	2.81	0.507
II.......	57	71.45	71.5	2.52	0.493

The preliminary observations were made on one subject by Dr. Carl Tigerstedt of Helsingfors during his short sojourn at the Nutrition Laboratory. A more extended investigation was carried out on a second subject with special emphasis upon change in velocity and the influence of food, including experiments with uncontrolled diet and diets containing a preponderance of protein, fat, or carbohydrate. A few experiments were prolonged for the purpose of studying the possible influence of fatigue.

The metabolism found for the standing relaxed position, with the subject in the post-absorptive condition, is given in table I, and for walking at moderate speed, without food, in table II. It will be seen from the latter table that the average value found in 110 periods with these two subjects was, in round numbers, 0.5 gram-calorie.

The results obtained in experiments after a meal showed that the ingestion of food raised somewhat the resting metabolism but was without material effect upon the forward progression constant of 0.5 gram-calorie per horizontal kilogrammeter.

In the prolonged experiments without food, in one of which the subject walked 22 kilometers, successive periods showed very little, if any, change in the constant, thus suggesting the absence of a fatigue effect. Singularly enough the 22-kilometer experiment with food showed a distinctly lower constant than the comparable experiment without food on the preceding day.

TABLE III

METHOD OF PROGRESSION	NO. OF PERIODS	(a) AVERAGE DISTANCE PER MINUTE	(b) AVERAGE RAISING OF BODY PER MINUTE	(c) AVERAGE NUMBER OF STEPS PER MINUTE	(d) LENGTH OF STEP $\frac{100\ a}{c}$	(e) HEAT (COMPUTED) PER HORIZONTAL KILOGRAM-METER
Without food		*meters*	*meters*		*cms.*	*gm.-cals.*
Walking:						
Slow............	57	71.5	2.94	111	64.4	0.493
Medium.........	6	106.3	5.87	131	81.1	0.585
Fast............	7	144.1	7.75	152	94.8	0.932
Running...........	15	147.5	13.75	182	81.0	0.806

The influence on the constant of an increase in the rapidity of walking and particularly of the change in type of locomotion from walking to running is shown in table III. In calculating these values, the metabolism in the standing relaxed position was used for the basal metabolism. It will be seen that with increased velocity the height to which the body was raised, the number of steps, and the length of each step were all increased. The constant for the motion of forward progression was also increased in value, especially at the highest speed.

A more profound effect on all the factors of locomotion is noted when the change was made from walking to running. With essentially the same speed for each method of progression, the height to which the body was raised in running was nearly double that in walking; the number of steps was increased 20% but the length was correspondingly decreased. Of most significance is the decided fall of 15% in the value of the constant, i.e., from 0.932 gram-calorie for rapid walking to 0.806 gram-calorie for running. Since in running the body is lifted much higher than in walking, this is surprising. On the other hand, in the walking experiments there was, as is customary with trained walkers, considerable arm motion which was absent in the running experiments. Basal experiments made while the subject was standing still but swinging the arms in essentially the same amplitude and rhythm as when walking showed a great increase in the resting metabolism. The use of this base line reduces the progression constant for walking to 0.780 gram-calorie with an average speed of 144 meters per minute. This debatable procedure seems to emphasize the fact that for the most economical transport of the body, with or without superimposed load, some type of gait which reduces to a minimum the elevation of the body and the extraneous arm motion is most desirable.

The details of this research are reported in Publication No. 231 of the Carnegie Institution of Washington.

THE PHYSIOLOGY OF THE NEW-BORN INFANT

By Francis G. Benedict and Fritz B. Talbot

NUTRITION LABORATORY, CARNEGIE INSTITUTION OF WASHINGTON

Presented to the Academy, November 2, 1915

The physiology of the first week of life is of especial importance owing to the unique position of civilized woman who, alone of all mammals, is usually for several days so completely exhausted by labor as to make her existence and even that of her child absolutely dependent upon the ministrations of others. The insufficiency of the breast secretion, the loss in weight of the infant, which almost invariably occurs, and the too frequent but rarely needed complete substitution of bottle feeding for breast feeding make a study of the actual needs of the new-born infant of unusual interest.

In a research carried out by the Nutrition Laboratory, in which a respiration chamber was used, measurements were made of the carbon-dioxide output and the oxygen intake, and records were secured of the pulse-rate, respiration-rate, and body-temperature of 105 new-born infants. The observations often began before the infant was 1 hour old, the child being placed in the respiration chamber as soon as it had been bathed and dressed. The metabolism was then studied, frequently in 1-hour periods, for several consecutive hours. The primary object of the research was to determine the basal minimum metabolism of the infant with special reference to sex, weight, age, and length.

From the data obtained in these observations, the respiratory quotients were computed which indicated the nature of the principal substance burned in the body, i.e., fat or carbohydrate. A critical study of these respiratory quotients shows that the new-born infant was not supplied with a superfluous amount of body carbohydrate to burn during the period of insufficient breast secretion. Indeed, the predominance of fat combustion suggests the possible necessity of supplemental feeding of carbohydrates.

Select periods of muscular repose were obtained in the majority of cases which supplied data for computing the basal metabolism or minimum heat-production. This minimum heat-production, computed on the 24-hour basis, ranged from 95 to 193 calories, but as the body-weight likewise varied, the values per kilogram of body-weight were also computed. These ranged from 32 to 52 calories per kilogram of body-weight per 24 hours. Even on the basis of per square meter of body-surface, supposedly the best basis for comparison, the values ranged from 459 to 732 calories per square meter of body surface per 24 hours.

Of the numerous comparisons made none showed sufficient regularity even to suggest a physiological law, until the factors of age and length were considered. On the basis of age the figures showed that frequently the lowest as well as the highest values occurred during the first 24 hours of life and it was clear that there were profound disturbances of the heat-regulating mechanism during this period. After the first day considerable regularity appeared in the results. When length was also considered as a variant, all infants between 1½ and 6 days of age, 48 in number, showed that the heat-production per square meter of body-surface per 24 hours per centimeter of length was remarkably constant. The values computed on this basis rarely varied more than ± 6% from the average value of 12.65 calories, thus establishing a physiological constant for new-born infants.

The increase in the metabolism due to severe muscular exercise, such as lusty crying, averaged 65% for all infants but actually exceeded 100% in 10 instances, with a maximum increase in one case of 211%. As an evidence of the possible increase above basal metabolism in a normal, healthy, new-born infant this is worthy of special note.

The pulse-rate showed average values as follows for the first eight days: 112, 114, 116, 116, 116, 122, 119, and 126 beats per minute. A profound disturbance of the rectal temperature accompanied the bath following birth and this probably explains the great fluctuations in the heat values for the first day. Indeed, it is suggested that the bath be deferred for a day to permit the deficient heat-regulating mechanism a longer period of adjustment.

The probable daily maintenance requirement of energy for a new-born infant, with the usual periods of activity, asleep, and awake, is estimated to be approximately 62 calories per kilogram of body-weight per 24 hours. This takes no account of the requirement for growth, which may be neglected in considering the energy requirement for the first week of life. The results of the research give opportunity for suggestions as to supplemental feeding and methods of conserving energy.

A detailed report of the investigation, together with a complete translation of the interesting article on the respiratory exchange of infants, published by Hasselbalch in 1904, is given in Publication No. 233 of the Carnegie Institution of Washington.

A COMPARISON OF METHODS FOR DETERMINING THE RESPIRATORY EXCHANGE OF MAN

By Thorne M. Carpenter

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Presented to the Academy, November 2, 1915

The principal methods of determining the respiratory exchange of man in short periods require the use either of a chamber in which the subject is confined or breathing appliances which are attached to the nose or the mouth or, as in the case of a mask, to the face. Apparatus with breathing appliances are of two classes. In one the subject breathes into and out of a closed current of air driven by a positive blower, the products of respiration being absorbed and oxygen being admitted. In the other type of apparatus valves are used to separate the currents of inspired and expired air and the latter is either measured by a meter or collected in a bag or a spirometer, a sample being taken and analyzed by means of a suitable gas-analysis apparatus.

Thus far no adequate comparison of these different types of apparatus has been made, the only attempt at such comparison being the compilation of results obtained with various respiration apparatus to show that these apparatus measured accurately the respiratory exchange. It therefore seemed desirable to compare the several types of apparatus for determining the respiratory exchange, with men as subjects and with as nearly identical conditions as possible.

The apparatus employed in this investigation were the following: Bed respiration calorimeter (chamber type with closed circuit); two forms of the Benedict universal respiration apparatus, i.e., tension-equalizer unit and spirometer unit (apparatus with breathing appliance, closed circuit type); Zuntz-Geppert apparatus (valves with meter and gas-analysis apparatus); Tissot apparatus (valves with an automatically counterpoised spirometer); Douglas apparatus (valves with rubber-lined cloth

Results of comparisons of the respiratory exchange obtained with different apparatus.

APPARATUS COMPARED	NUMBER OF		ARITHMETICAL AVERAGE OF ALL EXPERIMENTS					
	Expts.	Subjects	Carbon-dioxide output per min.	Oxygen absorbed per min.	Respiratory quotient	Pulse-rate per min.	Respiration-rate per min.	Ventilation per min.
			cc.	cc.				liters
Bed respiration calorimeter	36	16	190	223	0.850	58.5	15.0	
Tension-equalizer unit			185	227	0.815	59.5	13.2	
Spirometer unit	9	6	198	233	0.850	59.5	14.1	
Tension-equalizer unit			197	231	0.855	58.5	12.8	
Zuntz-Geppert	11	6	186	227	0.82	64.5	17.0	5.96
Tension-equalizer unit			190	224	0.85	63.0	15.9	
Zuntz-Geppert	22	9	176	220	0.80	58.5	12.3	4.45
Spirometer unit			182	219	0.83	58.5	12.5	4.76
Tissot	10	2	167	194	0.86	48.0	10.2	4.26
Tension-equalizer unit			165	193	0.855	47.0	10.1	
Tissot	17	7	192	242	0.795	60.5	13.9	5.00
Spirometer unit			190	233	0.815	60.5	12.4	4.96
Douglas	16	8	178	224	0.795	62.0	15.3	5.15
Spirometer unit			189	231	0.820	61.5	14.3	5.04
Mouth-breathing } Tension-equalizer unit	9	5	187	220	0.85	55.0	13.6	
Nose-breathing			183	219	0.835	54.5	13.2	
Mouth-breathing } Spirometer unit	5	4	190	225	0.845	64.5	14.5	4.96
Nose-breathing			185	220	0.84	63.0	14.7	4.88
Mouth-breathing } Tissot	5	3	197	247	0.80	58.0	14.2	4.68
Nose-breathing			191	246	0.775	57.5	15.0	4.87
Mask } Spirometer unit	5	3	201	253	0.80	62.5	15.3	6.16
Nosepieces			206	252	0.815	63.5	14.7	5.36
Mueller valves, Tissot spirometer	5	2	194	233	0.83	58.0	17.6	6.48
Spirometer unit			187	225	0.83	57.5	20.3	5.88
Mueller valves } Tissot spirometer	7	3	192	233	0.825	59.0	15.5	5.73
Tissot valves			188	231	0.815	57.5	20.9	5.64
As normally used } Spirometer unit	13	6	191	238	0.80	63.0	11.6	4.71
With added dead space			197	239	0.825	64.5	11.8	6.07
Automatically counterpoised } Tissot spirometer	7	4	187	231	0.81	59.5	14.0	4.83
Approximately counterpoised			188	229	0.82	59.0	15.1	4.91

bag); masks, rubber mouthpiece, glass and pneumatic nosepieces, Mueller valves, and Haldane gas-analysis apparatus (laboratory form and portable form).

The subjects were healthy young men, mostly medical students and laboratory assistants. The comparison of any two apparatus was made by determining the respiratory exchange of a subject with both apparatus on the same day, preferably in alternate periods. The subject was, in both tests, in the post-absorptive condition with as complete muscular repose as possible.

Measurements were made of the elimination of carbon dioxide and the absorption of oxygen from which the respiratory quotients were calculated. Records were also obtained of the average pulse-rate, the average respiration-rate, the total respiratory ventilation, and the volume per respiration. A record of the degree of muscular repose was secured by means of a special device. The accompanying table gives a general summary of the results obtained in the principal comparisons of apparatus and their modifications.

From a study of the details of the experiments, it is considered that all of the apparatus employed are suitable for the determination of the total carbon-dioxide elimination and the oxygen consumption and that the simplest and quickest method is the Benedict universal respiration apparatus. Of the apparatus with breathing appliances, the type involving the analysis of the expired air is considered the best for the determination of the respiratory quotient. The use of a rubber-lined cloth bag for collecting the expired air affords the most favorable opportunity for thoroughly mixing the portion collected, but care must be taken to select a bag which is not appreciably permeable to carbon dioxide. A spirometer for collecting the expired air is preferable to a meter, as all of the air can be collected and sampled. The Tissot valves were found to be the most reliable and efficient of the various kinds tested. In the majority of the experiments the respiratory exchange was the same irrespective of the kind of breathing appliances used. The Haldane gas-analysis apparatus is considered to be the best for the analysis of expired air but the analyses should be frequently controlled, using the constancy in composition of outdoor air as a basis.

Adequate control tests, in which such combustible materials as alcohol, ether, or other substances, whose composition is known, are advised as a means of checking the accuracy of respiration apparatus in general. A sufficient number of experiments with any given conditions and conservatism in the acceptance of results are also strongly recommended.

A detailed description of all of the apparatus used, the results of the individual experiments, and a critical discussion of the technique of determining the respiratory exchange of man in short periods are given in Publication No. 216 of the Carnegie Institution of Washington.

NEURO-MUSCULAR EFFECTS OF MODERATE DOSES OF ALCOHOL

By Raymond Dodge and Francis G. Benedict
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Presented to the Academy, November 2, 1915

In accordance with its widely distributed 'Tentative Plan'[1] the Nutrition Laboratory of the Carnegie Institution has organized and initiated an exhaustive experimental study of the physiological consequences of the ingestion of small doses of ethyl alcohol in man. The first year's work under the psychological part of that Plan was devoted to an investigation of the effects of alcohol on a selected group of interrelated processes covering the fundamental neural activities at various levels of the cerebro-spinal system, from the simplest reflexes of the lumbar cord to the most complex cortical arcs that we could accurately measure by available laboratory techniques.

The selection of the particular group of neuro-muscular processes for measurement was determined by the following experimental demands: (1) The systematic demand for coördinate data covering as many as possible of the fundamental psychophysiological operations. (2) The interpretative demand for the least possible inclusion of unknown and uncontrolled factors. (3) The practical demand for natural reaction forms which would be comparable in a large number of individuals without special practice, and would show relatively little practice effect as a result of the experimental repetition. (4) The technical demand for dependable quantitative methods of stimulation and registration.

Of the simple reflex arcs which are available for experimentation, the patellar reflex and the protective lid-reflex were chosen, chiefly because of their similar latency and the accuracy of their modern techniques. Our measurements of these reflexes include data concerning their latency, the extent of the muscle contraction, and the relative duration of the refractory phase. Of the more complex cortical arcs the following were selected: (1) eye-reactions to suddenly appearing peripheral stimuli, a thoroughly practiced phase of each individual's spatial adjustment; (2) speech reactions to visual word stimuli; and (3) free associations. The last two are characteristic phases of the individual's adjustment to his social environment. The fundamental perseveration tendency was measured by the partial memorization of series of words. Sensory changes were indicated by Martin's Faradic-threshold measurements. Motor coördination was studied in the velocity and accuracy of eye-movements, and in the reciprocal innervation of the antagonistic muscles of the middle finger. Finally, pulse records (chiefly electrocardiograms from body leads) were taken either continuously or at homologous points in the various experimental processes.

The subjects consisted of two groups: a main group of college graduates who were very moderate users of alcohol, and a smaller group of out-patients of the Psychopathic Hospital who had been under treatment for delirium tremens.

Normal base lines in all cases included two normal experimental days for each subject and for each kind of experiment. One normal day came before and one after the experimental days on which alcohol was administered. In addition a 'normal of the day' was recorded for each experimental process on the days in which alcohol was given. Two alcohol doses were used, one containing approximately 30 cc. and the other 45 cc. of absolute alcohol.

All the measurements show more or less rhythmic and arrhythmic variations. In our statistical theory we assumed that, given a sufficiently large number of measurements, the normal rhythmic and arrhythmic variations will tend to compensate each other, leaving the average experimentally conditioned change relatively unaffected. In our data these average results show two particularly significant marks of reliability: (1) Similar processes are similarly affected and in similar degree. (2) In general the larger dose of alcohol shows the greater experimental effect.

The effect of alcohol was calculated in all cases by comparing the differences between the 'normals of the day' and subsequent periods on the normal and on the alcohol days. The greatest percentile effect was found in the reflexes. In the patellar reflex alcohol increased the latent time 10% while it decreased the amount of quadriceps thickening 46%. In the protective lid-reflex it increased the latent time 7% while it decreased the extent of lid movement 19%. It increased the latent time of the eye-reactions 5%; that of the speech reactions 3%. Memory and the free associations were only slightly affected. Sensitivity to Faradic stimulation decreased 14% after alcohol. The number of finger-movements decreased 9%; and the velocity of the eye-movements decreased 11% as a consequence of the ingestion of alcohol.

Quite in contrast to the general depression of the neuro-muscular processes at all levels of the cerebro-spinal system was the effect of alcohol on the pulse-rate. Under all the experimental conditions alcohol produced a relative acceleration of the pulse. In only a few cases with the larger dose did this relative acceleration become a positive acceleration so that the pulse-rate in the periods subsequent to the ingestion of alcohol was faster than during the normal of the day; but in practically every instance alcohol prevented the regular pulse retardation that accompanied the successive periods of moderate mental and physical work on normal days. This relative acceleration averaged approximately 3%. While this effect is intrinsically small, its regularity, the large number of records, and its concomitant variation with the size of the dose give the results a high degree of probability.

The effort to determine which of the antagonistic heart-regulating mechanisms was responsible for the relative acceleration demanded an analysis of the pulse data. A comparison of the relative changes in the durations of systole and diastole (method of Hunt) led to a thoroughgoing consideration of the variations in the pulse cycles during the different experiments. The records show a consistent tendency of alcohol to decrease the mean variations amounting to an average of 19%. Since the rhythmic and arrhythmic pulse changes within the limits of our 12–second records could not have been conditioned by the relatively slow-acting accelerator, it seems necessary to regard the decrease of the mean variation after alcohol as caused by a decreased responsiveness of the inhibitor. There is some evidence in the records that this paralysis of the inhibitor is not an exclusive effect, and it is probable that various natural and experimental conditions might be found which would shift the preponderance of paralysis to the accelerating mechanism. The small amount of change, its purely relative character, and the probability that both regulating mechanisms are affected doubtless give the conditions for the confusion of the scientific traditions concerning the effect of alcohol on the pulse rate.

In conjunction with the pulse acceleration, the general neuro-muscular depression may be regarded as presumptive evidence of the effect of alcohol on organic efficiency. In none of our data is there any indication of a pure facilitation effect of alcohol. Contrary to the theory of Kraepelin, we not only found no facilitation of the motor processes, but the depression of their simplest forms in the finger and eye-movements seems to be one of the most characteristic effects of alcohol. Indeed it is exactly these effects that correlate most closely with the average of all the effects for the several subjects. Practically it seems to follow that these processes may serve as a readily accessible indicator of individual susceptibility to alcohol. Theoretically it seems to follow that the effect on the motor coördinations indicates a central tendency of alcohol.

A full presentation and discussion of the various techniques and the resulting measurements are given in Publication No. 232 of the Carnegie Institution of Washington.

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[1] Tentative plan for a proposed investigation into the physiological action of ethyl alcohol in man. Privately printed and distributed January 1, 1913.

VARIATION AND INHERITANCE IN ABNORMALITIES OCCURRING AFTER CONJUGATION IN PARAMECIUM CAUDATUM

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In view of the difficulty of interpreting from the standpoint of evolution the changes that occur in the complex phenomena of inheritance from two parents, the study of reproduction from a single parent becomes of great importance. One of the difficulties in such work is that the lower organisms, in which uniparental reproduction commonly occurs, present relatively few characteristics that are at the same time variable and hereditary, among closely related individuals; though this is a condition demanded for studies of heredity and evolution.

Certain abnormalities that appear after conjugation in the infusorian Paramecium appeared to offer a favorable opportunity in this respect. These abnormalities vary greatly in occurrence, character, and degree; at the same time they are partially heritable. This paper is a summary of an extended study of these abnormalities in relation to the problems of inheritance, variation, and racial change.

Among the progeny of a large proportion (from 36 to 81% in different experiments) of exconjugants of Paramecium caudatum, abnormalities appear frequently. These abnormalities consist of irregularities in body form and dimensions, of many diverse types. Among them are monsters due to partial or irregular fusion; single bodies some larger, some smaller than normal; and a great variety of abnormal shapes. These animals propagate by fission; some of the lines of individuals thus derived from the exconjugants are quite without abnormalities, in others under the same conditions the abnormalities reappear for generations. Thus the abnormalities are hereditary. But diversities appear also within the abnormal lines themselves. Some of the individuals of an abnormal line may be normal, others abnormal in various degrees. Different lines show diverse proportions of abnormal individuals. It thus becomes possible to test by selection the inheritance of degree and kind of abnormality within a single stock all derived by fission from a single parent.

In long continued uniparental reproduction of any sort, a remarkable constancy in hereditary characteristics has been generally reported. All the progeny thus coming from a single parent have seemed uniform in their hereditary characteristics, though they may differ in their bodily appearance. And this is quite in agreement with the known cytological processes accompanying the two types of reproduction. In biparental reproduction there is a reduction and recombination of the nuclear elements, of precisely the same sort as the variation and recombinations of characteristics in the progeny, in Mendelian inheritance. In uniparental reproduction, particularly of the vegetative kind, such nuclear reductions and recombinations are not known; and the uniformity of the progeny is in agreement with this. These relations, with others not necessary to recount here, have given origin to the conception of the *genotype*, as the hereditary constitution, in contradistinction to the bodily appearance. The genotype is commonly held not to change in vegetative reproduction, or but rarely, and then by marked sudden steps, or mutations. In biparental reproduction the genotype does indeed change, but seemingly by mere shiftings and recombinations, in numerically predictable ways; so that the relations here are quite in agreement with the condition sketched above for uniparental reproduction.

A somewhat rigid, stereotyped scheme of heredity naturally results from the view of the facts just set forth: in particular, evolution by gradual change, guided by natural selection, appears to be excluded. This becomes still more marked if we conclude with Bateson ('14) that all mutations consist in the dropping out of factors. On the other hand, certain investigators in genetics oppose strongly this rigid view, holding that, over and beyond Mendelian recombinations, hereditary variations of slight degree are frequently occurring, so that evolution may well be continuous and guided by selection. The recent papers of Castle give typical expression to this point of view.

If hereditary variations are frequently occurring, aside from Mendelian recombinations, it should be possible to find them in vegetative reproduction. Here we are freed from the mixing of types which makes these relations so difficult to interpret in biparental reproduction. The abnormalities in Paramecium were, therefore, studied mainly with relation to this question: Can we by continued selection of normal individuals on the one hand, of abnormal individuals on the other, break our single stock into two or more, differing in hereditary constitution?

The results of this study are as follows:

1. The diversities in abnormality occurring within a single line of descent (derived from a single exconjugant) are in some lines not hereditary, so far as can be determined by long continued selection. In a very large proportion of the races in which the abnormals were regularly discarded and only normals retained to carry on the race, the abnormal character persistently reappeared, the selected normals producing abnormal progeny. In all the abnormal races there is a wide variation in degree of abnormality of the individual, from those perfectly normal to the monsters so deformed that they would never be recognized as Paramecia if their history were not known. Yet, as stated above, in most cases the progeny of all these variations were alike, the daughter cells of normal individuals being often just as abnormal, or even more so, than the daughter cells of monsters. This of course agrees with the conditions found in most of the studies on inheritance in 'pure lines' or clones; the diversities within the lines are not inherited.

2. But in other lines, diversities within the line showed themselves to be heritable, so that selection gave very different results from those usually obtained in pure line work. By selection, single lines, derived by fission from a single parent, were divided into two or more races differing hereditarily. This was successfully accomplished in twenty-five races; from each of these were isolated two lines, one quite normal, the other continually producing abnormalities,—the two cultivated side by side.

Calkins and Gregory[1] have in some cases obtained four diverse races from the four primary daughter cells, or 'quadrants' of an exconjugant —these being the four individuals that receive the four macronuclei produced before fission occurs. It is to be noted that our selection resulting in the isolation of lines differing hereditarily in abnormality often has been brought about much later in the series of generations, so that the differentiation has often occurred within the compass of a single 'quadrant,' or indeed within a much narrower fraction of the descent. In several cases, differentiation through selection did not begin till after several weeks had passed, with production of a great number of generations. Thus the results of selection in the present case cannot be interpreted as due to a primary difference in the four original macronuclei produced during conjugation. Selection is effective when begun with progeny of a single individual that has appeared many generations after conjugation.

3. In a race of Paramecium which upon extended examination shows no hereditary abnormalities, conjugation results in the appearance of many lines which are hereditarily abnormal, others which are normal throughout.

4. In the diverse lines descended from the separate exconjugants of a conjugating culture, the two lines descended from the two individuals that have conjugated together tend to be alike in respect to normality or abnormality. That is, if the progeny of the exconjugant *a* are abnormal, the progeny of its mate *b* are more frequently abnormal than would be the case if the distribution of abnormal races were not affected by conjugation.

Our main result, therefore, is that in respect to these abnormalities, while some lines are constant in hereditary character, in others hereditable variations do occur within the line, so that by selection it is possible to break the single stock into a number of stocks differing hereditarily. The genotype in these cases therefore does not remain constant in uniparental reproduction. The condition on which evolution through selection depends is therefore realized in respect to these characters.

The complete paper of which this a summary appears in the *Journal of Experimental Zoölogy.*

[1] Calkins, G. N., and Gregory, L. H., Variations in the Progeny of a Single Exconjugant of Paramecium Caudatum, *J. Exp. Zoöl.*, **15**, 467–525 (1913).

THE INFLUENCE OF THE MARGINAL SENSE ORGANS ON FUNCTIONAL ACTIVITY IN *CASSIOPEA XAMACHANA*

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1. *The Influence of the Sense Organs on the Rate of Regeneration.* The conclusion of many of the earlier workers on the problems of regeneration that the nervous system had an important part to play in the processes of regeneration has been attacked by most of the more recent students of these problems. While Bardeen ('01), Herbst ('96), Goldstein ('04), Wolff ('95, '02) and others maintained that there was some direct influence of the nervous system, or some portion of it (sensory ganglia Herbst), and Child ('04) recognized the indirect influence of the nervous system, exhibited through muscular activity; Goldfarb ('09), Stockard ('09) *et al* have denied that there is even an indirect influence of the nervous system.

An especially favorable opportunity for studying this problem is afforded by the Rhizostomous medusa *Cassiopea xamachana* the disks of which can be separated from the oral arms and kept in dishes of sea water for an indefinite period. Such disks retain their capacity for regeneration, while on account of their shape and the location of the marginal sense-organs they permit of many types of operation which are possible only on some simple organism.

In order to determine the influence of the nervous system on the rate of regeneration the following experiments were carried out. Pairs of entire disks of the same size were compared from one of which all of the rhopalia were removed while from the other an equal amount of tissue was removed from the bell margin between the rhopalia. In these, as in all of the following experiments, the amount of regeneration was measured inward from the edge of a hole in the center of the disk from which a circular piece of tissue had been removed.

The active member of any pair of such disks, being compared, regenerated faster in 75% of the experiments. In 10% of the experiments the two disks regenerated at an equal rate, while in the remaining 15% the inactive member of the pair showed the higher rate of regeneration, thus confirming the observations of Zeleny ('07) and Stockard ('09) that under such conditions sometimes the active and sometimes the inactive member of any pair of disks regenerates faster.

The marked divergency among the results of this series of experiments indicated that there must be individual physiological differences among the medusa disks of sufficient amount to invalidate the conclusions drawn from the comparison of entire disks. In all of the later experiments, therefore, the two halves of *one and the same disk* were compared with one another. This comparison is made possible through the fact that in *Cassiopea* the muscles and nerve fibres are confined to the subumbrella surface of the disk so that when two narrow strips of the subumbrella ectoderm, on opposite sides of the disk, are removed the two halves are insulated from one another until a new layer of tissue containing nerve fibres has been regenerated over one of the denuded areas. About thirty-six hours would elapse before functional nerve fibres would be regenerated, so by scraping over the strips each day the insulation of the halves of a disk was readily maintained.

In the second type of experiments the two halves of a disk were insulated in the manner just described; the sense-organs were removed from one half while an equal amount of tissue was removed from between the sense-organs of the other half. The disks were then allowed to regenerate in jars of normal sea water, the insulation between the halves being maintained throughout the experiment. The physiological characteristics of the two halves of any disk would at the start of an experiment be identical, and the pulsations of the half on which the rhopalia remained were sufficiently strong to move the inactive half about in the water and thus reduce to a very small amount any difference in aëration.

In every instance where the rates of regeneration of the two halves of a medusa disk under these experimental conditions were compared it was found that the active half—*i.e.*, the half on which the rhopalia remained—regenerated at a more rapid rate than the inactive half. The results of two such series of experiments in which the insulated halves of 40 disks were compared are shown in curves *A* and *B*, figure 1.

The proportion between the amounts of tissue regenerated each day, when that from the inactive half is taken as the unit was respectively: first day, 1 : 1.53; second day, 1 : 1.44; third day, 1 : 1.41; fourth day, 1 : 1.38; and for the fifth day, 1 : 1.39.

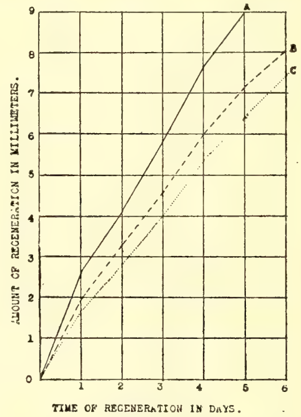

FIG. 1.—COMPARATIVE RATES OF REGENERATION OF SPECIMENS IN SEA WATER. *A*. HALF-DISKS WITH SENSE ORGANS ATTACHED. *B*. HALF-DISKS ACTIVATED WITH CIRCUIT WAVE OF CONTRACTION. *C*. HALF-DISKS WITHOUT SENSE ORGANS (INACTIVE).

Although the results from the experiments just described show conclusively that each half-disk on which the sense-organs were present regenerated faster than its companion half-disk without sense-organs, no evidence was afforded respecting the nature of the influencing factors. Since the muscular activity was the most striking difference between the two half-disks in this type of experiment other operations were undertaken to eliminate this factor. When disks with insulated active and inactive halves are put into a solution made up of 85 parts sea water and 15 parts 0.6 *m* $MgSO_4$ the activity first of the muscles, then of the rhopalia, and finally of the conducting portion of the nervous system is entirely suppressed. Under these experimental conditions the rate of regeneration of both half-disks becomes very much lowered. For the first few hours the half-disk on which the rhopalia remain regenerates faster, but after the full effect of the magnesium has become established the rate is the same for both halves.

As shown in the curves in figure 2 which represents the results of the comparison of the halves of 40 disks the difference in rate of regeneration is apparent only during the period before the rhopalia had come completely under the influence of the anesthetic.

These results, while confirming the previous observations that a half-disk under the control of the rhopalia regenerates faster than its companion half-disk from which these organs have been removed, were not sufficient, because of a lack of knowledge of the differential action of the magnesium ions on the several tissues involved, to determine whether the factor of muscular activity was or was not concerned in the initial difference in rate of regeneration. Fortunately another type of operation made known by Mayer ('06) makes it possible to determine the influence of muscular activity in a half-disk from which the rhopalia have been removed. When a series of cuts are made in the subumbrella tissues of a half-disk in such a manner that an endless labyrinth of tissue is formed, an entrapped wave of contraction can be initiated by an induction shock. This contraction wave will traverse the labyrinth of tissue for days before the muscles are overcome by fatigue so that the regeneration of new tissue to fill the open center of the disk will be completed before the contractions cease.

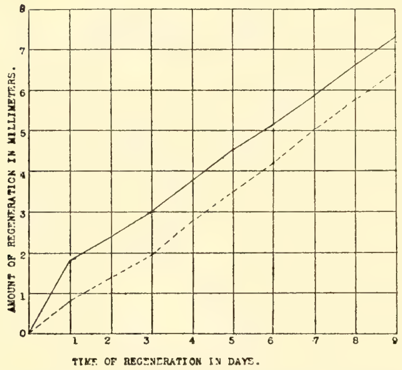

FIG. 2.—COMPARATIVE RATES OF REGENERATION OF SPECIMENS IN MAGNESIUM SOLUTION. SOLID LINE HALF-DISKS WITH SENSE ORGANS ATTACHED. BROKEN LINE HALF-DISKS WITHOUT SENSE ORGANS.

When the two insulated halves of a disk, from which all of the rhopalia have been removed while in the subumbrella tissues of one of them a circuit wave of contraction is maintained, are compared the actively contracting half-disk shows a slightly higher rate of regeneration than the inactive half. The difference in the rapidity of regeneration of the two halves in these experiments, as is shown by comparing curves *B* and *C*, figure 1, is not nearly as great as when a half-disk without rhopalia is compared with one on which the rhopalia remain (curves *A* and *C*, figure 1). When the fact that the rate of pulsation of a half-disk contracting under the influence of an entrapped wave is on the average more than three times the normal rate for a disk of the same size is taken into account it is at once apparent that some factor other than muscular activity must be the most important influence in determining the rate of regeneration. Added confirmation of this conclusion is obtained when the rates of regeneration of two insulated half-disks are compared, one of which is contracting normally under the influence of its rhopalia while an entrapped wave of contraction is maintained in the subumbrella tissues of the other (curves *A* and *B*, figure 1). In experiments of this type the rate of regeneration is throughout its course about 1.3 times greater for the half-disk on which the rhopalia remain than for the half of which the muscles are contracting under the influence of a circuit wave of contraction. In terms of muscular activity the last mentioned half-disk is developing more than three times the energy of the former but its rate of regeneration still remains lower than that of the half-disk under the control of the rhopalia. It is quite apparent, therefore, that there is no direct relationship beween the extent of muscular activity and the rate of regeneration, but that some other factor directly related to and controlled by the marginal sense-organs plays the most important part in determining the rate of regeneration. In the absence of the influence of the sense-organs regeneration can take place in an apparently normal manner, but always at a decidedly lower rate than that shown by a specimen under the control of the sense-organs.

2. *Influence of the Sense-Organs of Cassiopea on the Rate of Metabolism as Measured by Production of Carbon Dioxide.* The striking evidence of some sort of influence exerted by the rhopalia on the rate of regeneration in *Cassiopea*, as described above, naturally raises the question as to whether or not a similar influence on other activities not included in the former experiments can be detected. Since a measure of general metal bolic activity may be obtained by determining the relative rates of CO_2 production of different animals under various sets of experimenta- conditions a series of determinations of total metabolic activity of half-disks of *Cassiopea* under conditions similar to those characterizing the regeneration experiments were carried out. To accomplish this end the disks were separated into halves, the appropriate operations performed, and each one put into a separate jar containing 1000 cc. of sea water. The jars were then tightly closed by a rubber gasket and a clamped top, and, after a given interval of time, the relative amount of CO_2 produced was measured by titrating against normal sea water taken as a control at the time the jars were filled.

When a half-disk with its sense-organs was compared with the other half of the same disk from which the sense-organs had been removed the amount of CO_2 produced was always found to be greater for the normal half-disk. This result is parallel to that obtained from the comparison of the rates of regeneration and may be, at least in part, explained by the activity of the muscular system in the half-disk with sense-organs. When a comparison of the rates of metabolism is made between the halves of a disk from which all its sense-organs have been removed while the muscular system of one-half is contracting under the influence of an induced circuit wave of contraction the difference in the amounts of CO_2 produced is somewhat smaller than in the former experiment although the rate of pulsation in the activated half-disk is on the average 3.143 times as great as that of a half-disk pulsating under the control of its sense organs. It seems to be clearly established from this type of experiment that there is some other form of metabolic activity which is of greater importance as a source of CO_2 and which is more directly under the influence of the rhopalia than is the activity of the muscular system.

In the comparison of normally pulsating half-disks and those activated by a circuit wave of contraction a series of cuts, equal in extent to those used in forming the labyrinth in activated specimens, were made in the tissues of the half disks with sense organs in order to guard against any inaccuracy in the results due to inequality in the extent of the laceration of the tissues. The activity of the muscular tissue—rate of pulsation—was 3.143 times as great in the half-disk containing the circuit wave of contraction as in the normally contracting specimen, but after various periods of from 5 to 15 hours, when the titrations were made it was found that in every experiment except one the greater amount of CO_2 had been produced by the half disk on which the sense-organs remained. In the single experiment which proved an exception to the regular result the excess of CO_2 in favor of the activated specimen were very slight in comparison to the difference in rate of pulsation which for this specimen was 28 per minute for the half-disk with rhopalia and 118 per minute for the activated half-disk.

HERITABLE VARIATIONS AND THE RESULTS OF SELECTION IN THE FISSION RATE OF STYLONYCHIA PUSTULATA

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In organisms in which an admixture of two parents occurs at reproduction, the problems of evolutionary change become most complex and difficult. Heritable changes appear abundantly, but most of these are shown to be the definite working out of the numerical rules of inheritance. Whether any of the heritable changes that occur are of a different character is in dispute. In reproduction from a single parent these difficulties disappear; if evolutionary changes occur independent of biparental admixture it should be possible to demonstrate this in uniparental reproduction. Yet most recent work agrees that in such uniparental reproduction inherited variations occur rarely or not at all, and that selection has no effect in altering racial characteristics.

Most investigators, following Johannsen ('03, '09, '11), have found that 'pure lines' or 'clones' are hereditarily constant under selection. Johannsen's results were obtained with self-fertilized lines of beans. Similar ineffectiveness of selection has been found by Hanel ('08) and Lashley ('15) as to the number of tentacles in Hydra multiplying by budding; by Jennings ('08, '09, '10) for size in infusoria; by Barber ('07) (in the main), and by Winslow and Walker ('09), in bacteria; by East ('10) in the vegetative reproduction of the potato; by Agar ('13 and '14) in Cladocera and aphids multiplying parthenogenetically; and by various other investigators on diverse organisms. Some discordant results have been recorded, but most of these are ill-defined or uncertain; it is mainly in bacteria, with their immense difficulties for precise technique in pedigree work, that heritable variations or modifications have been described. The preponderance of evidence has been that in uniparental reproduction heritable variations do not occur (save as rare mutations of marked character), and that selection of slight individual variations is without effect in altering the hereditary characteristics.

These results have given origin to the concept of the Genotype (Johannsen), as a designation for the permanent heritable constitution of the race. The present paper deals with the inheritance of variations and the effect of selection in the case of a most delicately poised and readily modifiable physiological character, the rate of fission of an Infusorian multiplying without conjugation.

The questions here raised are: Can we, with respect to the character examined, get from a single genotype by selection two genotypes that differ characteristically from each other under identical conditions; and that retain these differences from generation to generation? Is selection of small variations such as appear within the pure strain or clone an effective evolutionary procedure?

For one hundred and thirty days two halves of a single clone (or set of individuals derived from the fission of a single parent) of Stylonychia pustulata were subjected to selection in opposite directions; this produced a marked and steadily increasing difference in the average fission rate of the two halves. Expressing the excess of generations produced by the fast-selected lines as a percentage of the total number of generations produced by both sets, the difference was 6.9% for the first thirty days; 12.8% for the next twenty days; 19.3% for the next thirty days, and 21.2% for the last fifty days. The records also show that for the fast lines the number of generations produced per line during the one hundred and thirty days ranges from 178 to 187, while for the slow lines the range is but from 116 to 128. The slowest fast-selected line produced 50 more generations than the fastest slow-selected line.

To determine whether the difference in fission rate thus produced is heritable, parts of the two sets were removed at intervals and subjected to culture without selection (by 'balanced selection,' in which unavoidable selections in one direction were always compensated by equal numbers of selections in the other direction). By long continued culture without selection, it was found that the difference was heritable. For example, after the two halves of the clone had been subjected to continuous opposite selection for 80 days it was found that the average difference per line per day had increased from 0.267 generation for the first thirty days to 0.415 generation for the last thirty days of that period. To test the permanence of this result these two sets of thirty lines were now subjected to ninety days of balanced selection, or ten days longer than the lines had been subjected to opposite selection. It was found that the average difference per line per day in favor of the progeny of the fast-selected set was for the three consecutive thirty-day periods of this experiment: 0.213, 0.256 and 0.284 generation.

Also, representatives of the two sets after 80 days of selection and 40 days of no selection ('balanced selection'), were subjected to mass culture for twelve days. Further balanced selection of these for fifty days showed that the inherited difference of fission rate still persisted. Thus the inherited difference produced by 80 days of selection had lasted for 102 days without selection.

Experiments with reversed selection showed that the inherited difference could be reversed in the same way that it is produced; the originally fast set was thus caused to become the slower one, and vice versa. Continuation of these two sets without selection showed again that the difference so produced was heritable.

Thus in this case the selection of small individual variations in fission rate has split the single clone (derived vegetatively from a single parent) into two heritably diverse divisions and the effect of selection was cumulative.

This experiment was now twice repeated. The first repetition was with the progeny of a single individual taken from one of the fast lines of the first set of experiments. Here again selection produced from the progeny of a single individual two sets differing hereditarily in average fission rate. Again, a new wild individual was obtained from a new mass culture. From this, two sets of thirty individuals each, all belonging to the seventh filial generation, were obtained; and subjected, in the manner previously described, one to 'fast' the other to 'slow' selection for thirty days. The excess of generations produced by the fast-selected lines, expressed as a percentage of the total number of generations produced by both sets, was 1.99% for the first ten days; 4.36% for the second ten days, and 7.10% for the third ten days; thus showing a gradual increase and indicating a cumulative effect of selection.

To test the permanence of this result these two sets of lines were now subjected to balanced selection for 21 days. On every day but one the lines that had been subjected to fast-selection averaged higher than the others, and the percentage that the difference in favor of the fast set is of the total number of generations produced by both together was practically constant; it was 3.97% for the first ten days and 5.01% for the last eleven days. The results are thus the same as in our first set of experiments.

It was found that if conjugation occurred in the fast set, and likewise in the slow set, the difference produced by selection continued to exist after conjugation.

This third series of experiments has entirely corroborated the results of the first and second series. In a second clone, unrelated to that used for the first and second series of experiments, opposite selection for thirty days produced a heritable difference of average fission rate, a difference that gradually increased as selection progressed, indicating again that the effect of selection on this physiological character is cumulative. This average difference persisted through twenty-one days of balanced selection, twenty-nine days of mass culture followed by conjugation, and then fifteen days of further balanced selection.

All the experiments thus give concordant results; through selection of individual differences in fission rate it is possible to divide a clone into two divisions differing hereditarily in rate of multiplication. The effects of selection are cumulative; the hereditary differences between the two divisions become greater the longer selection continues. By reversing the direction of selection the hereditary differences between the sets are reversed. The hereditary differences between the sets persist through the ordeal of conjugation.

Hence, in reference to the physiological character studied, the selection of small individual variations such as appear within the pure strain or clone is an effective evolutionary procedure.

How are we to account for the difference between the results here set forth and those of older investigators? In Stylonychia we are dealing with an organism which is large enough to be easily handled and followed individually, so that no question can arise as to the purity of the pedigrees (as sometimes occurs with reference to Bacteria). In this organism the facts as to the cumulative effects of selection are clear.

We are of course dealing with a delicate physiological characteristic, and this is perhaps more readily varied (even hereditarily) than the characters examined by most other investigators. Further, it is perhaps true that hereditary changes are more easily brought about in the Protozoa than in the more complex organisms, for in Protozoa the 'apparatus of heredity' is in close chemical contact with all the somatoplasm.

But a certain feature of the experimental procedure in the present case may have more importance than these conjectural considerations. It has been possible in my work to make a much greater number of actual selections (where plus and minus cases were both present to choose from), than in most of the work that has given negative results. And it has been found that a few selections give very slight results, and that a great number are required to give any marked differences between the sets. Thus, in my main experiment, on the average 39.86 plus selections were made in the fast-selected lines; 34.36 minus selections in the slow-selected lines. The difference between the two sets was thus the equivalent of some 74 selections extending through an average of 150 generations. This resulted in the production of a constant average difference per line of 0.42 of one fission per day.

Contrast with this great number of selections the *six* made by Johannsen in obtaining his negative results with beans, the three or four made by East with potatoes, the two made by Winslow and Walker with bacteria, and similar small numbers made by most other investigators along these lines; even indeed the selection through fifteen generations made by Agar, in Cladocera. It appears not at all inconceivable that in these organisms an equal number of selections, covering as great a number of generations, as were made in Stylonychia, would have given similar heritable effects. What all the work shows (and here my own is not in positive disagreement) is that heritable variations of considerable extent do not occur so frequently as was at one time supposed, so that a few selections are not sufficient for establishing a definite positive effect. But negative results from a few selections are not sufficient for disproving the occurrence of heritable small variations which may be gradually accumulated. This indeed has been admitted by many of those who have obtained negative results.

As a result of this work upon Stylonychia it is possible to give precise data as to the occurrence of heritable variations and their accumulation through selection, when sufficiently long continued. And this can hardly fail to have influence on the conception of the hereditary constitution or *genotype* as a fixed thing, changing only discontinuously by marked steps or mutations, that do not intergrade.

The full paper appears in the *Journal of Experimental Zoölogy.*

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HEREDITARY ANCHYLOSIS OF THE PROXIMAL PHALANGEAL JOINTS (SYMPHALANGISM)

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Presented to the Academy, October 30, 1915

There are many recognized forms of congenital malformations of the hands and feet. Walker in 1901 first described the type of deformity which is made the subject of this study, and showed that the condition had been transmitted through five generations, though the number of his recorded cases was too small to justify a definite conclusion on a Mendelian basis. Farabee in 1905, and Drinkwater in 1908, showed that another type of deformity of the hands, known as brachydactylism, was a dominant unit-character, transmitted in accordance with the Mendelian law.

The lesion in the condition under discussion consists of a congenital anchylosis, due apparently to the failure of formation of the joint between the proximal and middle row of phalanges. One or more, often all of the fingers are involved. The resulting deformity is known in the community as 'stiff fingers,' in contradistinction to the normal which are called 'crooked fingers.' This condition has been transmitted through seven generations, the progenitor of the family having migrated from Scotland to Virginia in 1700. There are connections of the family still in Scotland who carry the trait.

In the Virginia branch, which has been made the object of this statistical study, record has been secured of 312 descendants, among whom there were 84 affected persons, a few more than the 25% of the total number which would be expected. Excluding the incomplete families of the first three generations, in which were recorded few other than the affected persons carrying the trait, there are 72 completed families comprising 302 individuals. Of these 72 completed families, 44 of them were from the mating of unaffected parents with 152 unaffected children. Of the 28 families in which there was an affected parent, there were 150 children, 78 of them, or 52%, carrying the trait. It has been observed that the trait may be transmitted in outspoken form by a parent in whom it is inconspicuous, though never by an unaffected parent. The trait moreover is transmissible by either sex, and both hands and feet of the affected individuals may be involved.

The character, in short, behaves as a simple Mendelian dominant, with equal chance, among the offspring of affected individuals, that it will be or will not be inherited.

This paper will appear in full with photographs and charts in a forthcoming number of *Genetics*.

THE RELATIVE STIMULATING EFFICIENCY OF SPECTRAL COLORS FOR THE LOWER ORGANISMS

By S. O. Mast

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Presented to the Academy, November 10, 1915

The relation between color and reactions in organisms has for many years been a prominent problem in the study of behavior. The earlier investigators (Bert, Lubbock, Romanes, Graber and others) were interested in this problem largely from the point of view of comparative psychology. Their aim was to ascertain the relation between color-vision in man and in the various animals with the hope of thus elucidating the evolution of psychic phenomena.

Loeb studied the reaction to colors in plants and in animals for the purpose of showing that they are the same in both, hoping thus to demonstrate that there is nothing in the nature of psychic phenomena in animals. Other investigators, e.g., Engelmann, Wiesner, Strasburger, Verworn, Parker and Blaanw, were interested in the problem more from the point of view of comparative physiology.

I have for some time held the opinion that, aside from the importance of this problem in comparative psychology and physiology, it ought to yield results which would throw light on the nature of the chemical changes in the organisms associated with the reactions to light, especially those associated with changes in the sense of the reactions.

The chief difficulty encountered in the work has been connected with obtaining monochromatic light and measuring it in terms suited for comparison. Thus while a considerable number of organisms, both plants and animals, have been investigated in regard to the relation between wave-length and stimulation, only in a few are the results of such a nature that they can be compared with sufficient accuracy to warrant more than very tentative conclusions. Moreover, in a number of cases, otherwise excellent, only the region in the spectrum of maximum stimulation has been ascertained. In the following observations, to be published in full elsewhere, these difficulties and defects have been to a large extent eliminated.

It is well known that many of the simplest organisms respond very definitely to light. Some orient and travel fairly directly towards the light, others away from the light, while still others go toward it under some conditions and away from it under others.

In a field of light consisting of two horizontal beams crossing at right angles these organisms proceed toward or from a point situated between the two beams. The location of this point depends upon the relative effective illumination received by the organisms from these beams. If it is the same in quality and quantity, so that the stimulation is the same, the point lies approximately half way between them. (Details as to the process of orientation in these organisms may be found in my book *Light and the Behavior of Organisms*, John Wiley & Sons, New York, 1911.) Consequently whenever the organisms proceed toward or from a point thus located, it may be concluded that the stimulating effect of the light in the beams is equal, no matter how the light may differ, either in quantity or in quality. (This is literally true for only a few organisms, but the principle as applied holds for all, as will be demonstrated in the extended paper to follow.) It is, therefore, obvious that if the light in one beam is kept constant in quality, white for example, while that in the other is changed in color the relative stimulating efficiency of the different colors can be ascertained. To do this all that is necessary is to vary the luminous intensity of the white light for each change in the colored light until, in each case, the organisms proceed on the same path. The stimulating effect of the different colors will then be directly proportional to the various luminous intensities of the white light required to make the organisms under each of the different conditions, proceed in the same direction, e.g., if for green it required twice as much light from the white source to make the creatures take a given course as it does for yellow, then the stimulating effect of the green is twice as great as that of the yellow. To ascertain the relative efficiency in terms of wave-lengths and energy it is only necessary to use a spectrum having a known distribution of energy, and to make corrections in accord with this distribution.

In the experiments referred to below, two gas-filled street-series tungsten lamps with coiled filaments were used in series to produce two beams of light. One of these beams passed through a Hilger constant deviation spectrometer and the other through a Lummer-Brodhun rotating sector. The whole apparatus was so arranged that the two beams of light crossed at right angles in the field of observation. For every color tested the intensity of the illumination in the beam of white light was adjusted by varying the opening in the sector, until the course of the organisms bisected the angle between the two beams of light. In nearly all cases the successive regions selected in the spectrum differed by 10 $\mu\mu$. In this way the relative stimulating effect for the different regions of the spectrum was ascertained in fifteen different species as follows: Chlamydomonas, Trachelomonas and Phacus, each one species; Euglena, five species; Panderina, Eudorina, Gonium and Spondylomorum, each one species; earthworms, Arenicola (larvae) and blowfly (larvae) each one species. All but the last three are green, microscopic organisms, relatively very simple in structure.

The results obtained will be stated in terms of relative stimulative effects of the different regions of the spectrum tested without corrections for the difference in the energy of these regions. They are, however, of such a nature that the corrections mentioned will not result in marked alterations. These corrections will appear in the final paper.

For all but one of the microscopic organisms the results fall into two groups. In the one group the region of stimulation begins in the blue near the violet, between 430 and 440 $\mu\mu$. From here toward the red end of the spectrum the stimulating efficiency rises, at first slowly and then rapidly, to a maximum in the green near the yellow, between 530 and 540 $\mu\mu$; then it falls, at first rapidly and later more and more slowly, ending in the red at about 640 $\mu\mu$. In the other group the region of stimulation begins in the violet between 420 and 430 $\mu\mu$, only a short distance from the place where it begins in the first group. From here the efficiency rises very rapidly reaching a maximum in the blue between 480 and 490 $\mu\mu$. It then falls rapidly and ends in the green in the neighborhood of 520 $\mu\mu$. Three of the microscopic forms, Pandorina, Eudorina and Spondylomorum, belong to the first group, the rest to the second. To this group belong also Arenicola larvae and the earthworms. For the remaining microscopic form (Chlamydomonas) the maximum is in the green very near 510 $\mu\mu$; and for the blowfly larvae it is approximately at 520 $\mu\mu$. The distribution in the spectrum, of stimulating efficiency is, for this creature, essentially the same as the distribution of brightness for totally color-blind persons. No difference in the relative effect of the different wave-lengths was discovered in any given species under different conditions. It was the same for organisms collected in different regions at different periods of the day and tested under various conditions of illumination and temperature, and it was the same for negative and positive individuals.

These results show that stimulation in all of the organisms studied depends upon the wave-length of the light; that the stimulating efficiency is very much higher in certain regions of the spectrum than in others; but that the distribution of this in the spectrum differs greatly in certain organisms that are closely related in structure, e.g., Pandorina and Gonium, while it is essentially the same in others that are very different in structure, e.g., Euglena and earthworms. They show, moreover, that if the absorption throughout the spectrum is the same in the different organisms the chemical or physical changes associated with the reactions differ in some species which are closely related. And that the changes in the organism, whatever they may be, which cause changes in the sense of the reactions are not reversible; for if they were reversible one would expect the distribution of stimulating efficiency in the spectrum to differ in positive and negative specimens of the same species.

[This article is a contribution from the Nela Research Laboratory, National Lamp Works of the General Electric Company, Nela Park, Cleveland, Ohio.]

THE MISSION RANGE, MONTANA

By W. M. Davis

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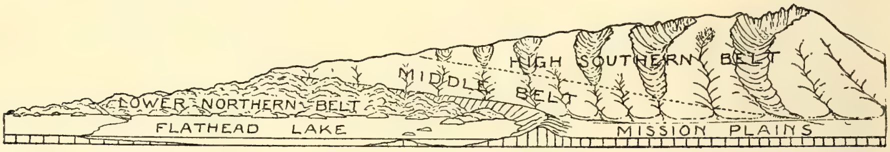
Presented to the Academy, November 12, 1915

The Mission Range, one of the smaller members of the Rocky Mountains in western Montana, believed to be composed of deformed rocks chiefly quartzites, has the appearance of a gently tilted and moderately dissected fault block, trending north and south and about 70 miles in length. The steeper face, probably representing the battered fault scarp, looks to the west. At the low northern end of the range, the gently undulating crest emerges from the surrounding plains which are about 3000 feet in altitude, and rises slowly southward, reaching a height of 9500 feet near its abrupt southern end, thus gaining a local relief of 6500 feet. The eastern side of the range is said to slope more gently than the western face. The nearly even crest and long eastern slope suggest that the mountain mass is an upraised fragment of a former worn-down mountain region.



The western face of the range, which I saw during a Shaler Memorial study of another Montana problem in 1913, may be divided as in the above figure into three oblique belts by two nearly parallel, south-dipping planes, about 1000 feet apart. The middle belt includes smoothly rounded summits and large-textured, full-bodied, waste-covered spurs, between wide-spaced, steep-pitching, apparently consequent ravines of normal erosion. All the high-reaching valleys of the southern belt expand in cirques of local glaciation at their heads, and continue downwards in narrowing troughs with oversteepened walls. These features are best developed at the high southern end of the range; there the cirques are huge cliff-rimmed cavities, strongly expanded southward; the mountain crest is sharpened to Alpine arêtes between opposing cirque walls; the long troughs, encroaching broadly on the normal forms of the rounded spurs, reach the foot of the mountains; and terminal moraines advance a short distance upon the piedmont plain, sometimes enclosing a lake in their loop. More than a score of glacial cirques and troughs may be counted, but their strength diminishes as the range crest lowers northward, and the northernmost cirques are of small dimensions, faintly developed, and reach only a few hundred feet down from the valley heads. The limiting plane which touches the lower end of the troughs rises slowly northward from the base of the range at the southern end and reaches the crest of the range north of mid-length.

The lower northern belt of the range shows innumerable bare crags, knobs, cliffs and ledges of small texture, due to scouring and plucking by the terminal portion of a broad and overwhelming glacier of Canadian origin. The limiting plane, marking the height of the invading glacier, touches the range crest about a quarter length from its low northern end and descends to the range base south of mid-length. The northern quarter of the range, lying entirely beneath this limiting plane, exhibits glaciated forms in a minutely irregular crest and a disorderly slope of scoured and plucked hills and hollows, all of less and less height northward, until the last visible knobs, deeply scored and channeled and more or less detached from one another, rise only 100 feet or so above the surrounding intermont plain of glacial gravels and silts, diversified by low morainic hills, which presumably conceals a farther northward extension of the range crest. The second quarter of the range shows similar small-textured forms up to the limiting plane, but above it the mountain crest and slope retain the simpler, large-textured forms of normal erosion, until, near mid-length, the valley heads begin to show the cirques of the high southern belt. Through this second quarter each normal valley is barred by a morainic embankment on the line of the limiting plane, and below the plane each spur is imperfectly truncated in a bold and rugged slope which presents a tumult of rocky cliffs and ledges, descending abruptly and without well-defined valley reentrants to the waters of Flathead Lake. The apex height of the rugged spur facets and the altitude of the somewhat lower morainic embankments decrease slowly and systematically southward; the facets become smaller and less continuous with one another; the embankments become longer, larger, and more nearly continuous, until they curve away from the range base and form a noble terminal moraine, 400 or 500 feet in height and a mile or more wide, which swings westward across the intermont depression and separates Flathead Lake on its northern concave side from a broad till plain of earlier glaciation on its southern convex side.

It is to the long and gradual southward rise of the mountain mass that the Mission Range owes the clear separation of features due to local glacial sculpture in the high southern belt from those due to general glacial sculpture in the low northern belt, by the oblique middle belt of normal sculpture. As far as I have seen and read, the range is unique in this systematic tripartite arrangement of normally and glacially sculptured forms. A fuller account of the range will be prepared for the *Bulletin of the American Geographical Society.*

DEFINITION OF LIMIT IN GENERAL INTEGRAL ANALYSIS

By Eliakim Hastings Moore

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Presented to the Academy, November 8, 1915

1. *General Analysis.* The problem of Science is the organization of and the study of the interrelations amongst the objects and phenomena of Nature. Analogous objects or phenomena are grouped into classes. In the progress of Science, with the discovery of new objects or phenomena or interrelations, the bases of classification initially of necessity superficial become more fundamental; thus in Physics, Electricity and Magnetism and later Light merge in Electromagnetism.

Mathematics with its source in Nature progresses in similar fashion. Hence, remembering that the objects or phenomena of Mathematics may be theories (doctrines), we may enunciate the following heuristic principle:

> The existence of analogies between central features of various theories implies the existence of a more fundamental general theory embracing the special theories as particular instances and unifying them as to those central features.

After the development of such a general theory, the fact that the various theories are instances of the general theory implies as an obvious consequence (and accordingly eclipses in importance) the analogies between the central features of the various theories. In illustration of the heuristic principle may be adduced the theories of General Analysis mentioned below.

Analysis is the branch of Mathematics devoted to the classification of and the study of the interrelations amongst numerically valued functions. A (single-valued) function τ or $\tau(p)$ is a table (or rule or process) assigning to every element or member p of a certain class or range $\mathfrak{P}$ a definite element q of a certain class $\mathfrak{Q}$. It is numerically valued in case the functional values $q, = \tau(p)$, are numbers real or complex. Although not always numerical, the independent variable p of a function τ considered in a theory of Classical Analysis is always of specified nature; e.g., the variable p may be a curve or a numerically valued function, $\tau(p)$ being the length of the curve p or the average value over its range of definition of the function p. However, the nature of the element p is not fully specified in Fréchet's theory[1] (1906) of sets of elements p of a class $\mathfrak{P}$ and of continuous functions on such sets. This general theory has as instances the theories for the linear continuum $\mathfrak{P}$ and for the n-dimensional space $\mathfrak{P}$ initiated by G. Cantor and for the class $\mathfrak{P}$ of continuous curves due especially to Arzelà. Fréchet conditions his class $\mathfrak{P}$ by a definite but undefined relation L (the concept of convergence of a sequence $\{p_n\}$: $p_1, p_2, \ldots, p_n, \ldots$, of elements p to an element p_0 of $\mathfrak{P}$ as a limit) possessing certain properties; these properties of the relation L are in the special theories immediate consequences of the current definitions of the relation L in those theories.

Those theories of Analysis in which at least certain of the functions involved are on a range $\mathfrak{P}$ of elements p of a nature not specified, or at least not fully specified, we may designate as theories of General Analysis. Thus, Fréchet's theory with basis ($\mathfrak{P}$; L) is a theory of General Analysis, as is likewise my theory[2] (1910) of classes of functions on a general range $\mathfrak{P}$.

A general range $\mathfrak{P}$ is an arbitrary particular range $\mathfrak{P}$ with abstraction of its particular features, e.g., Fréchet's range $\mathfrak{P}$ with abstraction of the feature L. Properties of functions, classes of functions, etc., on or connected with a general range $\mathfrak{P}$ (whose definitions accordingly involve no particular features of the range $\mathfrak{P}$) are of 'general reference,' while others are of 'special reference.' Thus, relative to a linear interval $\mathfrak{P}$, the continuity of a function is of special reference, while the property of uniformity of convergence of a sequence of functions and the property of the class of all continuous functions that the limit of a uniformly convergent sequence of the class belongs to the class are properties of general reference.

2. *General Integral Analysis.* A theory of General Analysis involving a numerically valued single-valued functional operation J (of the type of definite integration) on a class $\mathfrak{K}$ of functions κ whose range of definition involves a general range $\mathfrak{P}$ we may designate as a theory of General Integral Analysis. Thus, my theory[3] of linear integral equations is a theory of General Integral Analysis. This general theory has as instances the classical theory due to Fredholm and Hilbert-Schmidt for the case of continuous functions, and other classical theories.

3. *Definition of Limit in General Integral Analysis.* In a subsequent note I shall indicate a general theory of linear integral equations having as instance Hilbert's theory of functions of infinitely many variables.

The integration process J, undefined in my earlier theory, is in this theory defined. Its definition turns on the definition of limit which I wish to explain in this note.

By way of example consider on the range $\mathfrak{P}^{\mathrm{III}}$ ($p = 1, 2, 3, \ldots, n, \ldots$) a numerically valued function $\alpha(p)$ with absolutely convergent sum $J\alpha, = \alpha(1) + \alpha(2) + \alpha(3) + \ldots$. $J\alpha$ is the limit in the classical sense, as n increases without bound, of $J_n\alpha, = \alpha(1) + \alpha(2) + \ldots + \alpha(n)$. Here the definitions of $J_n\alpha$ and of limit are of special reference. However, taking (not the first n but) any finite set σ say of n elements p: $p_1 < p_2 < \ldots < p_n$, of the range $\mathfrak{P}^{\mathrm{III}}$, we secure definitions of $J_\sigma\alpha, = \alpha(p_1) + \alpha(p_2) + \ldots + \alpha(p_n)$, and of limit which are of general reference.

Indeed, consider at once a general class $\mathfrak{P}$ and a numerically valued (possibly many-valued) function F on the class $\mathfrak{S}$ of all finite sets σ of elements p of the range $\mathfrak{P}$. (In the example cited $F(\sigma) = J_\sigma\alpha$). We say that the number a is the limit as to σ of the function $F(\sigma)$, or that, as to σ, $F(\sigma)$ converges to a, in notation:

$$\underset{\sigma}{L}\ F(\sigma) = a, \tag{1}$$

in case for every positive number e there exists a set σ_e (depending on e) of such a nature that for every set σ including σ_e $|F(\sigma) - a| \leqq e$, in symbols:

$$e : \supset : \exists\, \sigma_e \ni \sigma^{\supset \sigma_e} . \supset . |F(\sigma) - a| \leqq e. \tag{2}$$

If for a set σ including σ_e $F(\sigma)$ is many-valued the understanding is that the final inequality holds for every value of $F(\sigma)$. (The notation e denotes a positive number; the notations n, m used below denote positive integers.)

The L_σ of (1) is a single-valued functional operation of the type of a definite integral, in that it reduces every function $F(\sigma)$ of the class of all functions convergent as to σ to a number a. Accordingly, the class $\mathfrak{P}$ being general, this definition of limit (even apart from its use in the theory mentioned) belongs to General Integral Analysis.

In order to obtain definitions of various modes of convergence in case the function F involves a parameter we notice the equivalent forms (3, 4, 5) of the definition.

$$n : \supset : \exists\, \sigma_n \ni \sigma^{\supset \sigma_n} . \supset . |F(\sigma) - a| \leqq 1/n, \tag{3}$$

$$\exists\, \{\sigma_m\} \ni n : \supset : \sigma^{\supset \sigma_n} . \supset . |F(\sigma) - a| \leqq 1/n. \tag{4}$$

viz., there exists a sequence $\{\sigma_m\} : \sigma_1, \sigma_2, \ldots, \sigma_m, \ldots$, of sets σ of such a nature that for every n and set σ including σ_n $|F(\sigma) - a| \leqq 1/n$.

$$\exists \{\sigma_m\} \cdot \ni n :\supset: \exists\, m_n \ni \sigma^{\supset \sigma_{m_n}} .\supset. |F(\sigma) - a| \leqq 1/n, \tag{5}$$

viz., there exists a sequence $\{\sigma_m\}$ such that for every positive integer n there exists a positive integer m_n (depending on n) such that for every set σ including σ_{m_n} $|F(\sigma) - a| \leqq 1/n$.

Now let the function F involve a parameter u on a range $\mathfrak{U}$ and suppose that $F(\sigma, u)$ converges as to σ for every u of $\mathfrak{U}$; the limit is a single-valued function, say φ, of u, in notation:

$$\mathop{\mathrm{L}}_{\sigma} F(\sigma, u) = \varphi(u) \qquad (u), \tag{6}$$

that is,

$$u .:\supset:. \exists \{\sigma_{um}\} \ni n :\supset: \exists\, m_{un} \ni \sigma^{\supset \sigma_{um_{un}}} .\supset. |F(\sigma, u) - \varphi(u)| \leqq 1/n, \tag{7}$$

viz., for every u of $\mathfrak{U}$ there exists a sequence $\{\sigma_{um}\}$ of sets σ (depending on u) such that for every positive integer n there exists a positive integer m_{un} (depending on u and n) such that for every σ including $\sigma_{um_{un}}$ $|F(\sigma, u) - \varphi(u)| \leqq 1/n$.

The convergence is semiuniform over the range $\mathfrak{U}$ in case a single sequence $\{\sigma_m\}$ is effective as the sequence $\{\sigma_{un}\}$ for every u of $\mathfrak{U}$, and it is uniform in case moreover for every n a single positive integer m_n is effective as the positive integer m_{un} for every u of $\mathfrak{U}$, that is, the notations:

$$\mathop{\mathrm{L}}_{\sigma} F(\sigma, u) = \varphi(u) \qquad (u;\ \text{semiunif.}); \tag{8}$$

$$\mathop{\mathrm{L}}_{\sigma} F(\sigma, u) = \varphi(u) \qquad (u;\ \text{unif.}), \tag{9}$$

have the respective meanings:

$$\exists \{\sigma_m\} \ni u .:\supset:. n :\supset: \exists\, m_{un} \ni \sigma^{\supset \sigma_{m_{un}}} .\supset. |F(\sigma, u) - \varphi(u)| \leqq 1/n; \tag{10}$$

$$\exists \{\sigma_m\} \ni n .:\supset:. \exists\, m_n \ni u :\supset: \sigma^{\supset \sigma_{m_n}} .\supset. |F(\sigma, u) - \varphi(u)| \leqq 1/n. \tag{11}$$

If v is a numerically valued single-valued function of u on $\mathfrak{U}$, we define semiuniformity and uniformity of convergence relative to v as scale function over the range $\mathfrak{U}$, in notation: as in (8, 9) with the parentheses replaced by (u; semiunif. $v(u)$), (u; unif. $v(u)$) respectively, by replacing in the definitions (10, 11) the final $1/n$ by $|v(u)|/n$. Thus, semiuniformity and uniformity are absolute, i.e., relative to the scale function $v(u) = 1$.

The definitions of uniformity (absolute and relative) may be simplified by omitting '$\exists\ m_n \ni$' and replacing 'σ_{m_n}' by 'σ_n'. This form of definition is suggested directly by (4); the more complicated form (5) with its redundant existential feature m_n serves however to suggest the definitions of semiuniformity (absolute and relative), and relative semiuniformity proves to be of importance in the applications.

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[1] M. Fréchet, Sur quelques points du calcul fonctionnel, *Palermo, Rend. Circ. mat.*, **22**, 1–74 (1906).

[2] E. H. Moore, Introduction to a Form of General Analysis, 1–150, *The New Haven Mathematical Colloquium*, Yale Univ. Press, 1910. Cf. also, E. H. Moore, On a Form of General Analysis with Application to Linear Differential and Integral Equations, *Atti* IV *Cong. Inter. Mat.* (Roma, 1908), **2**, 98–114 (1909).

[3] E. H. Moore, On the Foundations of the Theory of Linear Integral Equations, *Bull. Amer. Math. Soc.*, Ser. 2, **18**, 334–362, (1912). On the Fundamental Functional Operation of a General Theory of Linear Integral Equations, *Proc. Fifth Inter. Congr. Math.* (Cambridge, Aug., 1912) **1**, 230–255 (1913).

NOTICE OF SCIENTIFIC MEMOIR

The Variations and Ecological Distribution of the Snails of the Genus Io. By CHARLES C. ADAMS, New York State College of Forestry, Syracuse, N. Y. Second Memoir of Volume 12 of the Memoirs of the National Academy of Sciences, Washington, 1915. 1–184 p., 64 pl.

Io is a large gilled snail which lives only in the Tennessee River system. It is extremely variable, shows a remarkable distribution in the streams and in this it appears to be related to the physical history of the drainage. Throughout this Memoir emphasis is placed upon relating the changes of the animals to the changes in the environment. The general natural history of the snails is briefly summarized, the local races are described and the localities from which the collections studied were secured is given in detail. The shells were grouped in convenient classes for descriptive purposes. The diameter of the shell, its degree of globosity, and the degree of development of the spines were determined quantitatively. These qualities are discussed by streams and drainage systems. In the parallel flowing Powell, Clinch and (North Fork) Holston rivers, the shells are smooth or with low spines in the headwaters, and down stream have longer spines. This condition is quite remarkable and no previous detailed investigation has been made of a problem of this character.

The development or evolution of the gross environment is discussed. The author states "It is considered that a knowledge of the development and structure of the environment is as essential a part of the problem as is the development and structure of the animals themselves." An outline history of the Tennessee drainage is given. In the past this family of shells was thought to have originated in the Northwest (Laramie) but the author suggests an alternative hypothesis, that they originated in the southeast.

Large numbers of immature shells were secured in order to study the development of the shell and its sculpture. Certain shells are smooth or without sculpture in the early stages, some remain smooth, others develop nodules or spines either early or late in life, or even develop a sculpture and later lose it. Other shells are spinose from the start and remain so. The loss of sculpture occurs largely in one locality, in the vicinity of which drainage changes have probably occurred in relatively recent geological time.

In the general discussion and summary a brief review is given of inheritance in molluscs, as Mendelian inheritance applies to many of their characters. The results of recent experimental breedings are applied to the interpretation of the variations of these shells. Other topics discussed are agencies and means of dispersal, longitudinal distribution in streams, causes of variation, geographic range and variation of each of the forms of *Io*, the migration of the environment and migrations and relationships of the various forms. The illustrations include figures of the shells, showing the ontogeny of shell sculpture, and samples of the shell population at representative localities. Diagrams and tables illustrate the quantitative relations and there are several maps. The great diversity seen in these shells is probably related to the complex mixtures of many strains which has been made possible by the complex stream history.

REPORT OF THE AUTUMN MEETING

Prepared by the Home Secretary

The Autumn Meeting of the Academy was held in the American Museum of Natural History at New York City, on November 15, 16, 17, 1915.

Forty-eight members were present, as follows: C. G. Abbot, Becker, Boas, Boltwood, Britton, Castle, Cattell, Chittenden, W. B. Clark, F. W. Clarke, J. M. Clarke, Conklin, Coulter, Crew, Cross, Davenport, Day, Donaldson, Hague, Harper, Harrison, Hillebrand, Holmes, Howell, Iddings, Lindgren, Loeb, Lusk, Mall, Mendel, Michelson, Morgan, Morse, A. A. Noyes, H. F. Osborn, T. B. Osborne, Pickering, Ransome, Reid, Richards, Rosa, Schuchert, Theobald Smith, Van Hise, Walcott, Webster, Wheeler, H. S. White.

BUSINESS SESSION.

The Home Secretary announced that since the Annual Meeting in April the Academy had lost by death two members, John Ulric Nef, elected 1904, who died on August 13, 1915, and Frederick Ward Putnam, elected 1885, who died on August 18, 1915; also two foreign associates, Theodor Boveri, elected 1913, who died on October 15, 1915, and Paul Ehrlich, elected 1904, who died on August 20, 1915.

The Home Secretary made also the following announcements: That Mr. Edwin B. Frost had been elected Chairman of the Directors of the Bache Fund and Mr. Arthur Gordon Webster had been elected to succeed Mr. Ira Remsen.

That Mr. Theodore William Richards had been elected to succeed Mr. Ira Remsen as a member of the Directors of the Wolcott Gibbs Fund.

That the ad interim appointments of Acting Chairmen of the Sections of the Academy were as follows:

Mathematics: E. H. Moore, Acting Chairman.
Astronomy: G. C. Comstock, Acting Chairman.
Physics and Engineering: R. S. Woodward, Acting Chairman.
Chemistry: A. A. Noyes, Acting Chairman.
Geology and Paleontology: Arnold Hague, Acting Chairman.
Botany: J. M. Coulter, Chairman, 1915–1918.
Zoology and Animal Morphology: E. G. Conklin, Acting Chairman.
Physiology and Pathology: R. H. Chittenden, Acting Chairman.
Anthropology and Psychology: W. H. Holmes, Acting Chairman.

That the following members were appointed an Auditing Committee to audit the accounts of the Treasurer in accordance with Rule V. 5.

Arthur L. Day   William H. Dall, Chairman   F. W. Clarke

That the division of the members of the Board of Editors of the Proceedings into three groups, one-third of them retiring annually, in accordance with Rule V. 1, was approved as follows:

Dec. 1, 1916	Dec. 1, 1917	Dec. 1, 1918
W. B. Cannon	E. G. Conklin	J. J. Abel
J. McK. Cattell	C. B. Davenport	J. M. Clarke
J. M. Coulter	E. B. Frost	J. P. Iddings
Simon Flexner	W. H. Holmes	R. A. Millikan
R. G. Harrison	E. H. Moore	Alexander Smith

The Committee on the Henry Draper Fund recommended to the Academy that the following appropriations be made from available income: Four hundred dollars to Doctor Frank Schlesinger, Director of the Allegheny Observatory, to be applied toward the construction of a machine for measuring astronomical photographs; three hundred dollars to Doctor Joel Stebbins, Director of the University of Illinois Observatory, to be used in the further development of the photo-electric photometer and its application to the study of variable stars; two hundred dollars, to be applied toward the expense of publishing the Proceedings of the National Academy.

SCIENTIFIC SESSIONS

A public lecture was given on November 15 by Michael I. Pupin, of Columbia University, on Problems of Aerial Transmission.

Three public scientific sessions were held on November 16 and 17 at which the following papers were presented:

1. Edwin G. Conklin: Nature of cell polarity.
2. W. E. Castle: Is selection or mutation the more important agency in evolution?
3. A. E. Verrill: Inheritance of clubbed feet through five known generations; inheritance of abnormal or defective thumbs through four known generations.

4. Chas. B. Davenport: Heredity of stature.

5. E. C. MacDowell (introduced by Chas. B. Davenport): Parental alcoholism and mental ability—A comparative study of habit formation.

6. James B. Murphy (introduced by Jacques Loeb): Rôle of the lymphocytes in resistance to cancer.

7. Graham Lusk: The calorimeter as an interpreter of life processes.

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Errata

p. 325, l. 23, *for* four hundredth *read* three hundredth.
p. 325, last line, *delete* (instead of 150).
p. 498, l. 2, *for* 2.01 *read* 20.1.

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